UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Advanced Level

## THINKING SKILLS

Paper 3 Problem Analysis and Solution
9694/32
May/June 2012
1 hour 30 minutes

Additional Materials: Answer Booklet/Paper

## Electronic Calculator

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO NOT WRITE ON ANY BARCODES.
Calculators should be used where appropriate.

Answer all the questions.
Start each question on a new answer sheet.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 Study the information below and answer the questions. Show your working.
A trainee detective is given a pair of pages torn from a book, which includes a continuous section of double-page spreads mapping the whole of an unknown city, and asked to estimate what the population of the city is. The pages look like this:


He tries to work out how many similar pages constitute the city, before estimating how many people live on each page.

Initially, he assumes that the city is rectangular, and that the pages mapping the city are arranged in rows, numbered consecutively. There may be pages before and after the map section, dedicated to other material.

For example, a map might be distributed over the pages like this:

| 172 | 173 | 174 | 175 | 176 | 177 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 178 | 179 | 180 | 181 | 182 | 183 |
| 184 | 185 | 186 | 187 | 188 | 189 |
| 190 | 191 | 192 | 193 | 194 | 195 |

(a) Assume that the map section is as small as possible.
(i) What is the smallest number of pages the map section could cover?
(ii) What is the earliest page it could start on?
(iii) What is the latest page it could start on?
(b) If the detective's pages have come from the middle row of the map section, what is the largest possible number of pages in the map section?

The trainee detective quickly realises that this is a very clumsy estimate of the size of the city, because cities are not usually rectangular. He decides to refine his model, and consider cities which could be covered by a symmetrical, stepped reduction in the number of pages: one double page is removed from each end of the previous row, moving away from the middle line of pages until there are none left. For example:


From now on, he only considers cities of this type.
(c) Given this assumption, how many pages are there in the map section?
(d) The detective notices that the town hall lies on the right-hand edge of page 63. He assumes that this is at the centre of the city.

Given this assumption, on which page does the map section start?

2 Study the information below and answer the questions. Show your working.
Albert and his three colleagues were employed to paint the sixteen arches of Stoppard bridge. It took them four years to do, and when they had finished they needed to start again immediately. Two painters never work on the same arch at the same time.

A new type of paint was produced which takes the same effort to apply, and costs twice as much, but lasts for 16 years. Albert's colleagues immediately lost their jobs (were made redundant) and each received the redundancy payment. Albert continued to work on the bridge, and switched to using the new paint.

One painter's pay (per year) \$20000
Redundancy payment \$5000
Old paint (per painter per year) \$1000
New paint (per painter per year) \$2000
(a) How long was it before the bridge became unsafe because some parts had insufficient paint?
(b) Instead of getting rid of Albert's colleagues immediately, it had been suggested employing 12 more painters who would be employed for one year (and then making all the painters redundant). If this had been done, with everyone using the new paint, what would the cost have been over a 16 -year period?
(c) If the original four painters had switched over to using the new paint, they would all have been made redundant at the end of four years. How much would that have saved, compared with simply continuing with the old paint, over a 16-year period?
(d) A gradual reduction to having just Albert painting full time could have been arranged, without any arch being painted before it needed to be.
(i) Describe the way this could have been done, noting which paint would be used by each painter.
(ii) At what stage would Albert have become the only painter?
(iii) How much would this have cost over a 16-year period?

3 Study the information below and answer the questions. Show your working.
An entrepreneur called Tadeusz has acquired a minibus, and is keen to start a business taking paying passengers to a festival. He wants to fix a charge for the trip, so he decides to research how the charge might affect the number of passengers willing to pay. To make things easier, he is only going to consider prices which are whole numbers of dollars.

He asks five drivers about their experiences of trying to sell seats on buses for a similar journey.
Aaron says, "I charged $\$ 10$ and attracted 15 passengers."
Baal says, "I charged $\$ 20$ and managed to get 10 people."
Caelum says, "I charged $\$ 25$ and was able to get 9 people."
Daedalus says, "I charged $\$ 4$ and managed to attract 45."
Ephron says, "I charged $\$ 50$ and managed to get 7 people."
(a) Who took the most money for the journey, and how much did they take in total from their passengers?

Tadeusz decides that he will just take the information given by Baal and Caelum, and model the relationship between the price and the number of customers as a linear one, for simplicity. In other words, he assumes that a price increase of $\$ 5$ will lead to a decrease of 1 passenger, and that this model will continue to hold as the price gets higher or lower. He also assumes that the prices given are the highest that will attract that number of passengers: for example, a price of $\$ 21$ would attract only 9 passengers.
(b) According to this model, what price would generate the highest income?

Tadeusz's business partner Jeroboam is keen to maximise the income, but Tadeusz decides that it is better to drive with a fuller bus in order to get a good reputation, and for ecological reasons.

Tadeusz sees that, by using the information offered by Baal and Aaron instead, he can produce a model where the price which generates the highest income will attract more passengers than the previous model.
(c) Using only the information offered by Baal and Aaron to form a linear model, as has been done above with Baal and Caelum, calculate the price which produces the highest income, and the number of passengers this price will attract.

Tadeusz, knowing that Jeroboam disagrees with his business aims, plans to trick him into agreeing to a price which will attract a larger number of passengers. By choosing the information from a particular pair of drivers, Tadeusz tries to convince Jeroboam of a price which maximises the income (for this model) and promises 35 passengers.
(d) Which two drivers' information does Tadeusz use, and what are the expected takings according to this model?

Jeroboam laughs and points out that their minibus only takes 26 passengers. Jeroboam shows Tadeusz that a model derived from the information given by Daedalus and one of the other drivers suggests that they could earn $\$ 702$ from a full bus of 26 passengers.
(e) Show that Jeroboam's claim is correct, and state which other driver's information he uses for his model.

4 Study the information below and answer the questions. Show your working.
The island of Peladot lies off the coast of Amphibia, close to Scutiger.
A ferry makes regular crossings between Scutiger and Peladot.
Ferry Timetable - Weekdays

|  | Scutiger | Peladot | Peladot | Scutiger |
| :--- | :---: | :---: | :---: | :---: |
|  | Depart | Arrive | Depart | Arrive |
| MON | $07: 00$ | $07: 46$ | $08: 00$ | $08: 46$ |
|  | $09: 50$ | $10: 36$ | $10: 55$ | $11: 41$ |
|  | $11: 55$ | $12: 41$ | $13: 00$ | $13: 46$ |
|  | $14: 00$ | $14: 46$ | $15: 00$ | $15: 46$ |
|  | $16: 00$ | $16: 46$ | $17: 00$ | $17: 46$ |
|  | $18: 00$ | $18: 46$ | $19: 00$ | $19: 46$ |
| TUE/WED/THU | - | - | $06: 45 \mathrm{TH}$ | $07: 31 \mathrm{TH}$ |
|  | $07: 45$ | $08: 31$ | $08: 45$ | $09: 31$ |
|  | $09: 50$ | $10: 36$ | $10: 55$ | $11: 41$ |
|  | $11: 55$ | $12: 41$ | $13: 00$ | $13: 46$ |
|  | $14: 00$ | $14: 46$ | $15: 00$ | $15: 46$ |
|  | $16: 00$ | $16: 46$ | $17: 00$ | $17: 46$ |
|  | $18: 00$ | $18: 46$ | $19: 00$ | $19: 46$ |
|  | $20: 00 \mathrm{~W}$ | $20: 46 \mathrm{~W}$ | - | - |
| FRI | $07: 45$ | $08: 31$ | $08: 45$ | $09: 31$ |
|  | $09: 50$ | $10: 36$ | $10: 55$ | $11: 41$ |
|  | $11: 55$ | $12: 41$ | $13: 00$ | $13: 46$ |
|  | $14: 00$ | $14: 46$ | $15: 00$ | $15: 46$ |
|  | $16: 00$ | $16: 46$ | $17: 00$ | $17: 46$ |
|  | $18: 00$ | $18: 46$ | $19: 00$ | $19: 46$ |
|  | $22: 30$ | $23: 16$ | - | - |

KEY: $\quad \mathrm{W}=$ Wednesdays only
TH = Thursdays only
Fares
One-way tickets only:
Car (including driver) $\quad \$ 30$
Other Adults \$6
Children (under 5) Free
Children (5-15) \$3
Bicycle \$1
Return tickets are not available on the ferry, but permanent residents of Peladot can buy books of 20 tickets (of the relevant type) for the price of 12 and may take bicycles aboard free of charge.

The landing stage at Scutiger is a 3-minute walk from the railway station, which is served by the Amphibian Coast Line.

## Amphibian Coast Line Daily Timetable

Rana - Bufo

| Rana | $05: 45$ | $06: 55$ | $08: 10$ | $09: 10$ | $10: 10$ | $11: 10$ | $12: 10$ | $13: 40$ | $15: 10$ | $16: 10$ | $17: 25$ | $18: 25$ | $20: 40$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Andrias | $05: 57$ | $07: 07$ | $08: 22$ | $09: 22$ | $10: 22$ | $11: 22$ | $12: 22$ | $13: 52$ | $15: 22$ | $16: 22$ | $17: 37$ | $18: 37$ | $20: 52$ |
| Scutiger | $06: 11$ | $07: 21$ | $08: 36$ | $09: 36$ | $10: 36$ | $11: 36$ | $12: 36$ | $14: 06$ | $15: 36$ | $16: 36$ | $17: 51$ | $18: 51$ | $21: 06$ |
| Acris | $06: 22$ | $07: 32$ | $08: 47$ | $09: 47$ | $10: 47$ | $11: 47$ | $12: 47$ | $14: 17$ | $15: 47$ | $16: 47$ | $18: 02$ | $19: 02$ | $21: 17$ |
| Bombina | $06: 31$ | $07: 41$ | $08: 56$ | $09: 56$ | $10: 56$ | $11: 56$ | $12: 56$ | $14: 26$ | $15: 56$ | $16: 56$ | $18: 11$ | $19: 11$ | $21: 26$ |
| Siren | $06: 40$ | $07: 50$ | $09: 05$ | $10: 05$ | $11: 05$ | $12: 05$ | $13: 05$ | $14: 35$ | $16: 05$ | $17: 05$ | $18: 20$ | $19: 20$ | $21: 35$ |
| Craugastor | $06: 52$ | $08: 02$ | $09: 17$ | $10: 17$ | $11: 17$ | $12: 17$ | $13: 17$ | $14: 47$ | $16: 17$ | $17: 17$ | $18: 32$ | $19: 32$ | $21: 47$ |
| Scinax | $07: 07$ | $08: 17$ | $09: 32$ | $10: 32$ | $11: 32$ | $12: 32$ | $13: 32$ | $15: 02$ | $16: 32$ | $17: 32$ | $18: 47$ | $19: 47$ | $22: 02$ |
| Bufo | $07: 18$ | $08: 28$ | $09: 43$ | $10: 43$ | $11: 43$ | $12: 43$ | $13: 43$ | $15: 13$ | $16: 43$ | $17: 43$ | $18: 58$ | $19: 58$ | $22: 13$ |

Bufo - Rana

| Bufo | $05: 45$ | $06: 55$ | $08: 10$ | $09: 10$ | $10: 10$ | $11: 10$ | $12: 10$ | $13: 40$ | $15: 10$ | $16: 10$ | $17: 25$ | $18: 25$ | $20: 40$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Scinax | $05: 56$ | $07: 06$ | $08: 21$ | $09: 21$ | $10: 21$ | $11: 21$ | $12: 21$ | $13: 51$ | $15: 21$ | $16: 21$ | $17: 36$ | $18: 36$ | $20: 51$ |
| Craugastor | $06: 11$ | $07: 21$ | $08: 36$ | $09: 36$ | $10: 36$ | $11: 36$ | $12: 36$ | $14: 06$ | $15: 36$ | $16: 36$ | $17: 51$ | $18: 51$ | $21: 06$ |
| Siren | $06: 23$ | $07: 33$ | $08: 48$ | $09: 48$ | $10: 48$ | $11: 48$ | $12: 48$ | $14: 18$ | $15: 48$ | $16: 48$ | $18: 03$ | $19: 03$ | $21: 18$ |
| Bombina | $06: 32$ | $07: 42$ | $08: 57$ | $09: 57$ | $10: 57$ | $11: 57$ | $12: 57$ | $14: 27$ | $15: 57$ | $16: 57$ | $18: 12$ | $19: 12$ | $21: 27$ |
| Acris | $06: 41$ | $07: 51$ | $09: 06$ | $10: 06$ | $11: 06$ | $12: 06$ | $13: 06$ | $14: 36$ | $16: 06$ | $17: 06$ | $18: 21$ | $19: 21$ | $21: 36$ |
| Scutiger | $06: 52$ | $08: 02$ | $09: 17$ | $10: 17$ | $11: 17$ | $12: 17$ | $13: 17$ | $14: 47$ | $16: 17$ | $17: 17$ | $18: 32$ | $19: 32$ | $21: 47$ |
| Andrias | $07: 06$ | $08: 16$ | $09: 31$ | $10: 31$ | $11: 31$ | $12: 31$ | $13: 31$ | $15: 01$ | $16: 31$ | $17: 31$ | $18: 46$ | $19: 46$ | $22: 01$ |
| Rana | $07: 18$ | $08: 28$ | $09: 43$ | $10: 43$ | $11: 43$ | $12: 43$ | $13: 43$ | $15: 13$ | $16: 43$ | $17: 43$ | $18: 58$ | $19: 58$ | $22: 13$ |

Rail Fares To and From Scutiger

| To or From | Single | Return <br> (within one <br> week) | Return <br> same day <br> anytime) | Return <br> (same day <br> off-peak) |
| :--- | :---: | :---: | :---: | :---: |
| Andrias, Rana, <br> Acris, Bombina | $\$ 4.00$ | $\$ 7.00$ | $\$ 6.20$ | $\$ 5.60$ |
| Siren, Craugastor | $\$ 6.00$ | $\$ 10.50$ | $\$ 9.30$ | $\$ 8.40$ |
| Scinax, Bufo | $\$ 8.00$ | $\$ 14.00$ | $\$ 12.40$ | $\$ 11.20$ |

Children under 16: half price.
No charge for carriage of bicycles.
Off-peak return tickets are available only when all rail travel takes place between 09:30 and 16:30 on the same day from Monday to Friday and at any time on Saturday and Sunday.
(a) How many times each week does the ferry make the crossing from Scutiger to Peladot from Monday to Friday?
(b) Mr. and Mrs. Newton and their three children, aged 17, 14 and 12, have booked a holiday cottage on Peladot. They are going to take their car across on the ferry.

How much will a one-way ferry crossing cost the Newtons altogether?
(c) Mrs. Mander is on holiday on Peladot until Friday. On Tuesday she plans to cross to the mainland, travel by train to explore the Roman ruins at Craugastor, and then return to Peladot. She intends to buy an off-peak return ticket between Scutiger and Craugastor.
(i) What will be Mrs. Mander's total travel costs on Tuesday?
(ii) What is the greatest amount of time that Mrs. Mander will be able to spend in Craugastor?

Jeremy lives on Peladot, but works in Bombina. Because he is dependent on the ferry and the train to get to work, he is allowed flexible working hours. He works from Monday to Friday every week.

He crosses from Peladot to Scutiger on the first crossing each morning, and always returns on the 18:00 crossing from Scutiger. He cycles between home and the ferry, a journey of 12 minutes, and also between Bombina station and work, which takes him 7 minutes.
(d) (i) What is the earliest time that Jeremy can ever arrive at work?
(ii) What is the latest time that Jeremy can leave work in order to catch the 18:00 ferry at Scutiger?
(e) Jeremy always arrives at work as early as possible, and never leaves before 16:15. Travelling as economically as possible, what is the total cost of his travel to and from work each week?

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