THINKING SKILLS   **9694/31**

Paper 3 Problem Analysis and Solution   **October/November 2014**

1 hour 30 minutes

Additional Materials: Electronic Calculator

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Calculators should be used where appropriate.

The number of marks is given in brackets [ ] at the end of each question or part question.

Cambridge
International Examinations

[Turn over

1 *Study the information below and answer the questions. Show your working.*

A construction toy contains various square pieces which need to be packaged by laying them flat into a rectangular tray. The tray has a handle on one edge. For this question, only consider squares with sides parallel to the sides of the tray at all times.

It is possible to place squares with sides of length 1 unit to 5 units in a 5 x 12 rectangular tray as shown, but they do not fill the tray and so the pieces can slide during shipping.
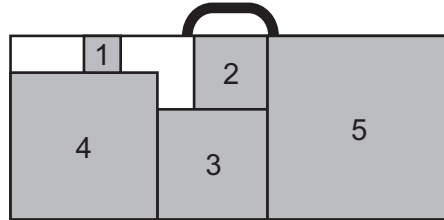

(a) (i) Which of these squares might be found in a different position after shipping? [2]

(ii) Draw a rearrangement of these pieces inside a 5 x 12 rectangle which would result in fewer pieces being able to move. [1]

Carrie manages to fit all the squares with sides from 1 unit to 11 units into a 19 x 27 rectangular tray.

(b) How many unit squares would be needed to fill all the gaps? [1]

Squares of side 1 to 7 can be placed in an 11 x 14 rectangular tray as shown.
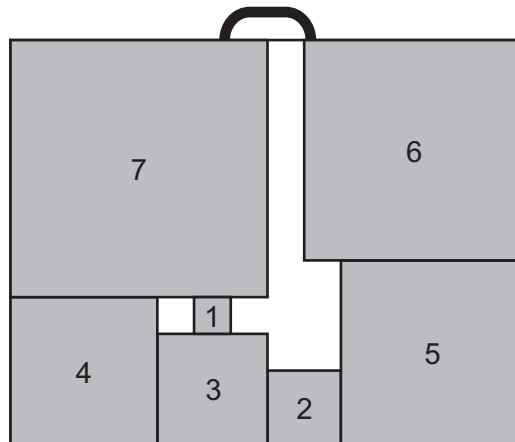
(c) Which one of these seven squares can never move, no matter how many of the others do? [1]

Having the contents move during transport is often undesirable. It may be worth adding one or more extra pieces to stop them doing so.

By moving just the smallest square in the configuration above, it would be possible to insert into the tray a single 'filler' piece made up from no more than 6 unit squares, so that no pieces can move.

(d) Design such a 'filler' piece, and show where the smallest square should be placed relative to it. [2]

(e) Draw another arrangement of the seven squares, without any extra pieces, within this 11 x 14 rectangle, so that none of the squares bigger than 3 x 3 can move. [3]

2 *Study the information below and answer the questions. Show your working.*

A bank is reviewing the security PINs that customers use for internet banking, and decides that it will only allow customers to use 4-digit Personal Identification Numbers (PINs) which cannot be easily tested by a particular PIN-cracking program. This program tests PINs that follow an easily memorable rule. An example of such a rule involves choosing a first digit and then adding a constant amount to it three times. Just two digits and one symbol are required to define a rule.

For example, **3 1 +** would generate the sequence starting at 3, and adding 1 each time: 3 4 5 6.

When such a sequence produces a number that is more than 1 digit, only the units digit is used. As an example: **5 4 +** would generate the sequence 5, 9, 13, 17, which produces the PIN 5 9 3 7.

(a) What 4-digit PIN would **8 7 +** produce? [1]

(b) How many different 4-digit PINs can be produced using two digits and an addition sign in this way? [1]

The program also tests 4-digit PINs that involve successive subtractions or successive multiplications. In all these cases only the units digit is ever taken from the sequence generated.
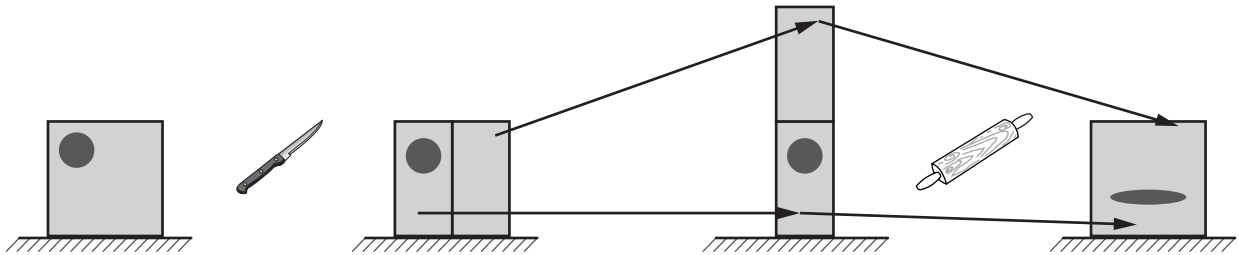
For example:
the rule **3 7 –** would generate the sequence 3, –4, –11, –18, which produces the PIN 3 4 1 8;
the rule **9 3 ×** would generate the sequence 9, 27, 81, 243, which produces the PIN 9 7 1 3.

(c) What rule would produce the PIN 2 6 4 2? [1]

(d) List all the rules that would produce the PIN 6 6 6 6. [2]

(e) In this part, consider only PINs with four different digits. Give an example of such a PIN which can be produced using two different rules, both using multiplication. State the rules. [2]

(f) List all of the 4-digit PINs of the form 3 1 _ _ which would not be allowed (i.e. are produced by one of the possible rules)? [2]

(g) Show that at least 97% of all possible 4-digit PINs are still allowed. [1]

3 *Study the information below and answer the questions. Show your working.*

When making puff pastry, a block of pastry is cut into two. One half is then placed on top of the other and the pastry is pressed down until it makes the original shape. This process of 'rolling out' is repeated many times.

We can model this by considering a cross-section of the pastry, viewed from the side, represented by a square of one unit by one unit. Points in this cross-section can be referred to using (x, y) coordinates.

In this model, the right-hand half of the pastry block is always placed on top of the left-hand half.

So, for example: (0.2, 0.2) moves to (0.4, 0.1)

and ($\frac{7}{8}$, $\frac{7}{8}$) moves to ($\frac{3}{4}$, $\frac{15}{16}$).

(a) (i) Give the coordinates of a point that ends up in the same place as it started, after one roll-out. [1]

(ii) Where does the point (0.4, 0.1) move to after a roll-out? [1]

(iii) Where does the point ($\frac{3}{5}$, $\frac{3}{5}$) move to after a roll-out? [1]

Rolling out mixes up the ingredients as well as introducing the desired texture to the pastry.

Ground spice has been sprinkled all over the top of the pastry.

(b) (i) How many layers of ground spice will there be after three roll-outs? [1]

(ii) How many roll-outs are needed before all points are within $\frac{1}{10}$ of a unit from some spice? [1]

A lump of butter is represented in the diagram above by the dark grey disk. Before the first roll-out, its left-hand edge was at $x = 0.1$ and its right-hand edge was at $x = 0.4$. As shown, it changed shape, but remained as one piece. However, it will be in two pieces after the second roll-out.

(c) (i) Into how many pieces will the butter have been cut after the fourth roll-out? [2]

(ii) Draw a pair of diagrams to show how two lumps of butter, of any simple shape, could combine to form one lump during a roll-out. One diagram should show the position of the two lumps before the roll-out, and the other diagram should show the single combined lump after the roll-out. [2]

Some points return to where they started after a number of roll-outs. A complete sequence of such points is called a cycle.

(d) (i) How many roll-outs in total are needed before the point $(\frac{1}{7}, \frac{4}{7})$ returns to where it started? List all the points in the cycle. [2]

   (ii) Give an example of a point on a different cycle of the same length. (This cycle must not include $(\frac{1}{7}, \frac{4}{7})$.) [1]

(e) (i) How many roll-outs in total are needed before the point $(\frac{1}{127}, \frac{64}{127})$ returns to where it started? [1]

   (ii) Identify a point which moves back to its starting position after 2 roll-outs. [1]

   (iii) Identify a point which moves back to its starting position after 5 roll-outs. [1]

4 *Study the information below and answer the questions. Show your working.*

The Chambet Open-air Shakespeare Festival will take place from Wednesday 3 July to Saturday 27 July. A total of 10 plays will be performed at three venues: Belmont Gardens, Corioli Park and Elsinore Common.

This is the festival's programme:

Belmont Gardens

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Week 1	1	2	As You Like It	As You Like It	As You Like It	Twelfth Night	Twelfth Night
Week 2	Twelfth Night	Love's Labour's Lost	Love's Labour's Lost	As You Like It	As You Like It	Twelfth Night	Twelfth Night
Week 3	Love's Labour's Lost	As You Like It	Twelfth Night	As You Like It	Twelfth Night	Love's Labour's Lost	As You Like It
Week 4	As You Like It	Twelfth Night	Twelfth Night	Love's Labour's Lost	As You Like It	Twelfth Night	28

Corioli Park

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Week 1	1	2	Romeo and Juliet	Romeo and Juliet	Othello	Othello	Romeo and Juliet
Week 2	Othello	King Lear	Romeo and Juliet	Othello	Timon of Athens	Romeo and Juliet	Othello
Week 3	Romeo and Juliet	Othello	King Lear	Timon of Athens	Romeo and Juliet	Romeo and Juliet	Othello
Week 4	Othello	King Lear	Timon of Athens	King Lear	Othello	Romeo and Juliet	28

Elsinore Common

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Week 1	1	2	The Tempest	Measure for Measure	The Tempest	Measure for Measure	The Tempest
Week 2	The Tempest	The Tempest	Measure for Measure	Measure for Measure	Cymbeline	The Tempest	The Tempest
Week 3	Measure for Measure	Measure for Measure	Cymbeline	The Tempest	The Tempest	Measure for Measure	Measure for Measure
Week 4	Cymbeline	The Tempest	Measure for Measure	Cymbeline	The Tempest	Measure for Measure	28

All performances start at 19:30

Ticket prices

First performance of each play: $15

All other performances during weeks 1 and 2: $18

All performances during weeks 3 and 4: $24

Special offer: See all 10 plays during weeks 3 and 4 for $175

(a) During the festival, which play will be performed

    (i) more times than any of the others? [1]

    (ii) fewer times than any of the others? [1]

Twelfth Night, Othello and The Tempest are the three plays scheduled for 8 July. This schedule is repeated on 14 July.

(b) Which **two** dates repeat the schedule of 11 July? [2]

Kate's two sons both appeared in Much Ado About Nothing at the previous festival, and she went to every performance. This time, Mark is in King Lear and Antony is in Love's Labour's Lost. She wants to see as many of their performances as she can, but, to avoid having to choose, she will not go to either play when both of them are performing on the same evening.

(c) (i) List all the dates on which Kate will watch either Mark or Antony performing at the festival. [2]

    (ii) What is the total cost of Kate's tickets? [2]

Henry lives in Chambet, and wants to see all 10 plays.

(d) What is the lowest possible total price that he could pay to see all 10 plays? [3]

Richard wants to see all of the plays, but he will not arrive in Chambet until 18 July, when only 10 evenings of the festival remain.

He particularly wants to save Twelfth Night for the final evening, and he wants to avoid going to the same venue on two consecutive evenings.

(e) (i) Explain why Richard must go to see Timon of Athens first (on 18 July). [1]

    (ii) In what order will Richard see the 10 plays? [3]

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