

Answer **both** parts A and B.

A INVESTIGATION

T-VALUES (20 marks)

You are advised to spend no more than 45 minutes on this part.

A grid of any length and width 10 is numbered 1, 2, 3,

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

The grid has a letter T placed on it, as shown.

The T has a horizontal bar of length 3 and a vertical bar of length 2.

The T shown is shape 1 because the number in the top left square of the T is 1.

T-values are found using this method.

Method	Calculation of T-value for shape 1
Square the number at the bottom of the T.	$22^2 = 484$
Multiply together the numbers at each end of the horizontal bar.	$1 \times 3 = 3$
Take the second answer from the first answer to find the T-value.	$484 - 3 = 481$

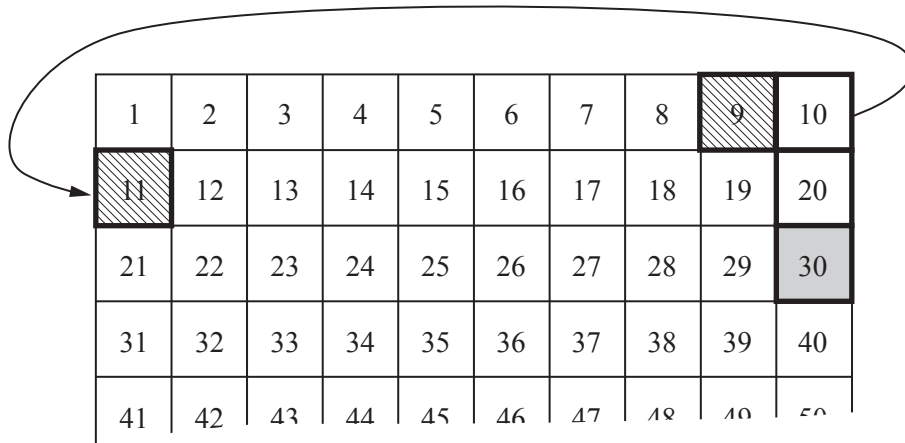
The T-value for shape 1 is $T_1 = 481$.

This investigation is about finding T-values.

1 (a) Complete this table.

Shape number n	Working	T-value T_n
1	$22^2 - 1 \times 3 = 484 - 3$	$T_1 = 481$
2	$23^2 - 2 \times 4 = 529 - 8$	$T_2 = 521$
3		$T_3 =$
4		$T_4 =$
5		$T_5 =$

- (b) When a T is placed at the end of a line, it still has a T-value.
The T “wraps round” like this.



Work out T_9 .

$T_9 = \dots\dots\dots$

- (c) $T_1, T_2, T_3, T_4, T_5, \dots$ form a sequence.
Find a formula, in terms of n , for T_n .

$T_n = \dots\dots\dots$

- (d) When $T_n = 2641$, find the value of n .

$n = \dots\dots\dots$

- (e) Explain why 843 cannot be a T-value.

.....
.....

- 2 The T is now placed on a new grid that is 11 squares wide.

1	2	3	4	5	6	7	8	9	10	11
12	13	14	15	16	17	18	19	20	21	22
23	24	25	26	27	28	29	30	31	32	33
34	35	36	37	38	39	40	41	42	43	44
45	46	47	48	49	50	51	52	53	54	55

- (a) Complete this statement for the numbers in the grid.

In each row the numbers increase by 1 and in each column the numbers increase by

- (b) Complete the squares in this T using expressions in terms of n .

n	$n + 1$	

- (c) Complete this working to show that $T_n = 44n + 529$.
The first line of working is started for you.

$$T_n = (n + \dots)^2 - n(n + \dots)$$

- 3 The T is now placed on a grid that is 12 squares wide.
Find a formula, in its simplest form, for T_n .

$T_n =$

- 4 The T is now placed on a grid that is w squares wide.
 - (a) (i) Show that $T_n = 4w^2 + 4(n + 1)w + 1$.

- (ii) Find the width of the grid when $T_9 = 1501$.

.....

- (b) Use **part (a)(i)** to explain why T_n must always be odd.

.....

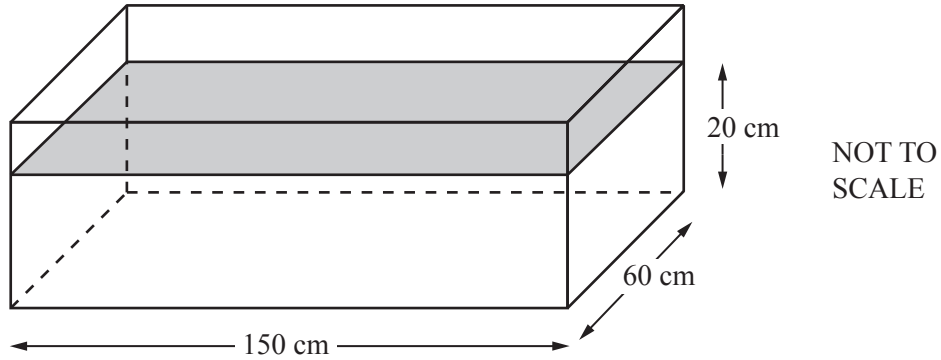
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B MODELLING**FILLING A BATH (20 marks)**

You are advised to spend no more than 45 minutes on this part.

Karen wants to work out how much it would cost to fill a bath every day for 1 year.

- 1 She assumes that a bath is a cuboid that is 150 cm long, 60 cm wide and 30 cm deep.



- (a) Find the number of litres of water needed to fill this bath to a depth of 20 cm.

.....litres

- (b) Water costs 20 cents per 100 litres.

- (i) Find the cost, in dollars, of filling a bath with water to a depth of 20 cm each day for 1 year (365 days).

\$

- (ii) Show that a model for the cost, $\$C$, of filling a bath to a depth, d cm, every day for

$$C = 6.57d.$$

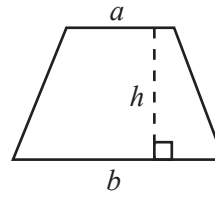
- (iii) In 1 year all baths are filled to the same depth, d cm.
The total cost of the water is $\$157.68$.

Use the model in **part (ii)** to find the value of d .

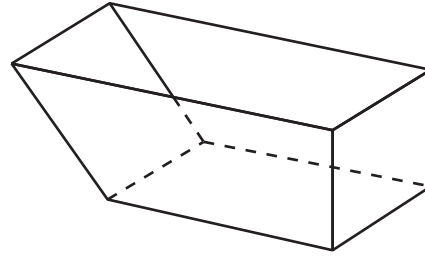
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You may use this information in the remainder of the task.

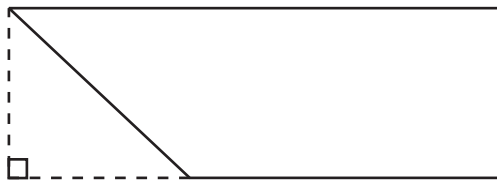
The area, A , of a trapezium is $\frac{h(a+b)}{2}$.



- 2 Karen notices that one end of her bath slopes.

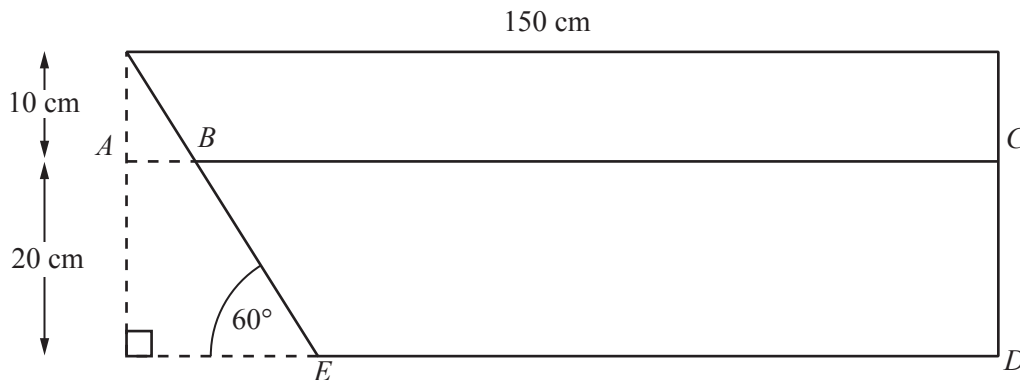


From the side, her bath looks like this.



NOT TO
SCALE

She estimates that the angle of slope is 60° .
The depth of water in the bath is 20 cm.



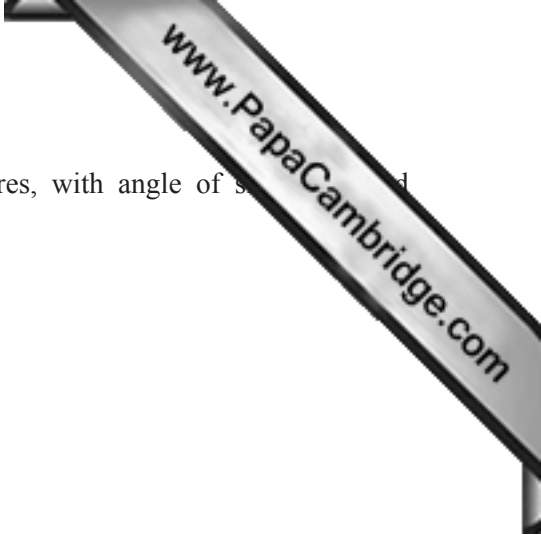
NOT TO
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(a) Show that $AB = 5.77$ cm, correct to 3 significant figures.

(b) $BC = 144$ cm, correct to 3 significant figures.

Find the volume of water in the bath, giving your answer in litres.

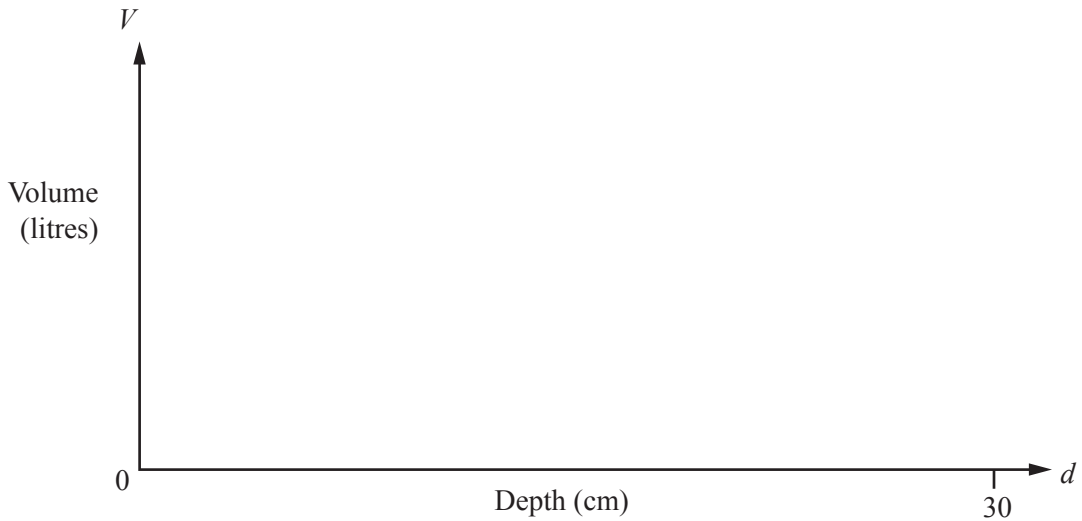
..... litres



- (c) Show that a model for the volume of water in the bath, V litres, with angle of slope 60° and depth d cm is

$$V = 0.03d \left(300 - \frac{(30-d)}{\tan 60^\circ} - \frac{30}{\tan 60^\circ} \right).$$

- (d) On these axes, sketch the graph of V for $0 \leq d \leq 30$.



- (e) Find the depth of water in the bath when the volume is 150 litres.

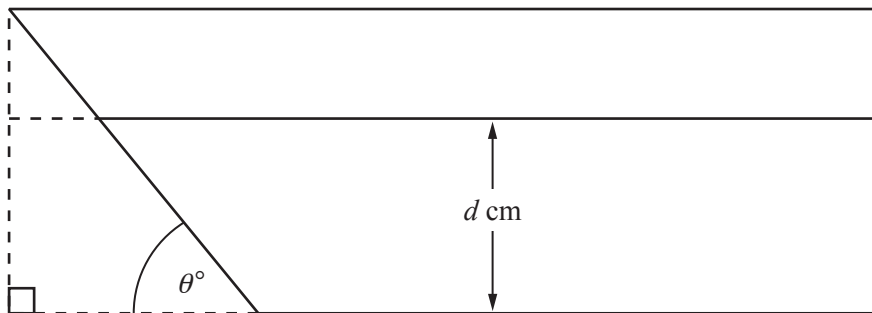
..... cm

- 3 The depth of water in her bath is d cm.
Water costs w cents per 100 litres.

(a) Change the model in **question 2(c)** to find the cost, $\$C$, of filling her bath every day for 1 year.

$C =$

(b) (i) Change your model in **part (a)** to find C when the angle of slope is θ° .



$C =$

Parts (b)(ii), (b)(iii) and (c) are printed on the next page.

- (ii) Describe the effect on the volume of water in her bath when θ decreases.

.....

- (iii) Why is the design of this bath not suitable when θ is small?

.....

- (c) Use your model from **part (b)** to find the cost of filling her bath every day for 1 year when

- $\theta = 50^\circ$
- $d = 25$ cm
- water costs 21 cents per 100 litres.

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