ADDITIONAL MATHEMATICS

Paper 4037/01

Paper 1

General comments

The paper was taken by candidates who gained a wide range of marks. There were many scripts which scored very highly, showing a good understanding of the syllabus and how to apply the techniques required both appropriately and correctly. However, there were also many candidates who gained marks in single figures only, who clearly found the paper extremely difficult.

Questions 4(ii), 5 and **6** caused the most problems for even the strongest candidates.

Comments on specific questions

Question 1

For the very first question on this paper candidates' attempts were surprisingly laboured in many cases. Those candidates who knew exactly what to do managed it in less than half a page but very often an answer was not reached despite laboured calculations occupying up to two sides.

One reason for either an incorrect or no solution at all was the fact that many candidates were unable to square a matrix. Some candidates simply squared each number giving no evidence of having tried to multiply two matrices together, whilst others did write the matrix **A** out twice and attempt unsuccessfully, using an incorrect form of row and column multiplication, to multiply out.

Another, more common, reason was the misuse of the identity matrix **I**. Many candidates reversed the zeros and ones whilst others spent a disproportionate amount of time finding and trying to solve the given equation using the inverse of matrix **A** instead of **I**.

Answers: $m = –3$, $n = –5$.

Question 2

Errors made were usually in the manipulation rather than knowledge of cot and cosec. Many candidates appear to need further practice particularly in deciding what they are trying to show and sorting out the terms appropriately. Most solutions started using the left hand side of the identity. Those that started from the right hand side were usually unable to produce the appropriate partial fractions required in the final part of the solution.

When asked to show something as in this question, candidates need to demonstrate the result to the satisfaction of the Examiner. Very often candidates got as far as $\frac{2\cos A}{\sin^2 A}$ and went straight to $2\text{cosec}A\cot A$. In this question, where the candidate is asked to show the given result, an Examiner could not be certain that the candidate understood the appropriate trigonometric relationships without an intermediate step.

Question 3

(i) A number of candidates lost marks because they did not express the answer in its simplest form. Too many candidates failed to rationalise correctly, using $\left(\sqrt{3}-1\right)$, rather than $\left(\sqrt{3}+1\right)$.

(ii) Some candidates went back to the original value of p, some used their answer to part **(i)** but some used a combination of the original value of p and their own answer to part **(i)**. They could then find a common denominator before rationalising or the other way round. This led to excessive calculation and generally an incorrect conclusion. Sometimes after a fair amount of working the rationalising of the denominator was not done leading to no marks awarded. Very few candidates completed this second part in a concise enough way. More often than not even if the question was successfully answered the calculation was far too circuitous because of a lack of manipulative skills and took up far too much of the candidate's valuable time.

Answers: **(i)** $2+\sqrt{3}$; **(ii)** $2\sqrt{3}$.

Question 4

Many candidates were not able to complete any part of this question correctly.

(i) Whilst many candidates realised that ${}^{9}C_{4}$ and ${}^{6}C_{4}$ were needed, the two combinations were usually added rather than multiplied.

(ii) Again the problem that the majority of candidates had, even though some had an idea of what was needed, was adding their combinations rather than multiplying.

Answers: **(i)** 1890; **(ii)** 1050.

Question 5

Very few completely correct solutions to this question were seen and it was not attempted at all by significant minority.

(i) The most common error was to combine both the two vectors without any reference to the time difference. A number of candidates reached 240**i** + 100**j**, the resultant velocity, and stopped there. The other major error was to add the wind vector to the velocity vector. There was evidence that some candidates were not able to cope with the negative component of the wind vector.

(ii) Many candidates chose to use $\tan\theta = \frac{900}{460}$ or similarly incorrect ratios in an attempt to get a bearing, clearly indicating that this topic is one which caused candidates difficulties.

Answers: **(i)** 300**i** + 40**j**; **(ii)** 082°.

Question 6

Very few completely correct solutions to this question were seen.

(i) Most candidates seemed unable to understand what was being requested of them. The majority of candidates used the given expression as the gradient in a straight line equation, often evaluated at (6, 20) but sometimes not. When integration was attempted, there were various inventive applications of the quotient rule, product rule or a mixture of both applied incorrectly to integration.

(ii) This part was attempted by most with about as much success as in the previous part. Most candidates realised that a gradient of 2 was involved, but were either unable to find the appropriate value of x and of y, or if they did, they failed to find the points where the normal met the coordinate axes.

Answers: **(i)** $y = 3\sqrt{4x+1}+5$; **(ii)** (0, 15), (30, 0).

Question 7

(i) Most candidates were able to make a reasonable attempt at the use of the given substitution and obtain a quadratic equation in *u*. Many went on to correctly obtain the solution to a power equation, but also tried to solve equations of the type $2^x = k$ with *k* being negative, introducing a spurious solution.

(ii) Many candidates were also able to make a reasonable attempt at this part of the question. Some candidates, although they were able to correctly evaluate $2\log_9 3$ and $\log_2 8$, had problems with the resulting equation and were unable to manipulate the logarithm correctly.

Answers: **(i)** 2.32; **(ii)** 4.

Question 8

Overall, this was a well answered question with many candidates gaining full marks.

(a) Some candidates failed to realize that they had to substitute both $x = 1$ and $x = 2$ into the given expression. However, the vast majority of candidates succeeded in doing this and then multiplied one of the two remainders by 4 to get either 47 (incorrect), or 32 (correct). Most candidates were able to achieve full marks.

(b) The factor ($x + 2$) was found by most candidates and an attempt made to divide the expression to find the quadratic factor. The most common error at this stage was to find a quadratic factor of $x^2 - 2x + 4$ and then to use the quadratic formula. Most candidates found the correct quadratic factor of $x^2 - 6x + 4$ and then went on to use the quadratic formula to find the other two solutions. However, although many candidates gained the three method marks, the accuracy marks were often lost because firstly candidates omitted to include the root $x = -2$ and secondly they failed to simplify the surd result far enough, even though guidance was given in the question

Answers: **(a)** 32; **(b)** -2, $3 \pm \sqrt{5}$.

Question 9

(i) Most candidates were able evaluate *xy* and x^2 and draw an appropriate straight line graph. However, there were some candidates who chose to use a non-linear scale on their graph which was deemed to be inappropriate.

(ii) Calculating the gradient of the resulting straight line graph and the intercept on the *y*-axis (or an equivalent method using substitution) proved to be straightforward for most candidates. Problems arose when candidates attempted to rewrite their $y = mx + c$ equation obtained for their straight line graph in the form $xy = mx^2 + c$ and from there to the form requested in the question

(iii) For most candidates finding the value of *y* for which $y = \frac{83}{x}$ proved difficult. Instead of using their graph together with $xy = 83$ and obtaining a value for x^2 and hence *y*, most candidates chose to substitute into an incorrect equation.

Answers: **(ii)** $y = 1.2x + \frac{24}{x}$; **(iii)** a value between 11.6 and 12.2.

Question 10

Nearly all candidates made some attempt at this question and for some weaker candidates it was their best attempted question.

(i) Candidates used various methods, the cosine rule, the sine rule, or basic trigonometry in the form of $2 \times 10 \sin 0.4$, the cosine rule being the most popular method. Premature rounding was a problem, whatever method candidates employed.

(ii) Some candidates caused difficulties for themselves by rounding 1.17 radians to 1.2 radians or by converting to degrees. Others confused degree methods with radian methods and arc length with area. There were some miscalculations of the angle *ABC*. The majority of candidates were able to calculate an arc length, but some candidates were unable to formulate a good plan and included extra parts and some incorrect subtractions. There were, however, many correct answers.

(iii) Many of the same problems that occurred in part **(ii)** also occurred in this part. Many candidates used $\frac{1}{2}r^2(\theta - \sin\theta)$ for either 10 or 7.79 without further work, while some candidates missed the 0.5 from the formula for sector area. There were again, however, many correct solutions.

Answers: **(ii)** 24.9 cm; **(iii)** 39.6 or 39.7 cm^2.

Question 11 EITHER

The great majority of candidates opted to do this alternative.

(i) Most candidates realised that they had to make use of the product rule when differentiating for the first time. Those that were unable to differentiate the exponential term correctly were still able to score for a correct method used, but were ultimately penalised later in the question. The application of the product rule for the second derivative seemed to cause more problems with many candidates omitting terms.

(ii) Most were able to equate the derivative obtained in part **(i)** to 0 and attempt to solve, although many candidates attempted to obtain a solution for the exponential term equated to 0.

(iii) Most candidates were able to apply a valid method for the determination of the type of turning point although there were some who chose to state the type of turning point with no attempt to justify their conclusion.

Answers: **(i)** $e^{2x} + 2xe^{2x}$, $4e^{2x} + 4xe^{2x}$; **(iii)** Minimum.

Question 11 OR

(i) Most candidates realised that differentiation of a quotient was involved and were able to make reasonable attempts at manipulation of the result, although there were some contrived results.

(ii) Most candidates were able to equate the derivative obtained in part **(i)** to 0 and attempt to solve the resulting equation. There were again instances of contrived answers from candidates who were unable to deal with logarithms correctly.

(iii) There were very few correct solutions to this part of the question. Most candidates did not attempt it. Those that did invariably failed to realise the connection with the previous part of the question and produced work of little or no merit.

Answer: **(iii)** $-\frac{1}{4x^2} - \frac{\ln x}{2x^2} + c$ or equivalent.

ADDITIONAL MATHEMATICS

Paper 4037/02

Paper 2

General comments

Candidates in some Centres produced high quality work displaying wide ranging mathematical skills, with well presented and clearly organised answers, but these were in a minority. However there were quite a substantial number of candidates who scored single figure marks and found the paper very difficult.

Amongst candidates with some of the requisite knowledge and skills, **Questions 1**, **5**, **7**, and **10** were found to be the most straightforward. It was very common for low marks to be obtained in **Question 6**. Full marks were rarely obtained on **Questions 2**, **3**, **4**, **8**, **9** and **11**. In **Question 8** almost all candidates assumed that integers alone were involved. In **Question 11** some candidates used unnecessarily long methods, especially in part **(iii)**.

Comments on specific questions

Question 1

There was a mixed response to this question. Able candidates had little difficulty with either part and a significant number of candidates were successful in part **(i)** but did not realise the relevance of 'as x increases from 10 to 10 + p' and left x in their answer. Quite often the last two marks were awarded after an error in **(i)**. One common error was to get an expression of the form kx^{-1} for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Many weaker candidates got into a complete mess with poor attempts at the product/quotient rule.

Answers: **(i)** $-\frac{1600}{x^3}$ **;** **(ii)** $-1.6p$.

Question 2

This question was generally poorly attempted. All sorts of poor algebra and technique were seen. Many candidates tried to 'expand' the bracket to get $\sin\left(\frac{x}{2}\right) - \sin 1 = \frac{1}{3}$ and others worked in degrees. Many who did get to 0.34… surprisingly were not able to go on and find x. The second value was not well dealt with and all sorts of mixtures were seen, e.g. $180 - 0.34$, $\pi \pm 0.34$, $2\pi \pm 0.34$ and other similar combinations. Some of the more able candidates did not manage, or attempt, to find the solution 15.2.

Answers: 2.68, 7.60, 15.2.

Question 3

Parts **(i)** and **(ii)** were generally done well although $9^{x+1} = 3^{2(x+1)}$ was often equated with 3^{2x+1}. Part **(iii)** proved beyond most candidates. Even some strong candidates 'cancelled' the 3^{2x} on the top line with just one 3^{2x} from the bottom line.

Answers: **(i)** 3^{2x+2}; **(ii)** 3^{2x}; **(iii)** $\frac{2}{3}$.

Question 4

Most candidates found this question difficult and very few candidates were able to give the three correct matrices in the right order. Amongst those with some idea of what to do, most success was achieved in putting together matrices for the numbers of models sold and the cost of each model. Unfortunately this would often be spoiled by presenting with these an incompatible matrix for the percentages or a matrix for the week numbers such as (1 2 3 4) either horizontally or vertically. Few seemed to really understand the rule for multiplying matrices using rows and columns, sometimes getting a (1 by 1) matrix from a (3 by 1) × (1 by 3), etc. There were some candidates with some understanding of the problem who broke it up into separate weeks or pairs of weeks to such an extent that, whilst ostensibly being presented in matrix form, matrix algebra was not really being used.

Answers: **(i)** $\begin{pmatrix} 0.3 & 0.3 & 0.2 \end{pmatrix} \begin{pmatrix} 8 & 12 & 4 \\ 7 & 10 & 2 \\ 10 & 12 & 0 \\ 6 & 8 & 4 \end{pmatrix} \begin{pmatrix} 300 \\ 500 \\ 800 \end{pmatrix}$ (for example); **(ii)** \$9690.

Question 5

Some candidates understood what was required in this question and produced completely correct solutions. However the majority scored part marks, some did not make any attempt to simplify $\left(\sqrt{2}\right)^3$ or $\left(\sqrt{2}\right)^5$, and some of these resorted to their calculator. A surprising number only gave the first 4 or 5 terms of the expansion in part **(i)**; this meant they were unable to get a full solution in the remainder of the question.

Answers: **(i)** $1+5x+10x^2+10x^3+5x^4+x^5$; **(ii)** $41+29\sqrt{2}$; **(iii)** 82.

Question 6

Candidates tended to make either very good or very poor attempts at this question. The vast majority simply did not realise it was a problem with two variables and simply wrote down expressions such as $\pi r^2+\pi r^2=\frac{29\pi}{2}$ and $2\pi r+2\pi r=10\pi$ leading to values of the radii of 2.5 m and 2.69 m. Those candidates who started correctly often scored full marks. For those who did not, it was mainly due to poor technique, with candidates writing $y=\frac{10\pi-2\pi x}{2\pi}$ and attempting to square that or substituting for π and getting decimals and equations with inaccuracies.

Answer: 1.5 m and 3.5 m.

Question 7

For those who had clearly studied vectors this was a reasonably straightforward question, and some good solutions were seen. However, even those who knew the vector methods often made errors in basic algebraic manipulation.

(i) Most candidates could find $\overrightarrow{BC}$ or $\overrightarrow{CB}$ though not all knew how to find the distance. Those who did could not always cope with squaring or taking the square root successfully. Two values were sometimes not found, though in this case the candidate usually picked the correct one. Other candidates made no further progress after writing $k^2-12k-364=0$.

(ii) The majority of candidates used the gradient of line, but not always successfully. Those using ratio of vectors sometimes confused the ratio and k, unfortunately using the same symbol k as in the question for their constant multiplier.

Answers: **(i)** 26; **(ii)** 16.

Question 8

Many candidates misunderstood the universal set and thought that x could only take integer values. The most common set of answers from able candidates was $x < 6$, followed by critical values of 4 and 7 and then $A \cap B$ = {5} and $(A \cup B)'$ = {7, 8, 9}. Of those who did use the correct universal several gave the answer to the last part as $7 < x < 10$ or simply $7 \varnothing x$.

Answers: **(i)** $4 < x < 6$; **(ii)** $7 \varnothing x < 10$.

Question 9

Those that realised that calculus was needed were usually able to gain the method marks. Obtaining the correct numbers proved much more difficult. Many candidates used degrees in both parts. The majority used degrees throughout.

(i) Most candidates knew to differentiate and $\sin\left(\frac{t}{2}\right)$ was often seen, but often the constant it was multiplied by was incorrect.

(ii) Not as many candidates knew to integrate and those that did were not as successful as in part **(i)**. t was not always found and, when it was, it was very often in degrees and not always correct. The weakest candidates sometimes 'simplified' the function at the outset to, for example, $4\cos t$.

Answers: **(i)** -1.92 ms^{-2}; **(ii)** 16 m.

Question 10

Many candidates scored full marks on this question.

(i) A mistake in finding X was to square incorrectly to get $4x - x^2 = 0$, giving an answer of (4, 0). Some candidates then went on to find M as (4, 4), which did not seem to worry them. Most candidates who attempted the question realised that to find M they had to differentiate and let $\frac{dy}{dx} = 0$. It was pleasing to see how many got the correct differential, though then solving the equation was not always straightforward.

(ii) Again it was pleasing to see how many got the right answer, or at least used the right method. A variety of incorrect limits from part **(i)** inevitably caused errors in the final answer. Other errors included not dealing with the $\frac{3}{2}$ correctly or omitting the $\frac{1}{2}$ in front of the x^2. Some candidates found the correct integral but did not deal with the arithmetic correctly when using the limits.

Answers: **(i)** (16, 0), (4, 4); **(ii)** $42\frac{2}{3}$.

Question 11

This question was quite well done on the whole, though candidates displayed poor technique on many occasions, often producing complicated equations involving decimals, when there was a neat method using a vector approach.

(i) Most candidates were successful in finding the equation of OB.

(ii) All sorts of methods were used for this part. Many who used the determinant method missed out the crucial factor of $\frac{1}{2}$, or even forgot that the beginning and end point in the determinant have to be the same. This method also led to some confusion in the signs.

(iii) Surprisingly few candidates could write *C* straight down. Many just divided (6, –3) by 3 to get (2, –1), even though the diagram showed *C* was in the second quadrant. Others tried to use simultaneous equations , such as $\sqrt{x^2+y^2}=\sqrt{5}$ (or the decimal equivalent) with their equation for *AC*. Sometimes candidates reduced the first equation to $x + y = 5$ (or equivalent).

(iv) Methods for the area were generally satisfactory, although again the determinant often had the factor $\frac{1}{2}$ missing. There were some elegant answers, but most were laborious often leading to some inaccuracy.

Answers: **(i)** $y = 2x$; **(ii)** (2, 4) **(iii)** (–2, 1); **(iv)** 40 units^2.

Question 12 EITHER

The first three parts of this question were not understood by a large proportion of the candidates. In parts **(i)** and **(ii)** very few candidates seemed to know what a range was; some confused it with domain. Quite a few candidates wrote the inverse of f as $\frac{x}{\ln}$, and inevitably stalled at that point. Parts **(iv)** and **(v)** produced more marks but the logarithmic and exponential functions were not well understood. It was not unusual to see ln alone or $e^x\left(\frac{x-2}{3}\right)$.

Answers: **(i)** all real values; **(ii)** all positive real values; **(iv)** $\frac{e^3-2}{3}$ or 6.03; **(v)** $2+3\ln 7$ or 7.84

Question 12 OR

This alternative was answered more frequently than **Question 12 EITHER**.

(**i**) Most candidates eliminated *y* and rearranged, not always successfully, but on the whole very well. Use of $b^2 - 4ac = 0$ was often correct but a few candidates did not understand the principle and used inequalities. Only a few used differentiation and, in general, they did not know what to do with the resulting equation.

(ii) A few candidates knew what they were doing but many could not cope with *a* not being 1. Those who could often obtained $c = \frac{11}{16}$.

(iii) Those using the formula were most successful here, realising and explaining well that no solution meant the curve did not cross *x*-axis. Other attempts were poor and badly explained.

(iv) This part was seldom answered correctly. Often there was a lot of working to find the inverse function without any conclusion.

Answers: **(i)** –2 or 6; **(ii)** $4\left(x+\frac{1}{4}\right)^2+2\frac{3}{4}$; **(iv)** $-\frac{1}{4}$.