Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/12**

Paper 1 **October/November 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\frac{6x}{2}$, $15 \times \frac{x^2}{4}$	B1 B1	OE In or from a correct expansion. Can be implied by correct equation.
	$\times (4 + ax) \rightarrow 3a + 15 = 3$	M1	2 terms in x^2 equated to 3 or $3x^2$. Condone x^2 on one side only.
	$a = -4$	A1	CAO
		4	

Question	Answer	Marks	Guidance
2	Attempt to find the midpoint M	M1	
	(1, 4)	A1	
	Use a gradient of $\pm\frac{2}{3}$ and *their* M to find the equation of the line.	M1	
	Equation is $y - 4 = -\frac{2}{3}(x - 1)$	A1	AEF
	Alternative method for question 2		
	Attempt to find the midpoint M	M1	
	(1, 4)	A1	
	Replace 1 in the given equation by c and substitute *their* M	M1	
	Equation is $y - 4 = -\frac{2}{3}(x - 1)$	A1	AEF
		4	

Question	Answer	Marks	Guidance
3	$(y =)\ \frac{kx^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\left(=\frac{k\sqrt{x}}{\frac{1}{2}}\right)\ (+c)$	B1	OE
	Substitutes both points into an integrated expression with a '+*c*' and solve as far as a value for one variable.	M1	Expect to see $-1 = 2k + c$ and $4 = 4k + c$
	$k = 2½$ **and** $c = -6$	A1	WWW
	$y = 5\sqrt{x} - 6$	A1	OE From correct values of both k & c and correct integral.
		4	

Question	Answer	Marks	Guidance
4(i)	Arc length $AB = 2r\theta$	B1	
	$\text{Tan}\,\theta = \frac{AT}{r}$ or $\frac{BT}{r}$ $\rightarrow AT\ or\ BT = r\tan\theta$	B1	Accept or $\sqrt{\left(\left(\frac{r}{\cos\theta}\right)^2 - r^2\right)}$ or $\frac{r\sin\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$ NOT $(90 - \theta)$
	$P = 2r\theta + 2r\tan\theta$	B1FT	OE, FT for *their* arc length + 2 × *their AT*
		3	

Question	Answer	Marks	Guidance
4(ii)	Area $\Delta AOT = ½ \times 5 \times 5 \tan 1.2$ **or** Area $AOBT = 2 \times ½ \times 5 \times 5 \tan 1.2$	**B1**	
	Sector area = $½ \times 25 \times 2.4$ (or 1.2)	***M1**	Use of $½r^2\theta$ with $\theta = 1.2$ or 2.4.
	Shaded area = 2 triangles – sector	**DM1**	Subtraction of sector, using 2.4 where appropriate, from 2 triangles
	Area = 34.3 (cm^2)	**A1**	AWRT
	Alternative method for question 4(ii)		
	Area of $\Delta\ ABT = ½ \times (5 \times \tan 1.2)^2 \times \sin(\pi - 2.4)$ (= 55.86)	**B1**	
	Segment area = $½ \times 25 \times (2.4 - \sin 2.4)$ (= 21.56)	***M1**	Use of $½r^2 (\theta - \sin\theta)$ with $\theta = 1.2$ or 2.4
	Shaded area = triangle – segment	**DM1**	Subtraction of segment from $\Delta\ ABT$, using 2.4 where appropriate.
	Area = 34.3 (cm^2)	**A1**	AWRT
		4	

Question	Answer	Marks	Guidance
5(i)	Use of Pythagoras $\to r^2 = 15^2 - h^2$	**M1**	
	$V = ⅓\pi(225 - h^2) \times h \to ⅓\pi(225h - h^3)$	**A1**	AG WWW e.g. sight of $r = 15 - h$ gets A0.
		2	

Question	Answer	Marks	Guidance
5(ii)	$\left(\frac{\mathrm{d}v}{\mathrm{d}h}=\right)\frac{\pi}{3}(225-3h^2)$	B1	
	Their $\frac{\mathrm{d}v}{\mathrm{d}h}=0$	M1	Differentiates, sets *their* differential to 0 and attempts to solve at least as far as $h^2 \neq 0$.
	$(h=)\ \sqrt{75},\ 5\sqrt{3}$ or AWRT 8.66	A1	Ignore $-\sqrt{75}$ OE and ISW for both A marks
	$\frac{\mathrm{d}^2h}{\mathrm{d}h^2}=\frac{\pi}{3}\ (-6h)\ (\rightarrow -\text{ve})$	M1	Differentiates for a second time and considers the sign of the second differential or any other valid complete method.
	$\rightarrow$ Maximum	A1FT	Correct conclusion from correct 2nd differential, value for h not required, or any other valid complete method. FT for *their* h, if used, as long as it is positive.
			SC Omission of π or $\frac{\pi}{3}$ throughout can score B0M1A1M1A0
		5	

Question	Answer	Marks	Guidance
6(a)	$(2x+1) = \tan^{-1}(⅓)$ (= 0.322 or 18.4 OR −0.339 rad or 8.7°)	*M1	Correct order of operations. Allow degrees.
	Either *their* 0.322 + π or 2π Or *their* −0.339 + $\frac{\pi}{2}$ or π	DM1	Must be in radians
	$x = 1.23$ **or** $x = 2.80$	A1	AWRT for either correct answer, accept 0.39π or 0.89π
		A1	For the second answer with no other answers between 0 and 2.8 **SC1** For both 1.2 and 2.8
		4	
6(b)(i)	$5\cos^2 x - 2$	B1	Allow $a = 5$, $b = -2$
		1	
6(b)(ii)	−2	B1FT	FT for sight of *their b*
	3	B1FT	FT for sight of *their a + b*
		2	

Question	Answer	Marks	Guidance
7(i)	$\left(\overrightarrow{PB}\right) = 5\mathbf{i} + 8\mathbf{j} - 5\mathbf{k}$	B2,1,0	B2 all correct, B1 for two correct components.
	$\left(\overrightarrow{PQ}\right) = 4\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$	B2,1,0	B2 all correct, B1 for two correct components.
			Accept column vectors. **SC** B1 for each vector if all components multiplied by –1.
		4	
7(ii)	(Length of PB =) $\sqrt{\left(5^2 + 8^2 + 5^2\right)} = (\sqrt{114} \approx 10.7)$ (Length of PQ =) $\sqrt{\left(4^2 + 8^2 + 5^2\right)} = (\sqrt{105} \approx 10.2)$	M1	Evaluation of both lengths. Other valid complete comparisons can be accepted.
	P is nearer to Q.	A1	WWW
		2	
7(iii)	$\left(\overrightarrow{PB}.\overrightarrow{PQ}\right) = 20 + 64 - 25$	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$ on *their* $\overrightarrow{PB}$ and $\overrightarrow{PQ}$
	$(Their\sqrt{114})(their\sqrt{105})\cos BPQ = (their\ 59)$	M1	All elements present and in correct places.
	$BPQ = 57.4(°)$ or 1.00 (rad)	A1	AWRT Calculating the obtuse angle and then subtracting gets A0.
		3	

Question	Answer	Marks	Guidance
8(a)(i)	21st term = 13 + 20 × 1.2 = 37 (km)	B1	
		1	

Question	Answer	Marks	Guidance
8(a)(ii)	$S_{21} = \frac{1}{2} \times 21 \times (26 + 20 \times 1.2)$ or $\frac{1}{2} \times 21 \times (13 + \textit{their}\ 37)$	M1	A correct sum formula used with correct values for *a*, *d* and *n*.
	525 (km)	A1	
		2	
8(b)(i)	$\frac{x-3}{x} = \frac{x-5}{x-3}$ oe (or use of a, ar and ar^2)	M1	Any valid method to obtain an equation in one variable.
	(a = or x =) 9	A1	
		2	
8(b)(ii)	$r = \left(\frac{x-3}{x}\right)$ or $\left(\frac{x-5}{x-3}\right)$ or $\sqrt{\frac{x-5}{x}} = \frac{2}{3}$. Fourth term $= 9 \times (\frac{2}{3})^3$	M1	Any valid method to find *r* and the fourth term with *their a* & *r*.
	$2\frac{2}{3}$ **or** 2.67	A1	OE, AWRT
		2	
8(b)(iii)	$S_\infty = \frac{a}{1-r} = \frac{9}{1-\frac{2}{3}}$	M1	Correct formula and using *their* '*r*' and '*a*', with $\lvert r \rvert < 1$, to obtain a numerical answer.
	27 **or** 27.0	A1	AWRT
		2	

Question	Answer	Marks	Guidance
9(i)	$f(x) = g(x) \rightarrow 2x^2 + 6x + 1 + k\ \ (= 0)$	*M1	Forms a quadratic with all terms on same side.
	Use of $b^2 = 4ac$	DM1	Uses the discriminant = 0.
	$(k =)\ 3½$	A1	OE, WWW
	Alternative method for question 9(i)		
	$4x + 8 = 2\ (\rightarrow x = -1½)$	*M1	Differentiating, equating gradients and solving to give $x =$
	Substitutes *their* x value into either $2x^2 + 6x + 1 + k = 0$ OR into the curve to find $y\left(= \frac{-13}{2}\right)$ then both values into the line.	DM1	Substituting appropriately for *their* x and proceeding to find a value of k.
	$(k =)\ 3½$	A1	OE, WWW
		3	
9(ii)	$2x^2 + 6x - 8\ (< 0)$	M1	Forms a quadratic with all terms on same side
	-4 and 1	A1	
	$-4 < x < 1$	A1	CAO
		3	
9(iii)	$(g^{-1}(x)) = \frac{x-1}{2}$	B1	Needs to be in terms of x.
	$(g^{-1}f(x)) = \frac{2x^2 + 8x + 1 - 1}{2} = 0 \rightarrow (2x^2 + 8x = 0) \rightarrow x =$	M1	Substitutes f into g^{-1} and attempts to solve it = 0 as far as $x =$
	0, –4	A1	CAO
		3	

Question	Answer	Marks	Guidance
9(iv)	$2(x+2)^2-7$	B1 B1	or $a=+2$, $b=-7$
	(Least value of f(x) or y =) -7 or $\geqslant -7$	B1FT	FT for *their* b from a correct form of the expression.
		3	

Question	Answer	Marks	Guidance
10(i)	$\frac{dy}{dx}=[0]+\left[(2x+1)^{-3}\right]\times[+16]$	B2,1,0	OE. Full marks for 3 correct components. Withhold one mark for each error or omission.
	$\int y\,dx = [x]+\left[(2x+1)^{-1}\right]\times[+2]$ (+c)	B2,1,0	OE. Full marks for 3 correct components. Withhold one mark for each error or omission.
		4	
10(ii)	At A, $x=½$.	B1	Ignore extra answer $x=-1.5$
	$\frac{dy}{dx}=2\rightarrow$ Gradient of normal $(=-½)$	*M1	With *their* positive value of x at A and *their* $\frac{dy}{dx}$, uses $m_1m_2=-1$
	Equation of normal: $y-0=-½(x-½)$ or $y-0=-½(0-½)$ or $0=-½\times½+c$	DM1	Use of *their* x at A and *their* normal gradient.
	B (0, ¼)	A1	
		4	

Question	Answer	Marks	Guidance
10(iii)	$\int_0^{\frac{1}{2}} 1-\frac{4}{(2x+1)^2}(\mathrm{d}x)$	*M1	$\int y\,\mathrm{d}x$ SOI with 0 and *their* positive x coordinate of A.
	[½ + 1] – [0 + 2] = (–½)	DM1	Substitutes both 0 and *their* ½ into *their* $\int y\mathrm{d}x$ and subtracts.
	Area of triangle above x-axis = ½ × ½ × ¼ $\left(=\frac{1}{16}\right)$	B1	
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)
	Alternative method for question 10(iii)		
	$\int_{-3}^{0} \frac{1}{(1-y)^{\frac{1}{2}}}-\frac{1}{2}(\mathrm{d}y)$	*M1	$\int x\,\mathrm{d}y$ SOI. Where x is of the form $k\left(1-y)^{-\frac{1}{2}}+c\right)$ with 0 and *their* negative y intercept of curve.
	$[-2]-\left[-4+\frac{3}{2}\right]=(½)$	DM1	Substitutes both 0 and *their* –3 into *their* $\int x\mathrm{d}y$ and subtracts.
	Area of triangle above x-axis = ½ × ½ × ¼ $\left(=\frac{1}{16}\right)$	B1	
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)

Question	Answer	Marks	Guidance
	Alternative method for question 10(iii)		
	$\int_0^{\frac{1}{2}} -\frac{1}{2}x+\frac{1}{4}-y\,dx$	***M1**	∫(*their* normal curve) with 0 and *their* positive *x* coordinate of A.
	Curve [½ + 1] – [0 + 2] = (−½)	**DM1**	Substitutes both 0 and *their* ½ into *their* ∫*y*d*x* and subtracts.
	$\int_0^{\frac{1}{2}} -\frac{1}{2}x+\frac{1}{4}dx = \frac{-x^2}{4}+\frac{x}{4} = \left[\frac{-1}{16}+\frac{1}{8}\right]-[0]\ \left(=\frac{1}{16}\right)$	**B1**	Substitutes both 0 and ½ into the correct integral and subtracts.
	Total area of shaded region = $\frac{9}{16}$	**A1**	OE (including AWRT 0.563)
		4	