Cambridge Pre-U

Cambridge Assessment International Education
Cambridge Pre-U Certificate

MATHEMATICS **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Question	Answer	Marks	Guidance
1(a)	$f(-2) = -8 + 4a + 6a - 12$	**M1**	Attempt $f(-2)$ Or attempt full division by $(x + 2)$ (as far as a 3 term quadratic quotient) Or set up sim eqns from $(x + 2)(x^2 + bx - 6)$
	$-8 + 4a + 6a - 12 = 0$ $10a = 20$	**M1**	Equate their $f(\pm 2)$ to 0 and attempt to solve for a Or attempt a using their attempt at division by $(x + 2)$ e.g. $2(4 - 5a) = -12$ or $R = 0$ Or attempt to solve $2 + b = a$ and $2b - 6 = -3a$ simultaneously
	$a = 2$	**A1**	Obtain $a = 2$
1(b)	$f(3) = 27 + 9a - 9a - 12$ or $f(3) = 27 + 18 - 18 - 12$	**M1**	Attempt $f(3)$, with numerical or algebraic a Or attempt full division / factorisation using $(x - 3)$ as far as attempting remainder; expect $f(x) = (x - 3)(x^2 + 5x + 9) + 15$
	$= 15$	**A1**	Obtain 15 (condone using incorrect a from **(i)**) No ISW if subsequently given as -15 A0 for $\frac{15}{x-3}$ unless it subsequently becomes 15 only If using division then attention must be drawn to the remainder for A1; 15 just appearing on the bottom line of a division is A0
2	$2\cos^2 x = \sin x(1 + 4\sin x)$	**M1**	Use $\tan x = \frac{\sin x}{\cos x}$ If using other identities then M1 for reducing to an equation in 2 trig ratios
	$2(1 - \sin^2 x) = \sin x + 4\sin^2 x$	**M1**	Use $\cos^2 x = 1 - \sin^2 x$ If using other identities then M1 for reducing to an equation in 1 trig ratio
	$6\sin^2 x + \sin x - 2 = 0$	**A1**	Obtain correct three term quadratic, **aef**
	$(3\sin x + 2)(2\sin x - 1) = 0$	**M1**	Attempt to solve quadratic in $\sin x$, then attempt at least one value for x, using $\sin^{-1}$
	$\sin x = \frac{-2}{3}$, hence $x = 221.8^\circ, 318.2^\circ$ $\sin x = \frac{1}{2}$, hence $x = 30^\circ, 150^\circ$	**A1**	Obtain two correct roots, in degrees, 1dp or better
		A1	Obtain four correct roots, in degrees, and no extras in range **SC** for answer only, allow B1 for 2 correct roots, B2 for 4 roots and no others

Question	Answer	Marks	Guidance
3(a)	$m = 3$	**B1**	Correct gradient **soi**
	$\log_{10}y = 3\log_{10}x - 1$	**M1**	Attempt equation of line, using coordinates correctly so M0 for using $x = 1$, $y = 2$ in $\log_{10}y = 3\log_{10}x + c$ Equation must be in terms of $\log_{10}x$ and $\log_{10}y$ Accept e.g. Y and X as variables, as long as X and Y defined (or implied) as $\log_{10}x$ and $\log_{10}y$
		A1	Obtain correct equation Allow log equivs for –1 e.g. $-\log_{10}10$ or $+\log_{10}0.1$ A1 BOD if the base of the logarithm is not shown, but A0 if a base other than 10 Accept e.g. $Y = 3X - 1$, as long as X and Y defined explicitly Correct equation must be seen in **(a)**
3(b)	$\log_{10}y = \log_{10}x^3 - 1$	**M1**	Use $m\log_{10}x = \log_{10}x^m$, either explicitly or as part of their solution (numerical m)
	$\log_{10}y = \log_{10}x^3 - \log_{10}10$ $\log_{10}y = \log_{10}(0.1x^3)$	**M1**	Attempt correct process to combine and then remove logs Could combine $\log_{10}y - \log_{10}x^3$, and then remove logs
	$y = 0.1x^3$ (for $x > 0$)	**A1**	Obtain correct equation **aef** with no logs Condone relationship expressed in words not an equation Condone no reference to domain If assuming that equation is $y = ax^n$: M1* correct linear form of $\log_{10}y = n\log_{10}x + \log_{10}a$ and compare coeffs with $y = 3x - 1$ M1d* attempt a A1 Obtain $y = 0.1x^3$ **oe** Assuming that equation is $y = ab^x$ is unlikely to gain any credit

Question	Answer	Marks	Guidance
4	$u^2 - 5u - 36 > 0$	**M1**	Attempt to solve / factorise equation, either in terms of x^2 or using a substitution
	$(u - 9)(u + 4) > 0$	**A1**	Correct roots / factorisation, either in terms of x^2 or their substitution
	$x^2 + 4 < 0$ is not possible as $x^2 \geqslant 0$	**B1**	State that this root / factor has no solutions (could still be their substitution not x^2) Allow e.g. $x^2 \neq -4$, but not just 'math error' Condone using $x^2 + 4 = 0$, but not $x^2 + 4 > 0$ (unless arguing that $x^2 + 4$ is always positive) Allow $x = \pm 2\mathbf{i}$ as long as then clearly discarded If B0, then other 5 marks are still available
	$x^2 > 9$	**A1**	Obtain $x^2 - 9 > 0$ or $x^2 > 9$ (or equiv with explicit substitution)
	$x < -3$, $x > 3$ **oe**	**M1**	Attempt to solve $x^2 > 9$ i.e. identify roots as ± 3 and select outside region www Can still gain M1 for a correct method to solve an incorrect inequality
		A1	A0 if extra real solutions, or if linked by 'and' (inc equiv incorrect set notation) If A0M0A0 as $x^2 > 9$ not seen then SR B1 for correct inequality Allow correct quartic graph, or testing either side of real roots, to imply A1M1A1
5	$\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$ or $\frac{\mathrm{d}r}{\mathrm{d}A} = \frac{1}{2\sqrt{\pi A}}$	**B1**	Seen or implied
	$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}t} \Rightarrow 3 = 2\pi r \times \frac{\mathrm{d}r}{\mathrm{d}t}$	**M1***	Attempt chain rule, with $\frac{\mathrm{d}A}{\mathrm{d}t} = 3$ (**oe**) and their attempt at $\frac{\mathrm{d}A}{\mathrm{d}r}$ or $\frac{\mathrm{d}r}{\mathrm{d}A}$ (from using differentiation)
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{3}{2\pi r} = \frac{3}{15}$	**M1d***	Attempt numerical value for $\frac{\mathrm{d}r}{\mathrm{d}t}$ using $2\pi r = 15$, or an attempt at a rearrangement of this
	0.2 cm s^{-1} i.e. rate of change is 0.2 cm s^{-1}	**A1**	Obtain 0.2 (cm s^{-1}) Condone missing units

Question	Answer	Marks	Guidance
6(a)	Mid-point AB is (2, 4)	**B1**	Correct mid-point **soi**
	Length $AB = 10$	**B1**	Correct length **soi** (or radius = 5) Allow BOD if 10 found but then identified as the radius
	$(x-2)^2 + (y-4)^2 = 25$	**B1ft**	Correct equation (**aef**), **ft** their midpoint and length
6(b)	$(x-2)^2 + (-2x-8)^2 = 25$	**M1***	Substitute into their circle equation (which must be of the correct format) to obtain equation in single variable
	$x^2 - 4x + 4 + 49 + 4x^2 + 32x + 64 = 25$	**M1d***	Expand and simplify to three term quadratic
	$5x^2 + 28x + 43 = 0$	**A1**	Obtain correct three term quadratic, condone no '= 0'
	$b^2 - 4ac = -76$	**M1**	Attempt to justify number of real roots, including use of quadratic formula Must be some attempt to evaluate discriminant / quadratic formula but does not have to be fully complete e.g. $b^2 - 4ac = 784 - 860$ as clearly negative M1 for attempting the complex roots, or stating correct values of $\frac{1}{5}\left(-14 \pm \mathrm{i}\sqrt{19}\right)$ M0 for 'x is complex' with no evidence
	no real roots, so no points of intersection	**A1**	Conclude appropriately www Must refer to both 'roots' (or solutions) and 'no intersection' **oe**

Question	Answer	Marks	Guidance
6(c)	$m_{radius} = \frac{4}{3}$	**B1**	Correct gradient of radius, using (5, 8)
	$m_{tangent} = -\frac{3}{4}$	**B1ft**	Correct gradient of tangent, ft their radius gradient having used (5, 8)
	$y - 8 = -\frac{3}{4}(x - 5)$	**M1**	Attempt equation of straight line, using their attempt at $m_{tangent}$ M0 if gradient of tangent has come from using A or B and not (5, 8)
	$3x + 4y - 47 = 0$ **www**	**A1**	Correct equation, in required form **OR (for first M1A1)** M1 attempt implicit differentiation, dealing with y term(s) correctly; expect $2(x-2) + 2(y-4)\frac{dy}{dx} = 0$ A1 obtain correct gradient, from correct derivative of correct equation **OR** M1 rearrange to $y = f(x)$ (allow BOD for $y = \pm f(x)$) and attempt differentiation, using the chain rule; expect $\frac{dy}{dx} = -(x-2)(25-(x-2)^2)^{-0.5}$ A1 obtain correct gradient, from correct derivative of correct equation, having used $y = +f(x)$ only
7(a)	$-3 + 4\lambda = 9$, hence $\lambda = 3$ **soi**	**M1**	Attempt value for λ Or attempt to eliminate λ from $a + 3\lambda = 2$ and $-3 + 4\lambda = 9$
	$a + 9 = 2$, hence $a = -7$	**A1**	Obtain $a = -7$
7(b)	$6 + 4b = 0$	**M1**	Attempt scalar product and equate to 0
	$b = -1.5$	**A1**	Obtain $b = -1.5$

Question	Answer	Marks	Guidance
7(c)	Lines intersect at (2, 9, 7)	B1	Identify point of intersection
	Length of one direction vector is 5 units	M1*	Attempt length of direction vector, including finding unit vector for direction Or attempt to find the number of direction vectors required e.g. $9\lambda^2 + 16\lambda^2 = 10^2$ Or attempt to find value of λ at the two points $(-7 + 3\lambda - 2)^2 + (-3 + 4\lambda - 9)^2 = 10^2$ (NB the last two methods are using λ in different ways)
	$2\mathbf{i} + 9\mathbf{j} + 7\mathbf{k} \pm 2(3\mathbf{i} + 4\mathbf{j})$	M1d*	Attempt their (2, 9, 7) ± 2 'direction vectors' Or substitute their λ into the equation of l_1 Use of λ must be commensurate with the method used to attempt λ ie using $\lambda = \pm 2$ in equation of l_1 is M0
	$-4\mathbf{i} + \mathbf{j} + 7\mathbf{k}$ and $8\mathbf{i} + 17\mathbf{j} + 7\mathbf{k}$	A1	Obtain both points Must be position vector (using either notation) but A0 if given as coordinates **SR** B1 if M0 but one correct position vector given
8(a)	f(0) = –2, f(1) = –2, f(2) = 4	M1	Substitute numerical values into f(x) until sign change seen
	f(1) < 0, f(2) > 0 so by sign change root lies between 1 and 2	A1	Correct conclusion, with correct reasoning Condone **either** correct inequalities **or** reference to sign change Must see 1 and 2 stated explicitly or as part of an interval eg [1, 2]
8(b)	$p = 1$, $q = 2$	B1	State or imply correct values for p and q
	$x_1 = 1.5$, $x_2 = 1.52753$	B1FT	Correct first iterate, for their starting value ($1 \to \sqrt{3},\ 2 \to \sqrt{2}$) Their starting value must be [1, 2], unless different conclusion from **(a)** Allow FT on incorrect p and q
	1.51964, 1.52187, 1.52124, 1.52142 …	M1	Use correct iterative process at least 3 times, with values being given to at least 4sf
	$\alpha = 1.521$	A1	Obtain 1.521 (must be exactly 4sf), following two iterations that are both 1.521 to 4sf Allow recovery for M1A1, and possibly B1 depending which iterate is incorrect Using Newton-Raphson is 0/4

Question	Answer	Marks	Guidance
8(c)	$F'(x) = \frac{x^2 - 2x(x+2)}{x^4}$ or $-\frac{1}{x^2} - \frac{4}{x^3}$	**M1**	Attempt quotient rule, or equiv, to find F'(x), where F(x) = $\frac{x+2}{x^2}$
		A1	Obtain correct derivative aef; ISW if subsequently spoilt by attempt to simplify
	F '(1.521) = – 1.57	**M1**	Attempt F '(their α), using α to 2sf or better, where F(x) = $\frac{x+2}{x^2}$ α must be the result of using the given iterative equation in **(b)**
	$\lvert -1.57 \rvert > 1$ so will diverge	**A1**	Correct conclusion, with correct reasoning and from using correct α (1.5 or better) Allow $-1.57 < -1$ or $1.57 > 1$ Allow -1.63 (or $\frac{-44}{27}$) from using $\alpha = 1.5$
9(a)	Crosses y-axis at (0, 2)	**B1**	Identify correct point on y-axis
	$2x - 3y - 3x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{2x-3y}{3x-3y^2}$	**M1**	Attempt implicit differentiation, implied by at least one $\frac{dy}{dx}$ term correct
		M1	Use product rule correctly; condone sign errors only
		A1	Obtain correct derivative, in any form; condone no '= 0'
	$\frac{dy}{dx} = \frac{1}{2}$	**A1**	Obtain $\frac{1}{2}$ **www**

Question	Answer	Marks	Guidance
9(b)	$2-3\frac{\mathrm{d}y}{\mathrm{d}x}-3\frac{\mathrm{d}y}{\mathrm{d}x}-3x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+6y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3y^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$	M1*	Attempt implicit differentiation on their $ky^2\frac{\mathrm{d}y}{\mathrm{d}x}$ term, dealing with the differential operators correctly
		A1	Obtain $6y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3y^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$
		B1ft	Obtain correct derivative of their $2x\pm3y\pm3x\frac{\mathrm{d}y}{\mathrm{d}x}$
	$2-3\times\frac{1}{2}-3\times\frac{1}{2}-0+12\left(\frac{1}{2}\right)^2+12\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$	M1d*	Substitute their x, y and $\frac{\mathrm{d}y}{\mathrm{d}x}$ values, explicitly or implied by correct final answer
	$12\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{6}$	A1	Obtain $\frac{-1}{6}$ www If using expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ M1* attempt quotient rule, including implicit diffn (at least one y term correct) A1ft obtain correct numerator, **ft** on sign errors in first derivative only A1 obtain $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\left(2-3\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(3x-3y^2\right)-\left(2x-3y\right)\left(3-6y\frac{\mathrm{d}y}{\mathrm{d}x}\right)}{\left(3x-3y^2\right)^2}$ (must see LHS) Then M1d* A1 as per main scheme
10(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-4x(k^2+x^2)^{-3}$	M1	Attempt chain rule to obtain $px(k^2+x^2)^{-3}$, any numerical p
		A1*	Obtain correct derivative **aef**
	$-4x(k^2+x^2)^{-3}=0\Rightarrow4x=0\Rightarrow x=0$ is the only solution	A1d*	Solve to get $x=0$ only, or verify that $x=0$ is the only stationary point Need to see some reference to 'only', 'no others' etc. A0 if an incorrect derivative is now being considered due to rearrangement error Reference to the denominator is not required, so **ISW** any comments

Question	Answer	Marks	Guidance
10(b)	$\mathrm{d}x = k\sec^2 u\,\mathrm{d}u$	**B1**	Any correct statement connecting dx and du
	$\int \frac{1}{(k^2+x^2)^2}\mathrm{d}x = \int \frac{1}{(k^2+k^2\tan^2 u)^2}k\sec^2 u\mathrm{d}u$	**M1**	Attempt integrand in terms of u, including use of their attempt at dx
	$= \int \frac{1}{(k^2\sec^2 u)^2}k\sec^2 u\mathrm{d}u$	**M1**	Use $1 + \tan^2 u = \sec^2 u$ to attempt integrand in a single trig ratio
	$= \int \frac{1}{k^3\sec^2 u}\mathrm{d}u = \int \frac{1}{k^3}\cos^2 u\mathrm{d}u$	**A1**	Simplify to obtain correct integrand in terms of either sec$u$ or cos$u$ only
	$= \frac{1}{2k^3}\int(\cos 2u + 1)\mathrm{d}u$	**M1**	Use cos2u identity, allowing $\frac{1}{2}(\pm \cos 2u \pm 1)$, and attempt to integrate, condoning sign errors only
	$= \frac{1}{2k^3}\left[\frac{1}{2}\sin 2u + u\right]$	**A1**	Obtain correct integral **www**
	$\frac{1}{2k^3}\left((\frac{1}{2}\sin\frac{\pi}{2} + \frac{\pi}{4}) - (\frac{1}{2}\sin 0 + 0)\right)$	**M1**	Attempt F($\frac{\pi}{4}$) – F(0), correct order and subtraction; condone no F(0) for M1 only Must be using $u = \frac{\pi}{4}$, or $x = k$ in an integral in terms of x which comes from a reasonable attempt to rewrite integral in terms of x
	$\frac{1}{2k^3}\left(\frac{1}{2}+\frac{\pi}{4}\right) = \frac{2+\pi}{8k^3}$ **AG**	**A1**	Rearrange to given answer, having used lower limit explicitly as AG If B0 for statement at start, then remaining 7 marks are all available
10(c)	$\left(\frac{1}{k^4}\times k\right) - \frac{2+\pi}{8k^3}$	**M1**	Attempt area of rectangle – area from **(ii)**
	$\frac{8}{8k^3} - \frac{2+\pi}{8k^3} = \frac{6-\pi}{8k^3}$	**A1**	Obtain correct area as a single term