Cambridge International Examinations
Cambridge Pre-U Certificate

MATHEMATICS (PRINCIPAL) **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2015**

2 hours

9919971260

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **3** printed pages and **1** blank page.

1 Show that $\dfrac{31}{6-\sqrt{5}} = 6 + \sqrt{5}$. [3]

2 The gradient of a curve is given by $\dfrac{dy}{dx} = 6x^2 + 2$. The curve passes through the point $(1, 3)$. Find the equation of the curve. [4]

3 The function f is given by $f(x) = |x - 2| + 3$ for $-5 \leqslant x \leqslant 5$.

(i) Sketch the graph of $y = f(x)$. [3]

(ii) Explain why f is not one-one. [1]

4 Find the volume of the solid generated when the region bounded by the x-axis, $x = 1$, $x = 2$ and the curve given by $y = x^3$ is rotated through 360° about the x-axis. [4]

5 **(i)** Show that the equation $\sin x - x + 1 = 0$ has a root between 1.5 and 2. [2]

(ii) Use the iteration $x_{n+1} = 1 + \sin x_n$, with a suitable starting value, to find that root correct to 2 decimal places. [3]

(iii) Sketch the graphs of $y = \sin x$ and $y = x - 1$, on the same set of axes, for $0 \leqslant x \leqslant \pi$. [2]

(iv) Explain why the equation $\sin x - x + 1 = 0$ has no root other than the one found in part **(ii)**. [1]

6 A cup of tea is served at 80 °C in a room which is kept at a constant 20 °C. The temperature, T °C, of the tea after t minutes can be modelled by assuming that the rate of change of T is proportional to the difference in temperature between the tea and the room.

(i) Explain why the rate of change of the temperature in this model is given by $\dfrac{dT}{dt} = -k(T - 20)$, where k is a positive constant. [2]

(ii) Show by integration that the temperature of the tea after t minutes is given by $T = 20 + 60e^{-kt}$. [6]

(iii) After 2 minutes the tea has cooled to 60 °C. Find the value of k. [3]

7 A curve is given parametrically by $x = 3t$, $y = 1 + t^3$ where t is any real number.

(i) Show that a cartesian equation for this curve is given by $y = 1 + \frac{1}{27}x^3$. [2]

A second curve is given by $y = x^2 + 4x - 19$.

(ii) Given that the curves intersect at the point $(3, 2)$, find the coordinates of all the other points of intersection between the two curves. [6]

8 The function f is given by $f(x) = \dfrac{x^2}{3x^2 - 1}$, for $x > 1$. Show that f is a decreasing function. [5]

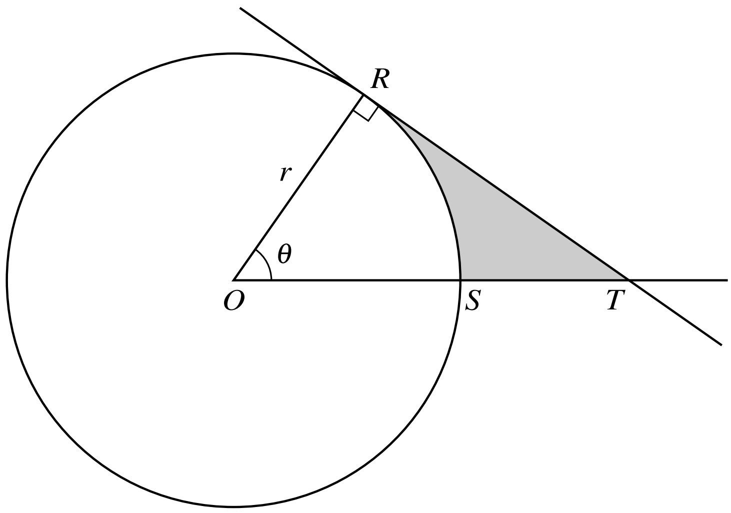
9 Find the equations of all the horizontal tangents to the curve with equation $y^2 = x^4 - 4x^3 + 36$. [8]

10 **(i)** Show that $\sin(2\theta + \frac{1}{2}\pi) = \cos 2\theta$. [2]

(ii) Hence solve the equation $\sin 3\theta = \cos 2\theta$ for $0 \leqslant \theta \leqslant 2\pi$. [6]

(iii) Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. Hence, by writing $\cos 2\theta - \sin 3\theta$ in terms of $\sin\theta$, use your answer to part **(ii)** to determine the solutions of $4x^3 - 2x^2 - 3x + 1 = 0$. [6]

11



The diagram shows a circle, centre O, radius r. The points R and S lie on the circumference of the circle, and the line RT is a tangent to the circle at R. The angle ROS is θ radians where $0 < \theta < \frac{1}{2}\pi$.

(i) Find expressions for the perimeter, P, and the area, A, of the shaded region in terms of r and θ. [7]

(ii) Hence show that $A \neq rP$. [4]

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