UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/13**

Paper 1 **May/June 2013**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

7566713344

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

1 Let $f(r) = r!(r-1)$. Simplify $f(r+1) - f(r)$ and hence find $\sum_{r=n+1}^{2n} r!(r^2+1)$. [5]

2 The roots of the equation $x^4 - 4x^2 + 3x - 2 = 0$ are α, β, γ and δ; the sum $\alpha^n + \beta^n + \gamma^n + \delta^n$ is denoted by S_n. By using the relation $y = x^2$, or otherwise, show that α^2, β^2, γ^2 and δ^2 are the roots of the equation

$$y^4 - 8y^3 + 12y^2 + 7y + 4 = 0.$$ [3]

State the value of S_2 and hence show that

$$S_8 = 8S_6 - 12S_4 - 72.$$ [3]

3 Prove by mathematical induction that, for every positive integer n,

$$\frac{d^n}{dx^n}(e^x \sin x) = (\sqrt{2})^n e^x \sin(x + \tfrac{1}{4}n\pi).$$ [7]

4 Show that $\frac{dy}{dx} = -\frac{4}{3}$ at the point A (1, −2) on the curve with equation

$$y^3 - 3x^2y + 2 = 0,$$

and find the value of $\frac{d^2y}{dx^2}$ at A. [8]

5 Show that $\int_0^1 xe^{-x^2}\,dx = \frac{1}{2} - \frac{1}{2e}$. [2]

Let $I_n = \int_0^1 x^n e^{-x^2}\,dx$. Show that $I_{2n+1} = nI_{2n-1} - \frac{1}{2e}$ for $n \geqslant 1$. [3]

Find the exact value of I_7. [3]

6 The linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} -2 & 5 & 3 & -1 \\ 0 & 1 & -4 & -2 \\ 6 & -14 & -13 & 1 \\ \alpha & \alpha & -2\alpha & -11\alpha \end{pmatrix}$$

and α is a constant. The null space of T is denoted by K_1 when $\alpha \neq 0$, and by K_2 when $\alpha = 0$. Find a basis for K_1 and a basis for K_2. [8]

7 Find the value of the constant λ such that $\lambda x e^{-x}$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 6e^{-x}. \qquad [4]$$

Find the solution of the differential equation for which $y = 2$ and $\frac{dy}{dx} = 3$ when $x = 0$. [6]

8 The curve C has parametric equations $x = \frac{3}{2}t^2$, $y = t^3$, for $0 \leqslant t \leqslant 2$. Find the arc length of C. [4]

Find the coordinates of the centroid of the region enclosed by C, the x-axis and the line $x = 6$. [7]

9 The square matrix $\mathbf{A}$ has an eigenvalue λ with corresponding eigenvector $\mathbf{e}$. The non-singular matrix $\mathbf{M}$ is of the same order as $\mathbf{A}$. Show that $\mathbf{Me}$ is an eigenvector of the matrix $\mathbf{B}$, where $\mathbf{B} = \mathbf{MAM}^{-1}$, and that λ is the corresponding eigenvalue. [3]

Let

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{pmatrix}.$$

Write down the eigenvalues of $\mathbf{A}$ and obtain corresponding eigenvectors. [4]

Given that

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

find the eigenvalues and corresponding eigenvectors of $\mathbf{B}$. [4]

10 Use the identity $2\sin P\cos Q \equiv \sin(P+Q) + \sin(P-Q)$ to show that

$$2\sin\theta\cos(\theta - \tfrac{1}{4}\pi) \equiv \cos(2\theta - \tfrac{3}{4}\pi) + \frac{1}{\sqrt{2}}. \qquad [3]$$

A curve has polar equation $r = 2\sin\theta\cos(\theta - \frac{1}{4}\pi)$, for $0 \leqslant \theta \leqslant \frac{3}{4}\pi$. Sketch the curve and state the polar equation of its line of symmetry, justifying your answer. [3]

Show that the area of the region enclosed by the curve is $\frac{3}{8}(\pi + 1)$. [6]

[Question 11 is printed on the next page.]

11 Answer only **one** of the following two alternatives.

EITHER

The line l_1 passes through the point A whose position vector is $4\mathbf{i} + 7\mathbf{j} - \mathbf{k}$ and is parallel to the vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The line l_2 passes through the point B whose position vector is $\mathbf{i} + 7\mathbf{j} + 11\mathbf{k}$ and is parallel to the vector $\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$. The points P on l_1 and Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. Find the position vectors of P and Q. [8]

Find the shortest distance between the line through A and B and the line through P and Q, giving your answer correct to 3 significant figures. [6]

OR

Show the cube roots of 1 on an Argand diagram. [1]

Show that the two non-real cube roots can be expressed in the form ω and ω^2, and find these cube roots in exact cartesian form $x + \mathrm{i}y$. [3]

Evaluate the determinant

$$\begin{vmatrix} 1 & 3\omega & 2\omega^2 \\ 3\omega^2 & 2 & \omega \\ 2\omega & \omega^2 & 3 \end{vmatrix}.$$ [3]

It is given that $z = (4\sqrt{3})\left(\cos \frac{4}{3}\pi + \mathrm{i} \sin \frac{4}{3}\pi\right) - 4\left(\cos \frac{11}{6}\pi + \mathrm{i} \sin \frac{11}{6}\pi\right)$. Express z in the form $r(\cos \theta + \mathrm{i} \sin \theta)$, giving exact values for r and θ. [5]

Hence find the cube roots of z in the form $r(\cos \theta + \mathrm{i} \sin \theta)$. [2]