UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2012 question paper

for the guidance of teachers

4037 ADDITIONAL MATHEMATICS

4037/12 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2012 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

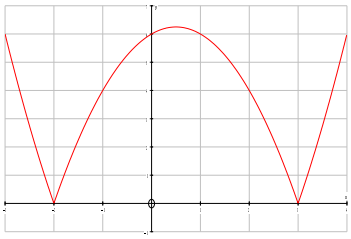
PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1	**(i)** $\frac{2}{21}(7x-5)^{\frac{3}{2}} \quad (+c)$	**B1** **B1, B1**	**B1** for multiplication by $\frac{2}{3}$, or div **B1** for $(7x-5)^{\frac{3}{2}}$, **B1** for $\frac{1}{7}$
	(ii) $\frac{2}{21}\left(16^{\frac{3}{2}}-9^{\frac{3}{2}}\right) \quad \left(=\frac{2}{21}(64-27)\right)$ $=\frac{74}{21}$ or awrt 3.52 or $3\frac{11}{21}$	**M1** **A1** **[5]**	**M1** for correct use of limits, must have attempted integration, must be using their $(7x-5)^{\frac{2n+1}{2}}$ from **(i)**
2	$4u^2 - 5u + 1 = 0$ $(4u - 1)(u - 1) = 0$ or $(4.2^x - 1)(2^x - 1) = 0$ $2^x = \frac{1}{4}, \quad 2^x = 1$ leading to $x = -2, 0$ Alternate scheme for one correct factor: $2^x = \frac{1}{4}$, leading to $x = -2$ $2^x = 1$, leading to $x = 0$	**B1, M1** **DM1** **A1** **A1** **[A1]** **[A1]** **[5]**	**B1** for $2^{2x+2} = 4u^2$ or 4×2^{2x} or $2^2 \times 2^{2x}$ or 2^2u^2 **M1** for attempt to obtain a 3 term quadratic equation in terms of either or, equated to zero. **DM1** for solution of quadratic equation **A1** for both **A1** for both
3	$\frac{\cos A}{\sin A}+\frac{\sin A}{1+\cos A}$ $=\frac{\cos A+\cos^2 A+\sin^2 A}{\sin A(1+\cos A)}$ $=\frac{(1+\cos A)}{\sin A(1+\cos A)}$ $=\frac{1}{\sin A}$ $\quad =$ cosecA Alternate solution: $\cot A+\frac{\sin A(1-\cos A)}{(1+\cos A)(1-\cos A)}$ $=\cot A+\frac{\sin A(1-\cos A)}{\sin^2 A)}$ $=\cot A+\frac{1-\cos A}{\sin A}$ $=\cot A-\cot A+\frac{1}{\sin A}$ leading to cosec$A$	**B1** **M1** **M1** **A1** **[M1]** **[M1]** **[M1]** **[A1]** **[4]**	**B1** for $\cot A = \frac{\cos A}{\sin A}$ **M1** for obtaining as a single fraction **M1** for use of $\cos^2 A + \sin^2 A = 1$ **A1** for correct simplification – answer given. **M1** for multiplying by $(1 - \cos A)$ **M1** for use of $\cos^2 A + \sin^2 A = 1$ anywhere **M1** for cancelling $\sin A$ **A1** for subtraction and simplification

4	Using $y = \frac{2-5x}{3}$ or, using $x = \frac{2-3y}{5}$	**M1**	**M1** for substitution to get an equa of one variable
	$5x^2 - 21x + 4 = 0$ or $3y^2 + 17y - 6 = 0$	**M1**	**M1** for attempt to form a 3 term quadra equation = 0
	$(5x - 1)(x - 4) = 0$ or $(3y - 1)(y + 6) = 0$	**DM1**	**DM1** for solution of quadratic equation
	$x = \frac{1}{5}$, $y = \frac{1}{3}$ $x = 4$, $y = -6$	**A1, A1** **[5]**	**A1** for each 'pair'
	Alternate substitutions: $x = \frac{2y}{3+y}$ or $y = \frac{3x}{2-x}$		
5	**(i)** $(2-x^2)\frac{3}{(3x+1)} - 2x\ln(3x+1)$	**B1** **M1** **A1**	**B1** for differentiating $\ln(3x + 1)$ correctly **M1** for correct attempt at product **A1** for all else correct
	(ii) $\frac{5x(-2\sec^2 2x) - 5(4 - \tan 2x)}{25x^2}$	**B1** **M1** **A1**	**B1** for differentiating $\tan(4 - 2x)$ correctly **M1** for correct attempt at quotient or product **A1** for all else correct
	or $\frac{5x(-2\sec^2 2x) - 5(4 - \tan 2x)}{(5x)^2}$	**[6]**	
6	**(i)** $\frac{8(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = 4(\sqrt{3}-1)$	**M1**	**M1** for rationalisation or attempt to form equation
	or $\frac{8}{\sqrt{3}+1} = a(\sqrt{3}-1)$, $8 = a\ (\sqrt{3}-1)(\sqrt{3}+1)$		
	$a = 4$	**A1**	
	(ii) $\sin 60 = \frac{\sqrt{3}}{2} = \frac{h}{4(\sqrt{3}-1)}$	**M1**	**M1** for use of sine or tangent and their value of a from **(i)** or $\frac{8}{\sqrt{3}+1}$
	$\tan 60 = \sqrt{3} = \frac{h}{2(\sqrt{3}-1)}$		
	Or $(4(\sqrt{3}-1))^2 = h^2 + (2(\sqrt{3}-1))^2$	**A1**	or Pythagoras, **A1** for rearranging and simplifying correctly to obtain given answer.
	$h = 6 - 2\sqrt{3}$ ANSWER GIVEN		
	(iii) Area $= \frac{1}{2}4(\sqrt{3}-1)(6-2\sqrt{3})$	**M1**	**M1** for valid method for area using their a from **(i)** or $\frac{8}{\sqrt{3}+1}$
	or $\frac{1}{2}4(\sqrt{3}-1)4(\sqrt{3}-1)\sin 60°$		
	$= 16\sqrt{3} - 24$	**A1** **[6]**	**A1** working must be seen

7	**(i)** 	**B1** **B1** **B1**	**B1** for shape **B1** for $x = -2, 3$ **B1** for $y = 6$
	(ii) $x^2 - x - 6 = 6$, leading to $x = -3, 4$ (www) $x^2 - x - 6 = -6$, leading to $x = 0, 1$ (www)	**B1** **B1** **B1** **[6]**	**B1** for one correct answer **B1** for a second correct answer **B1** for a third and fourth correct answer
8	**(i)** arc $AB = \frac{20\pi}{3}$ or 20.94, 20.9 $\tan\frac{\pi}{3} = \frac{AX}{10}$, AX = $10\sqrt{3}$, 17.3 (or XB) Perimeter = awrt 55.6 or $20\sqrt{3} + \frac{20\pi}{3}$	**B1** **B1** **B1**	**B1** for arc length correct **B1** for AX/XB **B1** for final answer
	(ii) Area of sector $AOB = \frac{1}{2}10^2\frac{2\pi}{3}$ or 104.7 or 105 Area of $OAXB = 100\sqrt{3}$ or 173.2 Shaded area = awrt 68.5 or $100\sqrt{3} - \frac{100\pi}{3}$	**B1** **M1** **M1** **A1** **[7]**	**B1** for sector area correct **M1** for valid attempt at area $OAXB$, using their BX from part **(i)** ($10 \times$ their BX) **M1** for area $OAXB$ – sector area used (independent) Must be considering a quadrilateral, not a triangle.
9	**(i)** 250	**B1**	**B1** for 250
	(ii) $8 = e^{\frac{x}{100}}$ $\frac{x}{100}$ = ln 'their 8' or $x = 100$ ln their 8 $x = 208$ or awrt 208	**B1** **M1** **A1**	**B1** for $8 = e^{\frac{x}{100}}$ **M1** for dealing with e correctly, using ln **A1** for awrt 208
	(iii) $\frac{dN}{dx} = \frac{1}{2}e^{\frac{x}{100}}$ $45 = \frac{1}{2}e^{\frac{x}{100}}$ $e^{\frac{x}{100}} = 90$, so $N = 4700$ (awrt 4700)	**B1, B1** **M1** **A1** **[8]**	**B1** for $e^{\frac{x}{100}}$, B1 for $\frac{1}{2}e^{\frac{x}{100}}$ or $\frac{50}{100}e^{\frac{x}{100}}$ **M1** for equating their $\frac{dN}{dx}$ to 45 and attempt to solve **A1** for 4700

10 (a) (i) $f'(x) = -(2+x)^{-2}$	**B1**	First **B1** may be implied by a co for $f''(x)$
$f''(x) = 2(2+x)^{-3}$	**B1**	If done by quotient rule, allow unsimp
(ii) $y = \frac{1}{2+x}$, $x = \frac{1}{y} - 2$	**M1**	**M1** for a valid attempt at the inverse
$f^{-1}(x) = \frac{1}{x} - 2$ or $\frac{1-2x}{x}$	**A1**	**A1** must be in correct form, allow $y = \ldots$
(iii) $f^2(x) = \left(\frac{1}{2+\frac{1}{2+x}}\right) = \frac{2+x}{5+2x}$	**M1**	**M1** for correct attempt at $f^2(x)$
	DM1	**DM1** for attempt at solution of $f^2(x) = -1$
Equating to – 1 leads to $x = -\frac{7}{3}$ or –2.33	**A1**	**A1** for $x = -\frac{7}{3}$ or equivalent
(b) (i) gh (x) or gh	**B1**	**B1** for either form
(ii) kg (x) or kg	**B1** **[9]**	**B1** for either form

11 (i) P (3, 1)	**B1, B1**	**B1** for each coordinate
Grad $AB = \frac{18}{12}$	**B1**	**B1** for gradient of AB
$\perp$ grad $-\frac{2}{3}$	√**B1**	√**B1** for perpendicular gradient
PQ: $y - 1 = -\frac{2}{3}(x-3)$ $\quad (2x + 3y = 9)$	√**B1**	√**B1** on their perp gradient and their point P Must be $y = \ldots$
(ii) Q (–15, 13)	**M1**	**M1** for use of $y = 13$ and their PQ equation.
	A1	**A1** for both coordinates (can be implied)
(iii) Area $= \frac{1}{2}\sqrt{18^2+12^2}\ \sqrt{8^2+12^2}$	**M1**	**M1** for a valid attempt at area $\frac{1}{2} \times PQ \times PB$
or Area $= \frac{1}{2}\begin{vmatrix} 3 & 11 & -15 & 3 \\ 1 & 13 & 13 & 1 \end{vmatrix}$		Matrix method using their coordinates correctly
or Area $= \frac{1}{2} \times 26 \times 12$		$\frac{1}{2} \times QB \times$ vertical perp height
$= 156$	**A1** **[9]**	

12 EITHER		
(i) velocity = 12**i** + 16**j** position = (54**i** + 16**j**) + (36**i** + 48**j**) = 90**i** + 64**j** ANSWER GIVEN	**M1** **A1**	**M1** for (3 × their velocity (must in numeric form)) + (54**i** + 16**j**)
(ii) (54**i** + 16**j**) + (12*t***i** + 16*t***j**)	**M1, A1**	**M1** for position vector + (their numeric velocity vector × time)
(iii) At 16 00, ship has 'travelled' (102**i** + 80**j**)	**B1**	**B1** for (102**i** + 80**j**)
boat needs to do this in 2 hours so velocity of boat (51**i** + 40**j**) speed $\sqrt{51^2 + 40^2}$ = 64.8	**M1** **A1**	**M1** for attempt at velocity of boat and speed
(iv) (51**i** + 40**j**) – (12**i** + 16**j**) = 39**i** +24**j**	**B1**	**B1**, allow unsimplified but must be correct
(v) $\tan\alpha = \frac{51}{40}$ angle = 51.9	**M1** **A1** **[10]**	**M1** for use of tan and their velocity vector

12 OR		
(i) $\overrightarrow{OQ}\ \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$	**B1**	Allow unsimplified
$= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$		
$\overrightarrow{PQ} = -\frac{5}{4}\mathbf{b} + \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$	√**B1**	Follow through on their $\overrightarrow{OQ}$, allow unsimplified
$= \frac{2}{3}\mathbf{a} - \frac{11}{12}\mathbf{b}$		
(ii) $\overrightarrow{QR} = \lambda\,\mathbf{a} - (\mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}))$	**M1**	**M1** for λa – their $\overrightarrow{OQ}$
$= \lambda\,\mathbf{a} - \frac{2}{3}\mathbf{a} - \frac{1}{3}\mathbf{b}$	**A1**	**A1** – allow unsimplified
(iii) $\overrightarrow{QR} = \mu(\overrightarrow{PQ} + \overrightarrow{QR})$	**M1**	**M1** for attempt to obtain $\overrightarrow{QR}$ in terms of $\overrightarrow{PQ}$
$(1-\mu)\overrightarrow{QR} = \mu\overrightarrow{PQ}$	**M1**	**M1** for attempt to simplify
$QR = \frac{\mu}{1-\mu}\left(\frac{2}{3}\mathbf{a} - \frac{11}{12}\mathbf{b}\right)$	**A1**	
(iv) Equating **b**'s $-\frac{11}{12}\frac{\mu}{1-\mu} = -\frac{1}{3}$	**M1**	**M1** for equating like vectors and attempt to solve
$\mu = \frac{4}{15}$	**A1**	**A1** for each
$\lambda = \frac{10}{11}$	**A1** **[10]**	