UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8483854260

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/05**

Paper 5 (Core) **May/June 2010**

1 hour

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, highlighters, glue or correction fluid.
You may use a pencil for any diagrams or graphs.

Answer **all** the questions.

You must show all relevant working to gain full marks for correct methods, including sketches.

In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.

At the end of the examination, fasten all your work securely together.
The total number of the marks for this paper is 24.

This document consists of **4** printed pages.

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UNIVERSITY *of* CAMBRIDGE International Examinations

Answer **all** questions.

INVESTIGATION **FERMAT'S LITTLE THEOREM**

The division $46 \div 5$ gives 9 with a remainder of 1.
A method for finding the remainder is

$46 \div 5 = \mathbf{9}.2$
Because $\mathbf{9} \times 5 = 45$, the remainder is $46 - 45 = 1$.

The division $921 \div 7$ gives a remainder of 4.
A method for finding the remainder is

$921 \div 7 = \mathbf{131}.571.....$
Because $\mathbf{131} \times 7 = 917$, the remainder is $921 - 917 = 4$.

The division $2^{11} \div 13$ gives a remainder of 7.
A method for finding the remainder is

$2^{11} \div 13 = \mathbf{157}.5384.....$
Because $\mathbf{157} \times 13 = 2041$, the remainder is $2^{11} - 2041 = 7$.

> Note: To calculate the value of 2^5
> **either** use the appropriate calculator key
> **or** use $2^5 = 2 \times 2 \times 2 \times 2 \times 2$.
> The value of 2^5 is 32.

1 Find the remainder in these divisions.

(a) $57 \div 6$

....

(b) $579 \div 13$

....

(c) $2^5 \div 7$

....

(d) $2^9 \div 17$

....

2 In 1640 the French mathematician Fermat found something interesting about the remainde[…] dividing by a **prime** number. Some of his results are shown in the table below.

Prime	Division	Remainder	Division	Remainder	Division	Remainder
3	$2^2 \div 3$					
5	$2^4 \div 5$		$3^4 \div 5$		$4^4 \div 5$	
7	$2^6 \div 7$	1	$3^6 \div 7$		$4^6 \div 7$	
11	$2^{10} \div 11$		$3^{10} \div 11$		$4^{10} \div 11$	1
	$2^{12} \div 13$					

Complete the unshaded boxes in this table. You may use the space below to show any working.

3 Use the patterns you have found in your table to complete the following statements.

(a) 7^{12} ÷ .... has a remainder of ....

(b) 3^{16} ÷ .... has a remainder of ....

4 From the table $2^6 \div 7$ has a remainder of 1.
This means that $2^6 - 1$ will divide by 7 exactly.
So $2^6 - 1$ has a **prime** factor of 7.

(a) Complete the following statements to show why $7^{12} - 1$ has a prime factor of 13.

.... has a remainder of 1.

This means that .... will divide by .... exactly.

So $7^{12} - 1$ has a **prime** factor of 13.

(b) **Write down** a prime factor of $3^{16} - 1$. ....

The investigation continues on the next page.

5 Complete the general statement below.

$a^{p-1} - 1$ has a prime factor of ....

This is called Fermat's Little Theorem.

6 When $p > 25$ and $a = 3$, write down a statement using Fermat's Little Theorem.

....

7 Write down a prime factor, other than 3, of 4 194 303.
[$2^{22} = 4\ 194\ 304$]

....