



## Cambridge International AS & A Level

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**FURTHER MATHEMATICS**

**9231/23**

Paper 2 Further Pure Mathematics 2

**October/November 2022**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.



1 Find the Maclaurin's series for  $\ln(1 + e^x)$  up to and including the term in  $x^2$ .

[5]

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- 2 (a) Show that the system of equations

$$x - y + 2z = 4,$$

$$x - y - 3z = a,$$

$$x - y + 7z = 13,$$

where  $a$  is a constant, does not have a unique solution.

[2]

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- (b) Given that  $a = -5$ , show that the system of equations in part (a) is consistent. Interpret this situation geometrically. [3]

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- (c) Given instead that  $a \neq -5$ , show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

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3 The curve  $C$  has parametric equations

$$x = e^t - \frac{1}{3}t^3, \quad y = 4e^{\frac{1}{2}t}(t-2), \quad \text{for } 0 \leq t \leq 2.$$

Find, in terms of  $e$ , the length of  $C$ .

[6]

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- 4 (a) Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\cosh^2 x - \sinh^2 x = 1. \quad [3]$$

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- (b) Show that  $\frac{d}{dx}(\tan^{-1}(\sinh x)) = \operatorname{sech} x$ . [3]

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- (c) Sketch the graph of  $y = \operatorname{sech} x$ , stating the equation of the asymptote. [2]

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- (d) By considering a suitable set of  $n$  rectangles of unit width, use your sketch to show that

$$\sum_{r=1}^n \operatorname{sech} r < \tan^{-1}(\sinh n). \quad [3]$$

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- (e) Hence state an upper bound, in terms of  $\pi$ , for  $\sum_{r=1}^{\infty} \operatorname{sech} r$ . [1]

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5 Find the particular solution of the differential equation

$$2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 4x^2 + 3x + 3,$$

given that, when  $x = 0$ ,  $y = \frac{dy}{dx} = 0$ .

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6 The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} 2 & -3 & -7 \\ 0 & 5 & 7 \\ 0 & 0 & -2 \end{pmatrix}.$$

- (a) Find a matrix  $\mathbf{P}$  and a diagonal matrix  $\mathbf{D}$  such that  $\mathbf{A}^5 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$ . [7]

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7 (a) State the sum of the series  $1 + w + w^2 + w^3 + \dots + w^{n-1}$ , for  $w \neq 1$ . [1]

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(b) Show that  $(1 + i \tan \theta)^k = \sec^k \theta (\cos k\theta + i \sin k\theta)$ , where  $\theta$  is not an integer multiple of  $\frac{1}{2}\pi$ . [2]

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(c) By considering  $\sum_{k=0}^{n-1} (1 + i \tan \theta)^k$ , show that

$$\sum_{k=0}^{n-1} \sec^k \theta \sin k\theta = \cot \theta (1 - \sec^n \theta \cos n\theta),$$

provided  $\theta$  is not an integer multiple of  $\frac{1}{2}\pi$ . [5]

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- (d) Hence find  $\sum_{k=0}^{6m-1} 2^k \sin\left(\frac{1}{3}k\pi\right)$  in terms of  $m$ . [2]

8 (a) Use the substitution  $u = 1 - (\theta - 1)^2$  to find

$$\int \frac{\theta - 1}{\sqrt{1 - (\theta - 1)^2}} d\theta. \quad [3]$$

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(b) Find the solution of the differential equation

$$\theta \frac{dy}{d\theta} - y = \theta^2 \sin^{-1}(\theta - 1),$$

where  $0 < \theta < 2$ , given that  $y = 1$  when  $\theta = 1$ . Give your answer in the form  $y = f(\theta)$ . [11]

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**Additional page**

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