Cambridge Pre-U

Cambridge Assessment International Education
Cambridge Pre-U Certificate

MATHEMATICS **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2018**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Question	Answer	Marks	Guidance
1(i)	$u_5 = 32 \times 0.75^4$	**M1**	Attempt u_5 using the recurrence relationship
	$u_5 = 10.125$	**A1**	Obtain 10.125 A0 for 10.1 if nothing better seen
1(ii)	$S_\infty = \dfrac{32}{1-0.75}$	**M1**	Attempt to use correct sum to infinity formula, with $a = 32$ and $r = 0.75$
	$= 128$	**A1**	
2(i)	$2(x+\frac{3}{2})^2+\frac{1}{2}$	**B3**	**B1** for each of $p = 2$, $q = \frac{3}{2}$, $r = \frac{1}{2}$
2(ii)	$x = -\frac{3}{2}$	**B1**	**FT** State $x = -\frac{3}{2}$ (ft on their q)
2(iii)	$2x^2 + 6x + 5 = k - 2x$ $2x^2 + 8x + 5 - k = 0$	**B1**	Equate and rearrange to obtain $2x^2 + 8x + 5 - k = 0$
	$64 - 4 \times 2 \times (5 - k) = 0$ $24 + 8k = 0$	**M1**	Use $b^2 - 4ac = 0$ to attempt to find k
	$k = -3$	**A1**	
	OR $4x + 6 = -2$	**B1**	Differentiate and equate to get $4x + 6 = -2$
	$x = -2$, $y = 1$ $1 = k + 4$	**M1**	Solve their $\frac{dy}{dx} = -2$ to find point of intersection and hence attempt to find k
	$k = -3$	**A1**	

Question	Answer	Marks	Guidance
3	$\ln 6^{2x-1} = \ln 3^{x+2}$ $(2x-1)\ln 6 = (x+2)\ln 3$	**M1**	Introduce ln on both sides, and drop powers Allow any log as long as bases are consistent (and also no base specified)
		A1	Obtain correct $(2x-1)\ln 6 = (x+2)\ln 3$ Allow correct equation with any log
	$2x\ln 6 - \ln 6 = x\ln 3 + 2\ln 3$ $2x\ln 6 - x\ln 3 = \ln 6 + 2\ln 3$ $x(\ln 36 - \ln 3) = \ln 6 + \ln 9$	**M1**	Attempt to make x the subject
	$x\ln 12 = \ln 54$	**M1**	Attempt correct processes to combine logs
	$x = \dfrac{\ln 54}{\ln 12}$	**A1**	Obtain $x = \dfrac{\ln 54}{\ln 12}$ Must now be ln not any other log If working in log to a different base then must justify change to ln No ISW if fraction incorrectly 'cancelled'
	OR $6^{2x} \times 6^{-1} = 3^x \times 9$	**M1**	Use index laws to split indices
	$36^x \times 6^{-1} = 3^x \times 9$	**M1**	Use index law to attempt same index
	$12^x = 54$	**A1**	Combine like terms to obtain $12^x = 54$
	$\ln 12^x = \ln 54$ $x\ln 12 = \ln 54$	**M1**	Introduce ln on both sides, and drop powers Allow any log as long as bases are consistent (and also no base specified)
	$x = \dfrac{\ln 54}{\ln 12}$	**A1**	Obtain $x = \dfrac{\ln 54}{\ln 12}$ Same additional guidance as A1 in main scheme

Question	Answer	Marks	Guidance
4	$u^2 + 2u - 6 = 0$	**M1**	Attempt to rewrite equation as a quadratic e.g. using $u = \sqrt{x}$
	$(u + 1)^2 - 7 = 0$ $u + 1 = \sqrt{7}$	**M1**	**Dep** on first M mark Attempt to solve resulting quadratic
	$u = -1 + \sqrt{7}$	**A1**	Allow unsimplified equiv Could still be $-1 \pm \sqrt{7}$
	$x = (-1 + \sqrt{7})^2$	**M1**	Recognise that root(s) need to be squared to obtain x
	$x = (1 - 2\sqrt{7} + 7)$	**M1**	Attempt correct method to square two term surd
	$x = 8 - 2\sqrt{7}$	**A1**	Obtain $8 - 2\sqrt{7}$ only
	OR $(x - 6)^2 = (-2\sqrt{x})^2$ $x^2 - 12x + 36 = 4x$	**M1**	Rearrange to appropriate form and attempt to square both sides M0 for squaring term by term
	$x^2 - 16x + 36 = 0$	**M1**	Gather like terms
		A1	Obtain correct quadratic
	$x = \dfrac{16 - \sqrt{256 - 144}}{2}$	**M1**	**Dep** on first M mark Attempt to solve quadratic – any valid method Condone ± in quadratic formula, but not +
	$x = 8 - 2\sqrt{7}$	**M1**	Attempt to simplify to required form
		A1	Obtain $8 - 2\sqrt{7}$ only
5(i)	$a(5 + 12\text{i}) + b(1 - 4\text{i}) = 7 + 36\text{i}$	**B1**	$u^2 = 5 + 12\text{i}$ soi
		B1	$v^* = 1 - 4\text{i}$ soi
	$5a + b = 7$ $12a - 4b = 36$	**M1**	Equate Re parts and Im parts Must be using given equation
	$12a - 4(7 - 5a) = 36$ $32a - 28 = 36$ $32a = 64$	**M1**	**Dep** on first M mark Attempt to solve simultaneously, using valid method
	$a = 2,\ b = -3$	**A1**	Obtain $a = 2$ and $b = -3$

Question	Answer	Marks	Guidance
5(ii)	Plot u and v	**B1**	Mark both points correctly on the Argand diagram Some indication of scale needed
	Sketch perpendicular bisector	**B1**	Attempt perpendicular bisector of their u and v Must be clearly intended as a perpendicular bisector e.g. drawn accurately or angle and mid-point clear on sketch
	Equation of perpendicular bisector is $y = x + 1$ using $x = 0$, gives $y = 1$	**M1**	Attempt to find point of intersection of perpendicular bisector with Im axis
	$0 + \mathrm{i}$	**A1**	Obtain $0 + \mathrm{i}$ Allow just i, or coord equiv e.g. (0, 1) **SR B1** for $0 + \mathrm{i}$, or equiv, with no justification (includes scale drawing)
6(i)	$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta = \frac{1}{2}r\sin\theta\, r\cos\theta$ $\frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{2}r^2\sin\theta\cos\theta$ $\theta - \sin\theta = \sin\theta\cos\theta$ $\theta = \sin\theta\,(1 + \cos\theta)$ **A.G.**	**B1**	Obtain area of segment as $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$
		B1	Obtain area of triangle as $\frac{1}{2}r\sin\theta\, r\cos\theta$
		M1	**Dep** on both B marks Equate areas and simplify
		A1	$\theta = \sin\theta\,(1 + \cos\theta)$ **A.G.**

Question	Answer	Marks	Guidance
6(ii)	1.296...	**B1**	Correct first iterate, from starting with 1 Allow 1.30
	... 1.224, 1.260, 1.243, 1.252, 1.247, 1.249...	**M1**	Correct iteration process at least 3 times Allow working to 3sf
	Hence root is 1.25	**A1**	Obtain 1.25 (must be 3sf)
	OR $\theta_{n+1} = \theta_n - \dfrac{\sin\theta_n + \sin\theta_n\cos\theta_n - \theta_n}{\cos\theta_n + \cos^2\theta_n - \sin^2\theta_n - 1}$	**B1**	Correct Newton-Raphson formula Allow any equiv e.g. with double angles
	1.338, 1.2535, 1.2488, 1.2488…	**M1**	Attempt to use N-R formula at least 3 times Must come from starting with f(θ) = 0
	Hence root is 1.25	**A1**	Obtain 1.25 (must be 3sf)
	1.245 < 1.251 1.255 > 1.246 or using $f(\theta) = \sin\theta\,(1 + \cos\theta) - \theta$ $f(1.245) = 5.623 \times 10^{-3}$ $f(1.255) = -9.235 \times 10^{-3}$	**M1**	Substitute 1.245 and 1.255 (or better) into both sides of equation, or equiv Allow their root ± 0.005 or better If their root is more accurate than 3sf then allow M1 for ± 5 in the next significant figure
	Inequality sign change, hence root is 1.25 to 3sf	**A1**	Conclude correctly, referring to sign change A0 if not confirming 3sf
7(i)	$\dfrac{dx}{dt} = 2t$, $\dfrac{dy}{dt} = 3t^2 - 2$ Hence $\dfrac{dy}{dx} = \dfrac{3t^2 - 2}{2t}$	**M1**	Attempt to find $\dfrac{dy}{dx}$ either from parametric equations or a Cartesian equation
		A1	Obtain correct $\dfrac{dy}{dx}$ Could be $\dfrac{dy}{dx} = \frac{3}{2}(x-1)^{\frac{1}{2}} - (x-1)^{-\frac{1}{2}}$
	$m = \frac{5}{2}$ $m' = -\frac{2}{5}$	**M1**	Attempt to find gradient of normal when $t = 2$
	$y - 4 = -\frac{2}{5}(x - 5)$	**M1**	Attempt to find equation of normal when $t = 2$
	$2x + 5y = 30$ **A.G.**	**A1**	Obtain $2x + 5y = 30$ **A.G.**

Question	Answer	Marks	Guidance
7(ii)	$2(t^2 + 1) + 5(t^3 – 2t) = 30$	**M1**	Attempt to solve equations for normal and curve simultaneously to obtain an equation in a single variable
	$5t^3 + 2t^2 – 10t – 28 = 0$	**A1**	Obtain correct 4 term cubic Could be $25x^3 – 179x^2 + 495x – 1125 = 0$
	$(t – 2)(5t^2 + 12t + 14) = 0$	**M1**	Attempt to factorise cubic, using $(t – 2)$ or $(x – 5)$ as a factor
		A1	Obtain $(t – 2)(5t^2 + 12t + 14) = 0$ or $(x – 5)(25x^2 – 54x + 225) = 0$
	$\Delta = –136$, hence no other roots	**A1**	Show that quadratic has no real roots and conclude – detail needed **NB** allow other methods, such as gradients of the two functions, as long as fully convincing
8(i)	$e^x \sin x - \int e^x \cos x dx$ or $-e^x \cos x + \int e^x \cos x dx$	**M1**	Attempt integration by parts once
		A1	Obtain correct expression
	$e^x \sin x - \left(e^x \cos x + \int e^x \sin x dx\right)$ or $-e^x \cos x + \left(e^x \sin x - \int e^x \sin x dx\right)$	**M1**	Attempt integration by parts again on their integral (parts consistent with first stage)
		A1	Obtain correct expression – must be entire expression, including 'uv' from first stage
	$2\int e^x \sin x dx = e^x \sin x - e^x \cos x$	**M1**	Attempt to rearrange
	$\int e^x \sin x dx = \frac{1}{2}e^x(\sin x - \cos x) + c$	**A1**	Obtain correct integral **A.G.**
8(ii)	$2 = \frac{1}{2}(0 – 1) + c$ so $c = \frac{5}{2}$	**M1**	Attempt to find c, using $x = 0$ and $y = 2$
	$y = \frac{1}{2}e^x(\sin x - \cos x) + \frac{5}{2}$	**A1**	Obtain correct equation aef
9(i)	$4\sin(2x + 30) = 2\sqrt{3}\sin 2x + 2\cos 2x$	**B1**	Correct expansion of first term
	$R\cos\alpha = 5$, $R\sin\alpha = 2\sqrt{3}$ $R^2 = 25 + 12$, so $R = \sqrt{37}$ $\tan\alpha = \frac{2\sqrt{3}}{5}$, so $\alpha = 34.7^\circ$	**M1**	Attempt correct process to find R, from expression of the form $a\sin 2x + b\cos 2x$
		M1	Attempt correct process to find α, from expression of the form $a\sin 2x + b\cos 2x$
	$f(x) = \sqrt{37}\cos(2x – 34.7^\circ)$	**A1**	Obtain $\sqrt{37}\cos(2x – 34.7^\circ)$ Allow 6.08, or better, for $\sqrt{37}$

Question	Answer	Marks	Guidance
9(ii)	Stretch in the y direction by a scale factor of $\sqrt{37}$	**B1**	**FT** Stretch in the y direction by a scale factor of their R Must use 'factor' or scale factor' Must be using their $R\cos(2x-\alpha)$
	Translation in the x direction by 34.7° Stretch in the x direction by sf $\frac{1}{2}$ OR Stretch in the x direction by sf $\frac{1}{2}$ Translation in the x direction by 17.4°	**M1**	Translation by ± (their α or $\frac{1}{2}\alpha$) and stretch sf $\frac{1}{2}$ or 2, both in x direction Allow informal language for 'translation', 'stretch' and direction (for the M1 only) Must be using their $R\cos(2x-\alpha)$
		A1	**FT** Translation in the x direction by their α or $\frac{1}{2}\alpha$
		A1	Stretch in the x direction by sf $\frac{1}{2}$ (must follow translation) or stretch followed by correct translation Must use 'factor' or scale factor'
9(iii)	$\cos(2x-34.7^\circ)=0.98639...$ $2x-34.7^\circ=-9.46^\circ$ or 9.46°...	**M1**	Attempt correct process to find at least one numerical value for $2x-\alpha$
	$2x=25.24^\circ$ or 44.16°... $x=12.6^\circ$ or 22.1°...	**M1**	**Dep** on first M mark Attempt correct process to find at least one positive numerical value for x
	$x=12.6^\circ$	**M1**	Attempt to find smallest value of x i.e. use $-\cos^{-1}\left(\dfrac{6}{R}\right)$
		A1	Obtain $x=12.6^\circ$ only, or better

Question	Answer	Marks	Guidance
10(i)	$\int -\frac{1}{2}\left(\frac{6-2u}{u}\right)^2 \mathrm{d}u$	**M1**	Attempt integrand in terms of u M0 if just du = dx
	$\int\left(-\frac{18}{u^2}+\frac{12}{u}-2\right)\mathrm{d}u$	**M1**	Attempt to simplify to three terms
		A1	Obtain correct three term expression, with fractions simplified Allow $\frac{12u}{u^2}$ if subsequently integrated as $k\ln u^2$
	$\frac{18}{u}+12\ln u-2u$	**M1**	Integrate expression of form $au^{-2}+bu^{-1}+c$ Allow errors in coefficients only
		A1	Correct integral in terms of u
	(18 + 12ln1 – 2) – (6 + 12ln3 – 6)	**M1**	**Dep** on first M mark Attempt F(1) – F(3), must be correct order, or use F(x = 1) – F(x = 0) Detail of use of limits needed as **A.G.**
	= 16 – 12ln3 **A.G.**	**A1**	Obtain 16 – 12ln3 **A.G.**
10(ii)	$\pi \times 4^2 \times 1$	**M1**	Attempt vol of cylinder
	16π – π(16 – 12ln3)	**M1**	**Dep** on first M mark Attempt vol of cylinder – π(16 – 12ln3)
	=12πln3	**A1**	Obtain 12πln3