Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **October/November 2017**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	$z^2 = 7 + 4\sqrt{3}$	B1	Accept $4 + 3 + 4\sqrt{3}$
	$a\left(7 + 4\sqrt{3}\right) + b\left(2 + \sqrt{3}\right) = 1 + \sqrt{3}$	M1	Equate both $\sqrt{3}$ terms and constant terms to obtain two equations in a and b.
	$7a + 2b = 1$ $4a + b = 1$	A1	Both correct. Accept equation with a multiple of $\sqrt{3}$
	Attempt to solve a pair of linear simultaneous eqns to $a =$ or $b =$	M1	**M1dep**
	$a = 1$ and $b = -3$	A1	
2	$2x^{1.5} + 6x^{-0.5} = x\left(x^{0.5} + 5x^{-0.5}\right)$	M1	Attempt to multiply by $x^{0.5} + 5x^{-0.5}$ or $x^{0.5}$ or divide by $x^{0.5}$
	$2x^{1.5} + 6x^{-0.5} = x^{1.5} + 5x^{0.5}$ or $x^{1.5} - 5x^{0.5} + 6x^{-0.5} = 0$ or $\dfrac{2x^2 + 6}{x + 5} = x$ or $\dfrac{2x + \frac{6}{x}}{1 + \frac{5}{x}} = x$	A1	Simplified numerical powers
	$x^2 - 5x + 6 = 0$	M1	**M1dep** obtain a three term quadratic. Allow errors in signs and coefficients but not powers
	$(x - 3)(x - 2) = 0$	M1	Solve a three term quadratic
	$x = 3$ or 2 only	A1	
3	Correctly obtain a value of $x = 2$	B1	Inequality not required
	Correctly obtain a value of $x = -\dfrac{1}{2}$	B1	Inequality not required
	$x > 2$ and $x < -\dfrac{1}{2}$	B1	**B1dep** mark final answer(s). Allow $2 < x < -\dfrac{1}{2}$

Question	Answer	Marks	Partial Marks
4	$x+4=y^2$	**B1**	
	$7y-x=16$ $7y-16+4=y^2$	**B1**	allow 2^4 for 16
	$y^2-7y+12 \to (y-3)(y-4)(=0)$ or $x^2-17x+60 \to (x-5)(x-12)(=0)$	**M1**	**Attempt** to eliminate x or y to obtain a three term quadratic.
	Solve a three term quadratic	**M1**	**M1dep**
	$\to y=3,\ x=5$ or $y=4\ x=12$	**A1**	Allow for values seen even if correct pairs not clear.
5(i)	${}^{10}C_4=210$	**B1**	
5(ii)	2 Mystery 2 others = ${}^5C_2 \times {}^5C_2$ =100 3 Mystery 1 other = ${}^5C_3 \times {}^5C_1$ = 50 4 Mystery = ${}^5C_4$ =5 Total 155	**B3**	**B1** for one combination, unsimplifiied **B1** for second combination, unsimplifiied **B1** for third combination, unsimplifiied and total
	<u>Alternative Method</u> All – 0 Mystery – 1 Mystery	 **B1**	 All minus 0 or 1 or both
	$=210-{}^5C_4-{}^5C_1 \times {}^5C_3$	**B1**	**B1dep** 1Mystery and 0 mystery unsimplified
	$=210-5-5\times 10=155$	**B1**	**B1dep** final answer
5(iii)	2M1C1R = ${}^5C_2 \times {}^3C_1 \times {}^2C_1=60$ 1M2C1R = ${}^5C_1 \times {}^3C_2 \times {}^2C_1=30$ 1M1C2R = ${}^5C_1 \times {}^3C_1 \times {}^2C_2$ $=15$ Total 105	**B3**	**B1** for one combination, unsimplifiied **B1** for second combination, unsimplifiied **B1** for third combination, unsimplifiied and total
6(i)	$\pi x^2 h=500 \to h=\dfrac{500}{\pi x^2}$	**B1**	Ignore units Condone r for x
6(ii)	$A=2\pi x^2+2\pi x h$	**M1**	Correct expression for A and insert for *their h.*
	$=2\pi x^2+2\pi x \times \dfrac{500}{\pi x^2} = 2\pi x^2+\dfrac{1000}{x}$	**A1**	Answer given Condone r for x.

Question	Answer	Marks	Partial Marks
6(iii)	Differentiate: at least one power reduced by 1	**M1**	
	$\frac{dA}{dx} = 4\pi x - \frac{1000}{x^2}$	**A1**	
	$\frac{dA}{dx} = 0 \rightarrow x = \sqrt[3]{\frac{1000}{4\pi}}$ isw or $\left(x = 4.3(0)\right)$	**A1**	
	$A = 2\pi(4.3)^2 + \frac{1000}{4.3} = 349\,\text{cm}^2$	**A1**	awrt 349
	$\frac{d^2A}{dx^2} = 4\pi + \frac{2000}{x^3}$ (>0) or a positive value $(\rightarrow \text{min})$	**B1**	Correct second differential (need not be evaluated) and conclusion. or Examine correct gradient either side of $x = 4.3$ and conclusion
7(i)	(Gradient or $\frac{dy}{dx}$) $= \frac{3x-1}{\sqrt{x}}$	**B1**	Gradient = Negative reciprocal. Can be implied.
	$= 3x^{\frac{1}{2}} - x^{-\frac{1}{2}}$	**B1**	± One correct term
	$y = 2x^{\frac{3}{2}} - 2x^{\frac{1}{2}}\,(+C)$	**M1**	at least 1 fractional power increased by1.
	$-10 = 2 - 2 + C \rightarrow C = -10$	**A1**	one term correct with simplified coefficients
	$y = 2x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - 10$	**A1**	For *C* from correct working.
7(ii)	$x = 4 \rightarrow y = 16 - 4 - 10 = 2$	**B1**	
	$\rightarrow \frac{dy}{dx} = 6 - \frac{1}{2} = 5.5$	**B1**	
	Eqn with *their* grad and point (4, ...)	**M1**	
	Eqn of tangent: $\frac{y-2}{x-4} = 5.5 \rightarrow y = 5.5x - 20$ oe	**A1**	Must be in the form $y = mx + c$ but accept $2y = 11x - 40$
8(i)	$2\mathbf{A} = \begin{pmatrix} 4 & 2 \\ 8 & 6 \end{pmatrix}$	**B1**	
	$(2\mathbf{A})^{-1} = \frac{1}{8}\begin{pmatrix} 6 & -2 \\ -8 & 4 \end{pmatrix}$	**B2**	**B1** for $\begin{pmatrix} 6 & -2 \\ -8 & 4 \end{pmatrix}$ **B1** for $\frac{1}{8}$

Question	Answer	Marks	Partial Marks
8(ii)	$4x+2y=-5$ $8x+6y=-9$	**B1**	
	Pre multiply $\begin{pmatrix}-5\\-9\end{pmatrix}$ by a 2×2 matrix.	**M1**	Allow recovery
	$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{8}\begin{pmatrix}6 & -2\\-8 & 4\end{pmatrix}\begin{pmatrix}-5\\-9\end{pmatrix}$	**M1**	Pre multiply *their* $\begin{pmatrix}-5\\-9\end{pmatrix}$ by *their* answer to (i)
	$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{8}\begin{pmatrix}-12\\4\end{pmatrix}=\begin{pmatrix}-1.5\\0.5\end{pmatrix}$	**A2**	**A1** for x value **A1** for y value oe Allow both unsimplified
9(i)	$\frac{\mathrm{d}}{\mathrm{d}x}(x\ln x)=x\times\frac{1}{x}+\ln x$ isw	**M1A1**	Product rule. One correct term + another term. Allow unsimplified.
9(ii)	$\int 1+\ln x\,\mathrm{d}x = x\ln x$	**M1**	Correct use of (i) and must be dealing with 2 terms. soi
	$\int \ln x dx = x\ln x - x + (C)$	**A1**	Correct answer with no working is fine.
9(iii)	$\int_k^{2k} \ln x\,\mathrm{d}x = [2k\ln 2k - 2k]-[k\ln k - k]$ $= k(2\ln 2k - \ln k - 1)$	**M1**	Insert limits and subtract correctly using *their* result from (ii) which must contain an ln function
	$= k\left(\ln(2k)^2 - \ln k - 1\right)$	**M1**	Uses $n\ln a = \ln a^n$ somewhere oe
	$= k\left(\ln\left(\frac{4k^2}{k}\right)-1\right)$	**M1**	Uses $\ln a - \ln b = \ln\left(\frac{a}{b}\right)$ or $\ln a + \ln b = \ln ab$ somewhere
	$= k(\ln 4k - 1)$	**A1**	Answer given Correct completion.

Question	Answer	Marks	Partial Marks
10(i)	$c=1\rightarrow 6(1)^3-7(1)^2+1=0\rightarrow(c-1)$ is a factor.	**B1**	Or correct division. Finding or using one correct factor.
	Attempt to factorise or use long division to obtain $6c^2\ldots\pm1$ or $6c^2\pm c\ldots$ respectively	**M1**	
	$(c-1)(6c^2-c-1)=0$	**A1**	
	$(c-1)(2c-1)(3c+1)=0$	**A1**	
	$c=1,\ \frac{1}{2},\ -\frac{1}{3}$	**A1**	**FT** From three different linear factors
10(ii)	$\frac{dy}{dx}=\sec^2x+6\cos x$	**B2**	**B1** for each term
10(iii)	$\frac{1}{\cos^2x}+6\cos x=7$	**B1**	**B1dep** Replaces $\sec^2x$ by $\frac{1}{\cos^2x}$
	$\rightarrow 6\cos^3x-7\cos^2x+1=0$	**B1**	**B1dep** Answer given so all steps must be correct.
10(iv)	$\cos x=1,\ \frac{1}{2},\ -\frac{1}{3}.$ $\rightarrow x=0,\ 1.05\left(\text{or } \frac{\pi}{3}\right),\ 1.91$	**A2**	**A1** for 2 values awrt **A1** for third value and no others in range. No credit for answers in degrees
11(i)	$y=0\rightarrow(x-4)(x+1)=0$	**M1**	Solve
	$\rightarrow A$ is $(4,0)$ nfww	**A1**	Indication somewhere that $x=4$ when $y=0$
11(ii)	$4+3x-x^2=mx+8$ $x^2+(m-3)x+4=0$	**M1**	Eliminate y.
	$b^2-4ac(=0)\rightarrow(m-3)^2=16$	**M1**	**M1dep** Use of discriminant
	$m=-1$	**A1**	Do not award if $m=7$ is not discarded
11(iii)	Obtain quadratic $x^2+(m-3)x+4=0$ using *their* m and attempt to solve.	**M1**	Working must be seen for any marks to be awarded. Must not be awarded if m is not obtained correctly
	Point B (2, 6)	**A1**	

Question	Answer	Marks	Partial Marks
11(iv)	Area under curve $=\int_2^4 \left(4+3x-x^2\right)\mathrm{d}x$ Integrate powers increased in at least 2 terms	**M1**	
	$=\left[4x+\frac{3}{2}x^2-\frac{1}{3}x^3\right]_2^4$	**A1**	
	$=\left[16+24-\frac{64}{3}\right]-\left[8+6-\frac{8}{3}\right]$ $=7\frac{1}{3}$	**M1**	**M1dep** Insert limits of *their* 2 and 4 and subtract in correct order. May be implied by $18\frac{2}{3}-\ldots$
	Intercept is (8,0) so area of triangle = $\frac{6\times 6}{2}=18$	**M1**	Area of triangle using *their* $B=\frac{(their\,8-x_B)}{2}\times y_B$ or Attempt to find other suitable areas to result in a complete method.
	Shaded area $=18-7\frac{1}{3}=10\frac{2}{3}$	**A1**	Accept 10.7. Must not be awarded if point *B* is not obtained correctly.