

| Surname | |
|---------------------|--|
| Other Names | |
| Centre Number | |
| Candidate Number | |
| Candidate Signature | |

AS FURTHER MATHEMATICS

Paper 2 - Discrete

7366/2D

Thursday 17 May 2018 Afternoon

Time allowed: 1 hour 30 minutes

For this paper:

- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (either Mechanics or Statistics). You will have
 - 1 hour 30 minutes to complete both papers.

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.



BLANK PAGE



INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book.
 Write the question number against your answer(s).
- Do not write on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO



Answer ALL questions in the spaces provided.

1 The table shows some of the outcomes of performing a modular arithmetic operation.

| | 2 | 3 |
|---|---|---|
| 2 | | 1 |
| 3 | 1 | |

Which pair are operations that could each be represented by the table?

Tick (✓) one box.

| | Addition mod 6 and multiplication mod 5 |
|--------|---|
| | Addition mod 6 and multiplication mod 6 |
| | Addition mod 4 and multiplication mod 5 |
| | Addition mod 4 and multiplication mod 6 |
| [1 mar | k1 |



2 The binary operation \otimes is given by

$$a \otimes b = 3a(5+b) \pmod{8}$$

where $a, b \in \mathbb{Z}$

Given that $2 \otimes x = 6$, which of the integers below is a possible value of x?

Circle your answer. [1 mark]

0 1 2 3



3 Alex and Sam are playing a zero-sum game.

The game is represented by the pay-off matrix for Alex.

Sam

Alex

| Strategy | S ₁ | S ₂ | S ₃ | |
|----------------|----------------|----------------|----------------|--|
| A ₁ | 2 | 2 | 3 | |
| A ₂ | 0 | 3 | 5 | |
| A_3 | -1 | 2 | -2 | |

3 (a) Explain why the value of the game is 2 [3 marks]

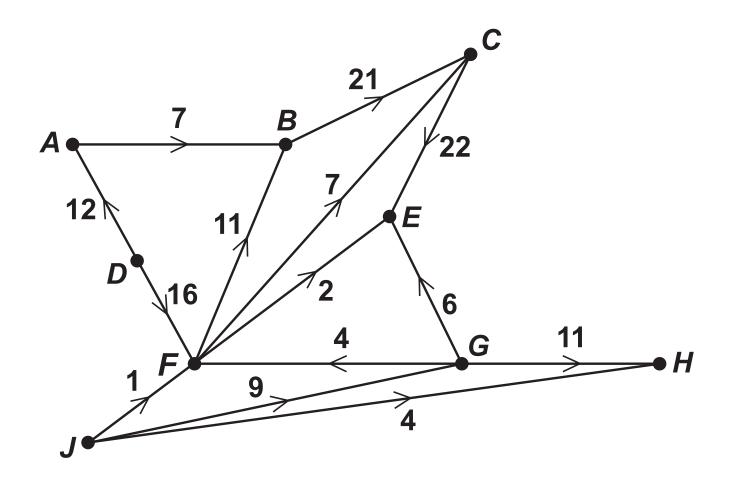


| (b) | Identify the play-safe strategy for each player. [1 mark] |
|-----|---|
| | |
| | |



4 The diagram shows a network of pipes.

Each pipe is labelled with its upper capacity in $\mbox{cm}^3\,\mbox{s}^{-1}$



4 (a) (i) Find the value of the cut given by $\{A, B, C, D, F, J\} \{E, G, H\}$. [1 mark]

| | | |
|-------------|-----------------|--|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |



| 4 | (a) | (II) | State what can be deduced about the maximum flow through the network. [1 mark] |
|----|-----|------|--|
| | | | |
| | | | |
| | | | |
| 4 | (b) | (i) | List the nodes which are sources of the network. [1 mark] |
| | | | |
| | | | |
| | | | |
| | | | |
| 4 | (b) | (ii) | Add a supersource S to the network. [1 mark] |
| [Τ | urn | ove | er] |



BLANK PAGE



| 4 | (c) | (i) | List the nodes which are sinks of the network. [1 mark] | |
|----|-----|------|---|--|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| 4 | (c) | (ii) | Add a supersink <i>T</i> to the network. [1 mark] | |
| • | (0) | (, | Add a capolonik 7 to the network. [1 mark] | |
| ГΤ | urn | ove | er] | |



A group of friends want to prepare a meal. They start preparing the meal at 6:30 pm

Activities to prepare the meal are shown in Figure 1 below.

Figure 1

| Label | Activity | Duration (mins) | Immediate predecessors |
|-------|---|--------------------|---------------------------|
| А | Weigh rice | 1 | - |
| В | Cook rice | 18 | А |
| ၁ | Drain rice | 1 | В |
| a | Chop vegetables | 10 | - |
| E | Fry vegetables | 12 | |
| F | Combine fried vegetables and drained rice | 1 | |
| 9 | Prepare sauce ingredients | 4 | ı |
| Н | Boil sauce | 12 | |
| 1 | Serve meal on plates | 2 | |

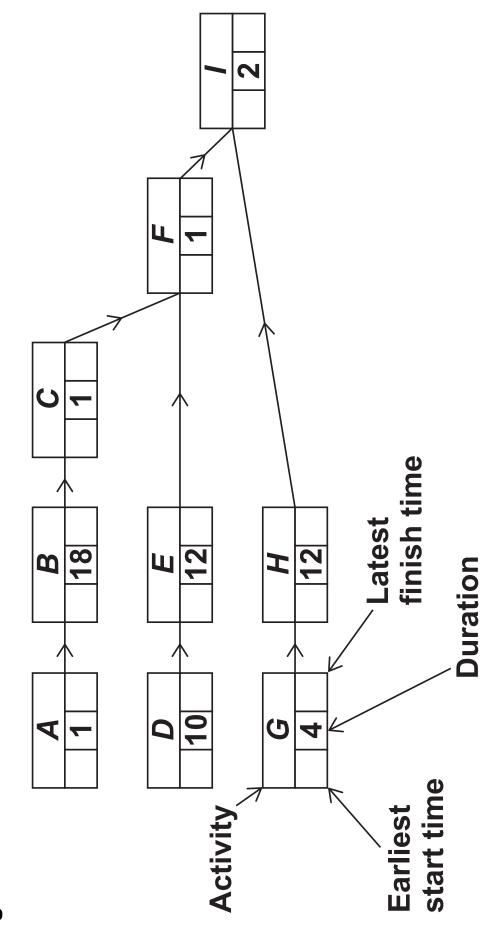


S

(a) (i) Use Figure 2 shown below to complete Figure 1 opposite. [1 mark] 2

5 (a) (ii) Complete Figure 2 showing the earliest start time and latest finishing time for each activity. [2 marks]

Figure 2



Question 5 continues on the next page



| meal | | | |
|--|--|--|--|
| t possible time at which the preparation of the meal [1 mark] | | | |
| J o | | | |
| tior | | | |
| ara | | | |
| rep | | | |
| e D | | | |
| t t | | | |
| nich | | | |
| X | | | |
| e at | | | |
| tin | | | |
| <u>e</u> | | | |
| possib mark] | | | |
| DO E | | | |
| est [1 | | | |
| arli ed. | | | |
| le e | | | |
| e th | | | |
| Determine the earli can be completed. | | | |
| tern be | | | |
| Det | | | |
| 5 (b) (ii) Determine the earlies can be completed. [| | | |
| @ | | | |
| ro. | | | |
| | | | |

Question 5 continues on the next page



The group of friends want to cook spring rolls so that they are served at the same time as the rest of the meal. This requires the additional activities shown in Figure 3.

Figure 3

| Label | Label Activity | Duration | Immediate predecessors |
|-------|---------------------------------------|----------|---------------------------|
| J | Switch on and heat oven | | Ι |
| K | Put spring rolls in oven and cook | | |
| 7 | Transfer spring rolls to serving dish | | |

It takes 15 seconds to switch on the oven.

The oven must be allowed to heat up for 10 minutes before the spring rolls are put in the oven.

It takes 15 seconds to put the spring rolls in the oven.

The spring rolls must cook in the hot oven for 8 minutes.

It takes 30 seconds to transfer the spring rolls to a serving dish.



5 (c)

| 5 (c) (i) Complete Figure 3 above. [1 mark] | Determine the latest time at which the oven can be switched on in order for the spring rolls to be served at the same time as the rest of the meal. [2 marks] | | | |
|---|---|--|--|--|
| (i) | (c) (ii) | | | |
| 2 | 2 | | | |
| | | | | |

6 An animal sanctuary has a rainwater collection site.

The manager of the sanctuary is installing a pipe system to connect the rainwater collection site to five other sites in the sanctuary.

Each site does not need to be connected directly to the rainwater collection site.

There are nine possible routes between the sites that are suitable for water pipes.

The distances, in metres, of the nine possible routes are given in the table below.

| From/To | Henhouse (<i>H</i>) | Goatshed (<i>G</i>) | Kennels (<i>K</i>) | Cattery (C) |
|--|--------------------------|-----------------------|-------------------------|-------------|
| Rainwater collection site (<i>R</i>) | 840 | 810 | 520 | 370 |
| Cattery (C) | _ | 680 | 610 | |
| Duckpond (<i>D</i>) | 480 | 310 | | • |
| Goatshed (<i>G</i>) | 150 | | • | |

Water pipe costs 60 pence per metre.

Find the minimum cost of connecting all the sites to the rainwater collection site.

Fully justify your answer. [5 marks]



| |
|------|
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |
| |



| | | | |
|---|------|------|------|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| , | | | |
| | | | |
| | | | |



| 1 | |
|---|--|
| | |
| | |
| | |
| | |



7 A linear programming problem has the constraints

$$1 \le x \le 6$$

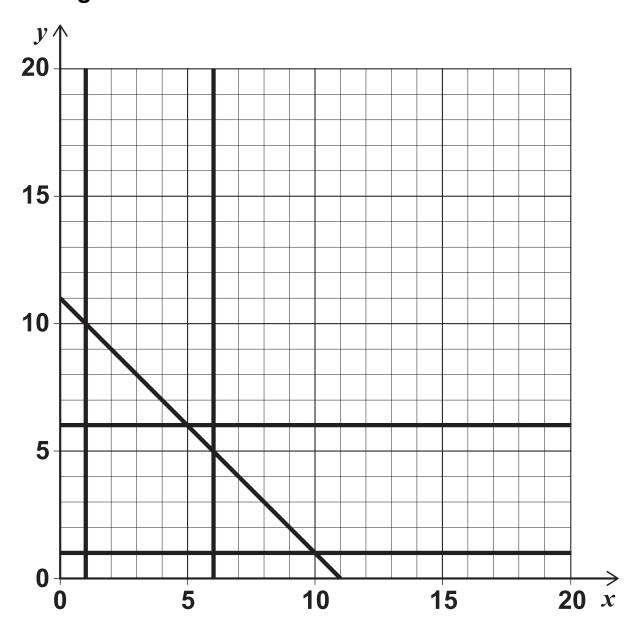
$$1 \le y \le 6$$

$$y \ge x$$

$$x + y \le 11$$

7 (a) (i) Complete Figure 4 to identify the feasible region for the problem. [2 marks]

Figure 4





| 7 | (a) (ii) | Determine the maximum value of $5x + 4y$ subject to the constraints. [2 marks] |
|---|----------|--|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |



| 7 | (b) | | The simple-connected graph ${\it G}$ has seven vertices. |
|---|-----|------|--|
| | | | The vertices of G have degree 1, 2, 3, v , w , x and y |
| 7 | (b) | (i) | Explain why $x \ge 1$ and $y \ge 1$ [1 mark] |
| | | | |
| | | | |
| | | | |
| | | | |
| 7 | (b) | (ii) | Explain why $x \le 6$ and $y \le 6$ [1 mark] |
| | | | |
| | | | |
| | | | |
| | | | |



| 7 | (b)(iii) | Explain why $x + y \le 11$ [2 marks] |
|---|----------|--|
| | | |
| | | |
| | | |
| | | |
| | | |
| 7 | (b)(iv) | State an additional constraint that applies to the values of x and y in this context. [1 mark] |
| | | |
| | | |
| | | |



| 7 | (c) | | The graph G also has eight edges. The inequalities used in part (a)(i) apply to the graph G . |
|---|-----|-----|---|
| 7 | (c) | (i) | Given that $v + w = 4$, find all the feasible values of x and y . [3 marks] |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |



7 (c) (ii) It is also given that the graph ${\it G}$ is semi-Eulerian.

On Figure 5, draw G. [2 marks]

Figure 5

END OF QUESTIONS



BLANK PAGE



| number | Additional page, if required. Write the question numbers in the left-hand margin. |
|--------|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



| Question number | Additional page, if required. Write the question numbers in the left-hand margin. |
|-----------------|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



| Question number | Additional page, if required. Write the question numbers in the left-hand margin. | | |
|-----------------|---|--|--|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |



| Question number | Additional page, if required. Write the question numbers in the left-hand margin. |
|--------------------|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |

| For Examiner's Use | | | |
|--------------------|------|--|--|
| Question | Mark | | |
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| TOTAL | | | |

Copyright information

For confidentiality purposes, from the November 2015 examination series, acknowledgements of third party copyright material will be published in a separate booklet rather than including them on the examination paper or support materials. This booklet is published after each examination series and is available for free download from www.aqa.org.uk after the live examination series.

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team, AQA, Stag Hill House, Guildford, GU2 7XJ.

Copyright © 2018 AQA and its licensors. All rights reserved.

PB/Jun18/7366/2D/E2

