Cambridge International Examinations
Cambridge Pre-U Certificate

MATHEMATICS **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2016**

MARK SCHEME

Maximum Mark: 80

Published

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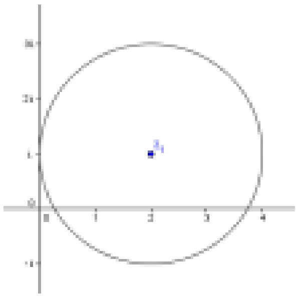
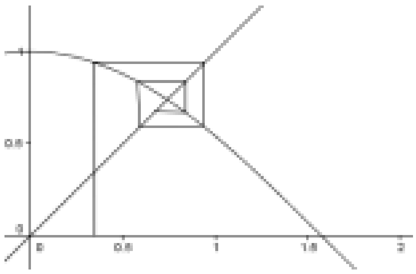
1 **(i)**	$f(-2) = -12$	M1 A1 [2]	Substitute $x = -2$, or any other complete method – must get as far as attempting the remainder but allow no more than 2 errors If using inspection then allow M1 for $(x + 2)(x^2 - 2x + k) - 2k$ Obtain –12 (no isw if then given as 12 or if given as $^{-12}/_{(x+2)}$) Must be identified as remainder so A0 if just left at bottom of division attempt
(ii)	12	B1FT [1]	FT on their **(i)**
2	$3^x = {}^5/_4$ $x = \log_3({}^5/_4)$	B1* M1d* M1d* A1 [4]	State $3^x = {}^5/_4$ Allow using logs before rearranging, as long as valid method to deal with $\log(4 \times 3^x)$ Take logarithms and apply at least one log rule correctly Rearrange to make x the subject Obtain correct answer aef Allow BOD if no base specified ISW decimal answer but not subsequent incorrect log work, such as $\log({}^5/_4)/\log(3) = \log({}^5/_{12})$
3	$\log_{10} y = 2x + 4$ $y = 10^{2x+4}$ $= 10^{2x} \times 10^4$ $= 10000 \times 100^x$ **AG** **OR** $y = 10000 \times 100^x$ $\log_{10} y = \log_{10}10000 + \log_{10}100^x$ $\log_{10} y = 2x + 4$ Conclude convincingly	M1 A1 M1 A1 [4]	State equation of form $\log_{10} y = mx + c$ State $\log_{10} y = 2x + 4$ Base 10 must be seen, or implied by later work Attempt correct process to remove logs Obtain $y = 10^{2x} \times 10^4$ and hence $y = 10000 \times 100^x$ M1 – take logs of both sides M1 – use one correct log rule A1 – obtain $\log_{10} y = 2x + 4$ A1 – relate to $y = mx + c$
4 **(i)**	$\|z_1\| = \sqrt{5}$ $\|z_2\| = 5$ $z_1 + z_2 = 5 + 5i$ $\|z_1 + z_2\| = \sqrt{50}$ $\sqrt{5} + 5 > \sqrt{50}$	B1 M1 A1 A1 [4]	Both correct Attempt $z_1 + z_2$ Could be implied by attempt at $\|z_1 + z_2\|$ Obtain $\sqrt{50}$ oe Conclude by approximating to sufficient accuracy or comparing surds – A0 if no clear comparison Could also use geometrical argument

(ii)		B1 B1 [2]	Circle Centre at 2 + i and radius of 2 soi Circle should be approximately correct i.e. have the y-axis as a tangent, and not pass through the origin
5 (i)	$\frac{3(x+1)+(x+2)}{(x+2)(x+1)} = \frac{4x+5}{x^2+3x+2}$ **OR** A(x+1) + B(x + 2) = 4x+ 5 so A = 3 and B = 1.	M1 A1 [2]	Attempt to add fractions using common denominator Simplify to obtain given answer M1 – use partial fractions on RHS A1 – obtain given answer
(ii)	$-\frac{3}{(x+2)^2} - \frac{1}{(x+1)^2}$	M1 A1 A1 [3]	Differentiate both terms on the LHS, or any other valid method Obtain one correct term Obtain fully correct $f'(x)$ Quotient rule: M1 – attempt quotient rule A1 – correct unsimplified expression A1 – correct simplified expression
(iii)	Denominators always +ve as $(x + k)^2 > 0$ Numerators always –ve, and $^{-ve}/_{+ve}$ is -ve	M1 A1 [2]	State, or imply, that "decreasing" implies $f'(x)<0$, and make some attempt to use this Conclude convincingly that $f'(x) < 0$ for all x (CWO, A0 if incorrect $f'(x)$)
6 (i)	Angle AOB $= \cos^{-1}\frac{16+6+20}{\sqrt{38\times 84}}$ $= 42.0^{\circ}$	M1 M1 M1 A1 [4]	Attempt $a.b$ for $\pm OA$ and $\pm OB$ (at least 2 elements correct) Use correct formula for their vectors Attempt evaluation, with correct two vectors Obtain 42.0° (allow 42°) or 0.733 rad If using cosine rule, then M1 – attempt sides (at least 2 correct) M1 – attempt cosine rule M1 – rearrange to attempt angle A1 – obtain 42.0°

(ii)	$BA = -6\mathbf{i} + \mathbf{j} + \mathbf{k}$ $\lvert BA\rvert = \sqrt{38}$ $\lvert BA\rvert = \lvert OA\rvert$ hence isosceles ($\neq \lvert OB\rvert$ not nec)	B1 M1 A1 [3]	State correct BA or AB Find one side length or one angle other than those found in part **(i)** If BA or AB has been stated then sufficient to just state $\sqrt{38}$ If BA or AB has not been stated then a minimum of $\sqrt{(36+1+1)}$ must be seen Conclude convincingly NB Angles and sides must be given in exact form to demonstrate equality B0M1A1 if BA or AB not explicit B0M1A1 if BA or AB incorrect, as long as of form $\pm 6\mathbf{i} \pm \mathbf{j} \pm \mathbf{k}$ If using cosine rule, then B1 – state correct cosine rule M1 – attempt evaluation A1 – conclude convincingly, including use of surd value for cos 42°
7 (i)	$f(0.7) = 0.0648 > 0$ $f(0.8) = -0.103 < 0$ Sign change hence root	M1 A1 [2]	Evaluate at both 0.7 and 0.8 Conclude by referring to sign change oe CWO
(ii)	Graph of $y = x$ and $y = \cos x$	B1 B1 [2]	Sketch both graphs… … in correct proportion to each other and intercepts correct
(iii)	$\frac{dy}{dx} = -\sin x$ since $0 < x < \pi/2$ the magnitude of $-\sin x$ is less than 1 therefore the iteration converges or e.g. <table><tr><td>x</td><td>0.7</td><td>0.8</td></tr><tr><td>dy/dx</td><td>–0.64…</td><td>–0.71…</td></tr></table>magnitude of gradient in the region is less than 1 therefore the iteration converges	B1 M1 A1 [3]	State correct derivative Consider magnitude of gradient, either in general terms or at specific value(s) Allow use of $^{\pi}/_{4}$ as a specific value Conclude using $\lvert F'(x)\rvert < 1$ Allow $-1 < F'(x) < 0$ A0 for $\lvert F'(x)\rvert \leqslant 1$ A0 for $\lvert F'(x)\rvert < 1$ from $0 \leqslant x \leqslant {}^{\pi}/_{2}$, unless end point clearly dealt with

(iv)		M1 A1 [2]	First two segments At least 5 segments Allow **(ii)** and **(iv)** on the same graph
(v)	$\cos(0.73905) - 0.73905 = +5.879\ldots \times 10^{-5}$ $\cos(0.73915) - 0.73915 = -1.085\ldots \times 10^{-4}$ By the sign change rule α lies in that interval and therefore rounds to 0.7391 to 4 dp.	M1 A1 [2]	Evaluate at both 0.73905 and 0.73915 (or values closer to the root) Conclude by referring to sign change CWO
8	$4^2 = r^2 + r^2 - 2r^2\cos\theta$ $r^2(1-\cos\theta) = 8$ Arc $PQ = r\theta = \theta\sqrt{\dfrac{8}{1-\cos\theta}}$	B1 M1 A1 M1 A1 [5]	State $4^2 = r^2 + r^2 - 2r^2\cos\theta$ Attempt to make r, or r^2, the subject Obtain a correct expression for r, or r^2 Attempt to eliminate r from $s = r\theta$ Obtain correct arc length, aef For expressions that involve f(½θ): B1 – correct expression involving r and ½θ (e.g. right-angled trig, Sine Rule etc.) M1 – attempt to eliminate r from $s = r\theta$ M1 – attempt to use a correct identity to link f(½θ) and cosθ A1 – obtain correct identity A1 – obtain correct arc length, aef

9	**(i)**		B1	$\sec x = \dfrac{1}{\cos x}$ oe seen anywhere
		$\dfrac{\sin x}{1+\sin x} \equiv \dfrac{\sin x(1-\sin x)}{(1+\sin x)(1-\sin x)}$	M1	Multiply top and bottom by $1 - \sin x$
		$\equiv \dfrac{\sin x - \sin^2 x}{1-\sin^2 x}$	A1	Obtain correct unsimplified expression
		$\equiv \dfrac{\sin x - 1 + \cos^2 x}{\cos^2 x}$	M1	Write denominator as $\cos^2 x$
		$\equiv \sec x \tan x - \sec^2 x + 1$	A1 [5]	Obtain correct simplified expression
		OR		
		$\sec x \tan x - \sec^2 x + 1 \equiv \dfrac{\sin x - 1 + \cos^2 x}{\cos^2 x}$		M1 – write with common denominator of $\cos^2 x$
		$\equiv \dfrac{\sin x - \sin^2 x}{1-\sin^2 x}$		M1 – attempt expression in terms of $\sin x$ only
		$\equiv \dfrac{\sin x(1-\sin x)}{(1+\sin x)(1-\sin x)} \equiv \dfrac{\sin x}{1+\sin x}$		A1 – obtain correct unsimplified expression A1 – obtain correct simplified expression
	(ii)	$\displaystyle\int_0^{\frac{1}{4}\pi} \frac{\sin x}{1+\sin x}\,\mathrm{d}x = \int_0^{\frac{1}{4}\pi} \sec x \tan x - \sec^2 x + 1\,\mathrm{d}x$	M1	Attempt integration of given expression (at least two terms)
			A1	Obtain at least two correct terms (allow if third term not yet integrated)
		$= \left[\sec x - \tan x + x\right]_0^{\frac{1}{4}\pi}$	A1	Obtain fully correct integral
		$= (\sqrt{2} - 1 + \frac{1}{4}\pi) - (1 - 0 + 0)$	M1	Attempt correct use of limits (correct order and subtraction) in their integration attempt
			B1	State or imply $\sec \frac{1}{4}\pi = \sqrt{2}$
		$= \frac{1}{4}\pi + \sqrt{2} - 2$ **AG**	A1	Obtain given answer convincingly
			[6]	Allow non 'hence' methods

10 (i)	$u = \frac{1}{x}$ and $\frac{du}{dx} = -\frac{1}{x^2}$	M1*	Attempt to link du and dx, to obtain kx^{-2}
	So $\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}dx = \int -\sin u \, du$	A1	Correct integrand in terms of u
	$= \cos u + c = \cos\left(\frac{1}{x}\right) + c$	M1d*	Attempt integration of their f(u) – of form $a\sin u$
		A1 [4]	Correct integral in terms of x, including $+ c$
(ii)		M1	Attempt correct use of limits in *their* integral from part **(i)** Allow M1 for muddles with fractions, such as $\cos(^1/_\pi)$
	$\int_{1/2\pi}^{1/\pi} \frac{\sin\left(\frac{1}{x}\right)}{x^2}dx = -2$	A1	Obtain –2 cwo
	$\int_{1/3\pi}^{1/2\pi} \frac{\sin\left(\frac{1}{x}\right)}{x^2}dx = 2$	A1 [3]	Obtain 2 cwo
(iii)	$\int_{1/(n+1)\pi}^{1/n\pi} \frac{\sin\left(\frac{1}{x}\right)}{x^2}dx = \cos(n\pi) - \cos((n+1)\pi)$	B1	Correct general expression in terms of n (no FT on incorrect integral)
	$\cos(n\pi) = 1$ if n is even and -1 if n is odd	M1	Consider values of $\cos(n\pi)$, or another relevant expression e.g. $-2\sin(n\pi + {}^\pi/_2)$
	So the integral is either $1 + 1 = 2$ if n even or $-1 - 1 = -2$ if n odd	A1 [3]	Fully convincing argument (including relevant subtractions) from cwo

11 (a)	$f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$	M1 A1	Attempt $^1/_h$ [f(x + h) – f(x)] Obtain correct expression (allow unsimplified denominator of $x+h-x$)
	$=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}.\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$	M1	Multiply top and bottom by $\sqrt{(x+h)}+\sqrt{x}$
	$=\lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$	A1	Simplify expression as far as possible
	$=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{\sqrt{x}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$	A1 [5]	Complete proof by considering lim $h\to 0$
	OR $f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$		M1 – attempt $^1/_h$ [f(x + h) – f(x)]
	$=\lim_{h\to 0}\frac{\sqrt{x}\left(1+\frac{1}{2}\frac{h}{x}-\frac{1}{8}\frac{h^2}{x^2}+...\right)-\sqrt{x}}{h}$		A1 – obtain correct expression (allow unsimplified denominator of $x+h-x$)
	$=\lim_{h\to 0}\frac{\sqrt{x}+\frac{1}{2}\frac{h}{\sqrt{x}}+h^2(...)-\sqrt{x}}{h}$		M1 – attempt binomial expansion with h/x A1 – simplify expression as far as possible
	$=\lim_{h\to 0}\frac{1}{2}\frac{1}{\sqrt{x}}+h(...)$ $=\frac{1}{2\sqrt{x}}$		M1 – complete proof by considering lim $h\to 0$ Could also go via $^{\delta x}/_{\delta y}$, from $x=y^2$
(b) (i)	$y-\sqrt{a}=\frac{1}{2\sqrt{a}}(x-a)$, $y-\sqrt{b}=\frac{1}{2\sqrt{b}}(x-b)$	M1	Attempt equations of both tangents
	$\frac{1}{2\sqrt{a}}(x-a)+\sqrt{a}=\frac{1}{2\sqrt{b}}(x-b)+\sqrt{b}$	A1 M1	Obtain both correct equations Eliminate one variable and attempt to solve – as far as a correct equation in which x appears only once Allow M1 if solving normals not tangents
	$x=\sqrt{ab}$ **AG**	A1	Obtain $x=\sqrt{ab}$, detail required
	$y=\frac{1}{2}\left(\sqrt{a}+\sqrt{b}\right)$ **AG**	A1 [5]	Obtain $y=\frac{1}{2}\left(\sqrt{a}+\sqrt{b}\right)$, detail required
(ii)	Any valid solution e.g. a = 4 and b = 16	M1 A1 [2]	State a pair of values that give one integer coord State a pair of values that give both integer coords