

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Ordinary Level

MARK SCHEME for the November 2004 question paper

4037 ADDITIONAL MATHEMATICS

4037/01

Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.



Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol \surd implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

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| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear) |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed) |
| CWO | Correct Working Only – often written by a 'fortuitous' answer |
| ISW | Ignore Subsequent Working |
| MR | Misread |
| PA | Premature Approximation (resulting in basically correct work that is insufficiently accurate) |
| SOS | See Other Solution (the candidate makes a better attempt at the same question) |

Penalties

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| MR -1 | A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through \surd " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. |
| OW -1,2 | This is deducted from A or B marks when essential working is omitted. |
| PA -1 | This is deducted from A or B marks in the case of premature approximation. |
| S -1 | Occasionally used for persistent slackness – usually discussed at a meeting. |
| EX -1 | Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting. |

November 2004

GCE O LEVEL

MARK SCHEME




MAXIMUM MARK: 80

SYLLABUS/COMPONENT: 4037/01

**ADDITIONAL MATHEMATICS
(Paper 1)**



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| | | | |
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| 1 | $OA = i + 9j$ $OB = 5i - 3j$ $OC = k(i + 3j)$ AB or $BA = 4i - 12j$ AC or $CA = (k - 1)i + (3k - 9)j$ CB or $BC = (5 - k)i - (3k + 3)j$ Ratio of i to j is the same $\rightarrow k = 2$ [or $m = -3 = (3k + 3) \div (k - 5) = (3k - 9) \div (k - 1)$ $\rightarrow M1 A1 \rightarrow k = 2 M1 A1$] [or $(1, 9) \rightarrow (5, -3) y = -3x + 12$ M1 A1 subs $(k, 3k)$ or solve with $y = 3x$ M1A1] | M1 A1 M1 A1 [4] | For one relevant vector . Must be “_” Any 2 of these correct unsimplified. Using ratio idea (not DM). Co. If use of $AB = a - b$ and correct answer obtained, allow full marks. Drawing M2 A2 |
| 2 (i) |  | B1 B1 | Co.co |
| (ii) |  | B1 B1 [4] | Co.co |
| 3 | Change to powers of 2. $2^{3x} \div 2^y = 2^6 \rightarrow 3x - y = 6$ Change to powers of 3. $3^{4x} \times 3^{-2y+2} = 3^4 \rightarrow 4x - 2y + 2 = 4$ $\rightarrow x = 5$ and $y = 9$ | M1 A1 M1 A1 A1 [5] | Needs to try all terms as powers of 2 and have \pm . Co Needs to try all terms as powers of 3 and have \pm . Co with bracket sorted. Co |
| 4 |  | M1 A1 A1 M1 A1√ A1 [6] | Any use of $\frac{1}{2}r^2\theta$ for M mark. A1 unsimplified, but complete. Co. M1 for any use of $s = r\theta$. A1√ unsimplified and √ on his r or x only. Co – allow anything rounding to 32.0 |

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| 5 | $(a + x)(1 - 2nx + \dots) = 3 - 41x + bx^2$ term in $x^3 = {}_n C_2 (\pm 2x)^2$ $\rightarrow a = 3$ $1 - 2an = -41 \rightarrow n = 7$ Coeff of x^2 is $3 \times 84 - 1 \times 14$ $\rightarrow 238$ | B1 B1 M1 A1 M1 A1 [6] | Wherever it comes. Co – anywhere Must use 2 terms. Must use sum of 2 products. Co. |
| 6 | $f(x) = 5 + 3\cos 4x$ (i) $a = 3$, period = $\frac{1}{2}\pi$ (ii) max/min $x = \frac{\pi}{4}$ or $\frac{2\pi}{4}$ or $\frac{3\pi}{4}$ \rightarrow max of 8 \rightarrow min of 2 $(\frac{\pi}{4}, 2)$ $(\frac{2\pi}{4}, 8)$ $(\frac{3\pi}{4}, 2)$ | B1 B1 B1 B1 B2, $1\sqrt{\quad}$ [6] | Co. allow 90° for period. When “8” is used as stationary value. When “2” is used as stationary value. $\sqrt{\quad}$ for $5 \pm$ his “a”. [B0 if degrees here] Ignore inclusion of max/min at 0 or π . |
| 7 | (a) $8 \times 8!$ or $\frac{8}{9} \times 9!$ or $9! - 8!$ $\rightarrow 322\,560$ (b) 2G, 1B ${}_5 C_2 \times {}_3 C_1 = 10 \times 3 = 30$ 3G, 0B ${}_5 C_3 = 10$ total = sum of these = 40 | M1 A1 [2] M1 A1 B1 A1 [4] | Must be ${}_n C_r$ – knows what to do. Ans only is ok for 2 marks. Needs to be a product of ${}_n C_r$'s. Co. Anywhere. Co. |
| 8 | (i) $y = (3x + 11)/(x - 3)$ Makes x the subject. $f^{-1}(x) = (3x + 11)/(x - 3)$ f and f^{-1} are the same functions. \rightarrow Graph has $y = x$ as line of symmetry. (ii) $g(x) = \frac{1}{2}(x - 3)$ $g^{-1}(x) = 2x + 3$ $\rightarrow 2x + 3 = (3x + 11)/(x - 3)$ $\rightarrow 2x^2 - 6x - 20 = 0 \rightarrow x = -2$ or 5 (iii) $gf(x) = -2 \rightarrow f(x) = g^{-1}(-2)$ $\rightarrow x = -2$ | M1 A1 B1 [3] B1 M1 A1 [3] B1 [1] | Good algebra in making x the subject. Co accept any mention of $y = x$. Anywhere. Algebra must lead to quadratic. Co. However obtained. |

| Page 3 | Mark Scheme | Syllabus | Paper |
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| <p>9 (a)</p> $s^2 = 3c^2 + 4s$ <p>Use of $s^2 + c^2 = 1$ $\rightarrow 4s^2 - 4s - 3 = 0 \rightarrow s = -\frac{1}{2}$ or $\frac{3}{2}$ $\rightarrow x = 210^\circ$ and 330°</p> <p>(b)</p> <p>$\cot = 1/\tan$ used $\tan 2y = 4$</p> <p>$2y = 1.326 \rightarrow y = 0.66$ or $2y = \pi + 1.326$ or $2\pi + 1.326$</p> <p>$\rightarrow y = 2.23$ $\rightarrow y = 3.80$ or 3.81</p> | <p>M1 DM1</p> <p>A1 A1√ [4]</p> <p>M1</p> <p>A1</p> <p>A1√ A1√ [4]</p> | <p>Used to eliminate cos completely. Scheme for quadratic.</p> <p>Co. √ for 2nd value from incorrect sine. (A1√ not given for extra values <u>in the range</u>, but could be given if soln of quadratic led to 2 values of sine < 1.)</p> <p>Use of $\cot = 1/\tan$ even if “2” removed incorrectly. Not for tan and 2y split. Co (must be radians) – not for 0.67.</p> <p>For (i) + $\frac{1}{2}\pi$ For (i) + π or (ii) + $\frac{1}{2}\pi$</p> <p>[S–1 for extra values in the range] [sc All answers in degrees B1.]</p> |
| <p>10</p> <p>(i)</p> $y = x^3 \ln x$ $dy/dx = 3x^2 \ln x + x^3 (1/x)$ $= 3x^2 \ln x + x^2$ <p>(ii)</p> $dy/dx = 0 \quad \ln x = -\frac{1}{3}$ <p>(iii)</p> $\delta y = dy/dx \times \delta x = (e^2 + 3e^2)p$ $= 4e^2 p \text{ or } 29.6p$ <p>(iii)</p> $d/dx (x^3 \ln x) = x^2 + 3x^2 \ln x$ <p>Integrating $\rightarrow x^3 \ln x = \frac{1}{3}x^3 + \int 3x^2 \ln x dx$ $\int x^2 \ln x dx = \frac{1}{3}(x^3 \ln x - \frac{1}{3}x^3)$</p> | <p>M1</p> <p>A1 [2]</p> <p>M1 A1 [2]</p> <p>M1 A1 [2]</p> <p>M1 A1 [3]</p> | <p>M1 correct “uv”. A1 ok unsimplified.</p> <p>Not DM – setting his dy/dx to 0 + attempt to solve.</p> <p>Use of small increases. Allow for use of dy/dt. $\Delta x = p$ essential for M mark. Alg expression with “p” ok for M1.</p> <p>\int is reverse of diff used. A1 needs $\frac{1}{3}x^3$ co</p> <p>Integration by parts ok. M1 A1 A1.</p> |
| <p>11</p> $4y = 3x + 1 \text{ and } xy = 28x - 27y$ <p>Sim equations. $\rightarrow x^2 - 10x + 9 = 0$ or $y^2 - 8y + 7 = 0$ $\rightarrow (9, 7)$ [(1, 1) was given]</p> <p>P(1, 1), Q(9, 7) \rightarrow gradient of PQ = $\frac{3}{4}$ Gradient of perp bisector is $-\frac{4}{3}$ M (mid-point of PQ) = (5, 4)</p> <p>Eqn of perp bis. $y - 4 = -\frac{4}{3}(x - 5)$ $3y + 4x = 32$ meets $y = 4x$ at R(2, 8)</p> <p>Area of $\Delta PQ = \frac{1}{2} \times PQ \times MR$ $= \frac{1}{2} \times 10 \times 5$ (or matrix method) $\rightarrow 25$</p> | <p>M1 DM1</p> <p>A1</p> <p>M1 M1</p> <p>A1 M1</p> <p>M1</p> <p>A1 [9]</p> | <p>Complete elimination of x or y Soln of quadratic (by scheme)</p> <p>Co for (9, 7)</p> <p>Use of $m_1 m_2 = -1$ with his PQ Use of $(\frac{1}{2}(x_1 + x_2), \frac{1}{2}(y_1 + y_2))$</p> <p>Co – unsimplified ok. Simultaneous eqns. Must be with a perp Any correct method.</p> <p>Co.</p> |

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| 12 | <p>EITHER</p> <p>(a) $N = 20\,000e^{-0.05n}$</p> <p>(i) $n = 10, N = 12\,130$ or more places.</p> <p>(ii) $2000 = 20\,000e^{-0.05n}$ $e^{-0.05n} = 0.1$</p> <p>Take logs $n = 45.1 \rightarrow 2006$</p> <p>(b) Put $y = 3^x$ $3^{x+1} = 3y$ or $3^{x-1} = \frac{1}{3}y$ $3y - 2 = 8y/3 \rightarrow y = 6$ $3^x = 6, x \lg 3 = \lg 6 \rightarrow x = 1.63$ [or \div by 3^{x-1} M1 A1 $3^{x-1} = 2$ M1 A1] [or \div by 3^{x+1} M1 A1 $3^{x+1} = 18$ M1 A1]</p> | <p>B1 [1]</p> <p>M1</p> <p>M1 A1</p> <p>[3]</p> <p>M1 A1 A1 A1</p> <p>M1 A1</p> <p>[6]</p> | <p>Co</p> <p>Isolating exponential – or taking logs to get 3 terms.</p> <p>Taking logs. Co. needs 2006, not 2005.</p> <p>Used. For each expression. Co.</p> <p>Taking logs for his 3^x. co.</p> |
| 12 | <p>OR</p> <p>$y = e^{\frac{x}{2}} + 3e^{-\frac{x}{2}}$</p> <p>(i) $dy/dx = \frac{1}{2}e^{\frac{x}{2}} - \frac{3}{2}e^{-\frac{x}{2}}$ $= 0$ when $e^x = 3$</p> <p>$y = \sqrt{3} + 3 \div \sqrt{3} = 2\sqrt{3}$</p> <p>(ii) $d^2y/dx^2 = \frac{1}{4}e^{\frac{x}{2}} + \frac{3}{4}e^{-\frac{x}{2}} > 0$, MIN</p> <p>(iii) $\int \left(e^{\frac{x}{2}} + 3e^{-\frac{x}{2}} \right) dx = 2e^{\frac{x}{2}} - 6e^{-\frac{x}{2}}$ [] at 1 – [] at 0 $= 4 + 2\sqrt{e} - 6\sqrt{e} = 3.66$</p> | <p>B1 B1</p> <p>M1</p> <p>A1</p> <p>[4]</p> <p>M1 A1</p> <p>[2]</p> <p>M1 A1</p> <p>M1 A1</p> <p>[4]</p> | <p>Anywhere –</p> <p>Setting his dy/dx to 0 and reasonable attempt at making e^x the subject. Co. Decimal check – A0.</p> <p>M1 Reasonable attempt by any method. A1 Correct deduction but needs second differential correct.</p> <p>Knowing to integrate for area + any attempt with exponentials. A1 co. DM0 if [] at 0 is ignored. Co.</p> |

DM1 for quadratic equation.

Formula Equation must be set to 0. Formula must be correct and correctly used, but allow for numerical and algebraic errors.

Brackets Equation must be set to 0. Must be an attempt to get two linear brackets. Each bracket must then be equated to 0 and solved.