UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

9694 THINKING SKILLS

9694/33 Paper 3 (Problem Analysis and Solution), maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

1 (a) How many passwords could she create in this way, excluding the one she ha used?

There are 4 ways to select the fifth letter, 3 the sixth, etc. So (4 × 3 × 2 × 1) – 1 ways.
23.

1 mark for 4! = 24 and 1 mark for – 1.
OR 1 mark for a systematic attempt to list the passwords [i.e. if solution involves one letter held while the others are permuted, and then another letter given a similar treatment. Not necessary for the list of permutations to be complete].

(b) How many possible passwords could Agnès make (including any already used by others)? [2]

Agnès has 3 choices for the first, second and third, 6 for the fourth, 5 for the fifth, 2 for 6^{th}, 7^{th}, and 8^{th}. 3 × 3 × 3 × 6 × 5 × 2 × 2 × 2 = 6480.

1 mark for 3 × 3 × 3 × 6 × 5 × 2 × 2 × 2 with missing or incorrect answer.
OR 1 mark for the correct eight numbers combined according to an incorrect process (for example 3 + 3 + 3 + 6 + 5 + 2 + 2 + 2 = 26).

(c) (i) How many possible passwords could Fred make? [1]

Fred has 3 × 3 × 3 × 6 × 6 × 2 × 2 × 1 = 3888.

(ii) Give an example of a password that could be used by Agnès but not Fred. [1]

A valid password starting with W, ending in M or with second letter Z is necessary and sufficient.
E.g. WSCBJIOP, AXCFJKOM, QZCRHILP.

(iii) How many passwords could be made by Fred but not Agnès? [2]

To determine the number that only Fred can produce, it is simplest to calculate the number that are common. The differences between keyboards are the position of W and Z, the location of M, and the positions of Q and A. As Q and A use the same finger, they can be ignored.
The common set is 2 × 2 × 3 × 6 × 5 × 2 × 2 × 1 = 1440, so there are 3888 – 1440 = 2448 unique to Fred.

1 mark for 1440.
OR 1 mark for logic but faulty arithmetic. Creditworthy answers must delineate the correct collection of cases for the problem to be answered.

(d) (i) How many times larger is the collection of acceptable passwords which include such an error than the collection of those that could come from the random generator? [1]

There are seven possible switches of adjacent letters, (and each one cannot be produced from any other switch) so there are 7 times as many.
Allow 8 [for identifying the total acceptable password increase, error or no error].

(ii) An attacker knows the rules for generating and checking the passwo[rd]. difference does allowing a transposition make to the (very small) cha[nce] guessing the password of a particular user?

It makes no difference since all variants are treated as equivalent.

2 (a) If a 50-year-old man and a 60-year-old man both make a $100 payment at the same time, how much longer will the 50-year-old have to wait than the 60-year-old before getting at least $100 back? [2]

Answer: 4 years.
Allow sensible rounding of 4.17.

If 2 marks can not be awarded, award 1 mark for seeing:
100/4.02 → 25 years
or 100/4.83 → 21 years
or 100/4.02 – 100/4.83 → 24.87 – 20.70 → 4.17 → rounded to 5 years.

(b) Simon believes that if he dies at the average age for a male, he will have received the same amount in total whether he were to purchase an annuity at 50 or 60. What would the average age of death have to be for Simon to be right? [2]

$4.83(x - 60) = 4.02(x - 50)$
$0.81(x - 60) = 40.2$
$x = \underline{109.63}$
Allow sensible rounding (including 110).

If 2 marks can not be awarded, award 1 mark for attempting to formulate the problem algebraically or by trial and improvement.

(c) The figures in the table are lower for females than for males, and yet are fair, in that they have the same total expected payment. What must be the reason for this? [1]

(With all other parameters being the same,) the average age of death must be greater for females.

(d) The rate is less for males than females for joint life policies, although it was greater for males than females for policies only covering the policyholder. Suggest a reason for this. [2]

If the person taking out the annuity and partner were always of the same age then the figures should be equal; as they are not it must be because there is an expected age difference.
On average, the male in a pair is older.
SC: It may be the case that couples in which the man takes out the policy tend to live longer.
If 2 marks cannot awarded, award 1 mark for answers which show some appreciation that the age difference between the husband and wife is the (likely) cause.
SC: award 1 mark for an answer which offers an explanation relating to possible incentives that might be offered to attract females to take out the policies.

(e) (i) Which option gives them the highest return next year?

The highest immediate return would be for John to put all the money into a sing policy for him. (2 x $4.36).

(ii) Of all the many different options for annuities available to them, the one which will give Janet and John the highest expected total return is the 'female and husband' annuity. Explain why this is the case. [2]

The only reason that one can be better than the others is if there is some extra information known to them but not those setting the rates. In this case we (only) know that they are the same age, which means that John is younger than would be expected for a spouse and can thus expect to live for longer.
This means that she should put all the money into the female + spouse option: $4.19

If 2 marks can not be awarded, award 1 mark for simply pointing out that the assumed age difference between man and wife does not apply to Janet and John / any reference to the non-averageness of John and Janet as causes of the anomaly. Individual life expectancies in a marriage are not independent of each other.

3 (a) Give an example of five weights which have a *mean* of 64kg. [1]

Any five numbers which add up to 320, all of which are between 55 and 100 inclusive (units not needed).

(b) What is the largest range that the weights from 1968 could have had, given that the claims made by the first newspaper are correct (and assuming that it used the mean in its calculation of the average)? [3]

Largest = 22. 45 is the largest range achievable if the mean is to be 64 [55/55/55/55/100], but it must half this (or less) if the range is to double by 1988.

3 marks for the correct answer.
If 3 marks cannot be awarded, 2 marks for an answer of 22.5.
1 mark for appreciating that the greatest possible range is 45.

(c) Give an example of five weights which have a mean of 72 kg, and a median of 64 kg. [2]

Five weights between 55 and 100 which add up to 360, and a median of 64.
E.g. 64/64/64/84/84.

1 mark for weights with a mean of 72.
1 mark for weights with a median of 64.

(d) Give an example of five weights for the year 1968 and five weights for the year 1988 which could have supported the claims by both of the newspapers (assuming that the first newspaper had used the mean in its calculations, and the second had used the median). [2]

Possible answer: 1968 = 60, 61, 64, 64, 71 and 1988 = 64, 64, 64, 82, 86.

1 mark for answers in which the range is doubled.
If this first mark is awarded, award the two further marks if the medians are 64 (1 mark), and the means increase from 64 to 72 (1 mark).

(e) Show that the third newspaper could not have been referring to the mean, when it refers to the average above. [2]

Mean of the five athletes goes from 64 to 72, and cox is 55.
So mean of oarsmen is ((5 x 64) – 55)/4 = 66.25 (or 66) in 1968.
And is ((5 x 72) – 55)/4 = 76.25 (or 76) in 1988.
Which is not more than 11kg

Alternative explanation: the mean of the five athletes increasing from 64 to 72 entails a total increase of (360 – 320) 40kg. If the mean weight of the four oarsmen were to increase by 11, this would entail an increase of (4 x 11) 44 kg. So the 3rd newspaper report's use of the mean would conflict with the previous two.

2 marks if the candidate shows the underlined figures.
If 2 marks cannot be awarded, award 1 mark for a correct calculation of the means of the rowers for a particular valid collection of weights, OR an attempt to calculate the change in total weight implied by the oarsmen's increase referring to the mean.

(f) Find five weights for year 1968 and five weights for the year 1988 which are consistent with all three newspaper reports. [4]

correct answers:
1968: 55 [cox], 64, 64, 65 & 72.
1988: 55 [cox], 64, 64, 88 & 89.

Award 4 marks for these weights, given unambiguously.

If 4 marks cannot be awarded, give 1 mark for each (awarded independently of each other) of the following four requirements being fulfilled:
(i) two sets of five weights which include a cox (55/56kg), have the right median (64kg), and have a mean which increases from 64 to 72.
(ii) two sets of 5 weights whose range doubles.
(iii) two sets or weights for the four oarsmen which increase by (more than) 11.
(iv) two sets or weights for the four oarsmen whose range (more than) triples.

4 (a) At what time will the last film finish each day?

Answer: 22:59 award 2 m

If 2 marks cannot be awarded, award 1 mark for evidence that Lupine Jives will be the last film to finish.

(b) How many times will each of the five films be shown during this seven-day period? [3]

Answer: Colossal	22 times	
Wolf Gang	29 times	
Illinois	28 times	award 3 marks
Mahatma	36 times	
Lupine Jives	30 times	

Deduct 1 mark for each incorrect figure. (Minimum 1 mark if candidates get at least one film correct.)
Penalise oversight of 'not Mon's and 'extra shows' once only.

(c) (i) Which two films will Dustin watch this Thursday? [2]

Answer: Mahatma (17:40 to 19:03) award 1 mark
and Illinois (20:20 to 22:02) award 1 mark

Times need not be given.
Alternatively, award 1 mark for a second film that correctly follows an incorrect first film; e.g. Illinois (18:00 to 19:42) and Wolf Gang (20:30 to 22:08) OR: Colossal (17.25 start) and Mahatma (19.35 start) OR Wolf Gang (18:15 to 19:53) and Illinois.

(ii) How long after the second film finishes will he have to wait for a bus to take him home? [2]

Answer: 8 minutes (22:02 to 22:10) award 2 marks

2 marks for a fully correct follow through from part **(c)(i)**.
If 2 marks cannot be awarded, award 1 mark for sight of correct finish time for second film (may be seen in part **(c)(i)**) OR for a correct waiting time/finishing time consistent with an incorrect finishing time (seen) of a film.

(d) (i) What is the time of the latest bus that Nicole can catch from Redford? [1]

Answer: <u>14:35</u>

(ii) What is the earliest time that they will be able to leave the Red Carpet?

Answer: 20:58 or 21.00 (the bus departure time) award 3 m

If 3 marks cannot be awarded, award 1 mark each for evidence of the following (max 2):

- If they see Colossal last they won't leave until 22:39.
- It is possible to see either Illinois or Mahatma before the 17:25 showing of Colossal and the other one afterwards.
- The 19:35 showing of Mahatma will finish before the 20:20 showing of Illinois.

Alternatively, award up to 2 marks for the second-best solution (such as Mahatma, followed by Colossal followed by Illinois: finishes at 22.02, first bus leaving at 22.10).
Award 1 mark for a finish time consistent with a correct alternative ordering (the two remaining alternatives (I – M – C and M – I – C) both finish at 22:39, catching the 23:20 bus.

(e) Between which two villages and at approximately what time do the buses pass for the last time each day? [2]

Answer: Between Redford and Hepburn.

Accept any time or time interval between 21:07 and 21:10 (inclusive).