Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/62**

Paper 6 Investigation and Modelling (Extended) **October/November 2020**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 4) and part **B** (Questions 5 to 9).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.
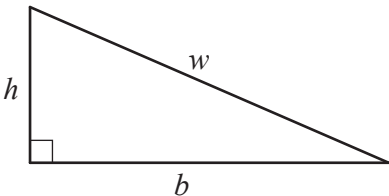
Answer **both** parts **A** and **B**.

A   INVESTIGATION (QUESTIONS 1 to 4)

AREA OF RIGHT-ANGLED TRIANGLES   (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation looks at finding the area of a right-angled triangle using its perimeter.

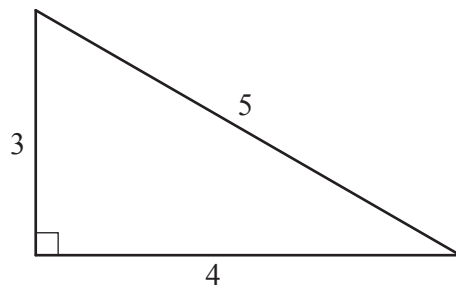In this investigation all lengths are in centimetres.

w is the hypotenuse of the triangle,
b is the base of the triangle,
h is the height of the triangle.

Perimeter, P, of this triangle. $P = b + h + w$

Area, A, of this triangle. $A = \frac{1}{2}bh$

1   (a)

NOT TO
SCALE

(i) Find the perimeter of this triangle.

.... [1]

(ii) Find the area of this triangle.

.... [1]

(b)

NOT TO
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Complete the table for right-angled triangles with sides *b*, *h* and *w*.

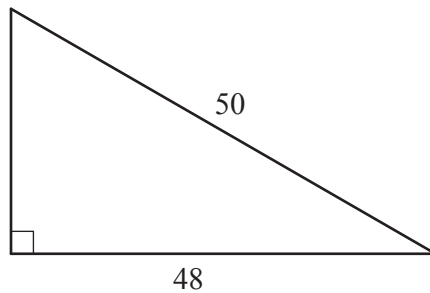| *b*	*h*	*w*	Perimeter, *P*	Area, *A*
12	5	13	30	30
84	13	85		
24		25	56	84
60	11		132	

[3]

2 **(a)**
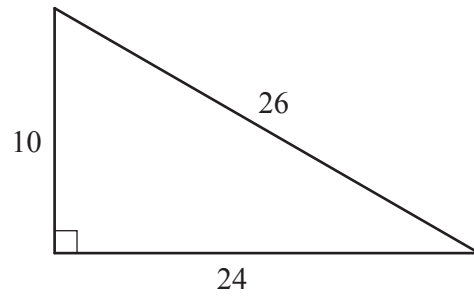
This triangle has perimeter $P = 60$.

Show that the calculation $\dfrac{60}{2} \times \left(\dfrac{60}{2} - 26\right)$ gives the correct area for this triangle.

[3]

(b)

This triangle has perimeter $P = 112$.

Show that the calculation $\dfrac{112}{2} \times \left(\dfrac{112}{2} - 50\right)$ gives the correct area for this triangle.
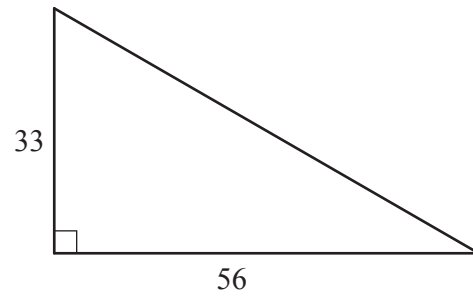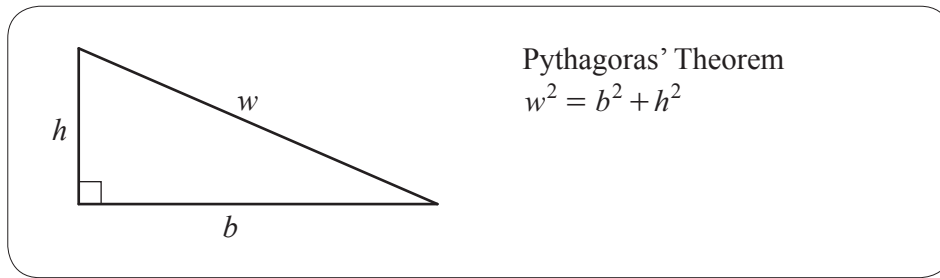
[3]

3 **(a)** Complete the table.

b	h	w	P	A	Calculation		
24	10	26	60	120	$\frac{60}{2} \times \left(\frac{60}{2} - 26\right)$	$=$	120
12	9	15	36	54	$\frac{36}{2} \times \left(\frac{36}{2} - 15\right)$	$=$	54
48		50	112		$\frac{112}{2} \times \left(\frac{112}{2} - 50\right)$	$=$	
15	8	17		60		$=$	60
21		29	70	210		$=$	
	12	37		210		$=$	

[4]

(b) Write an expression for the area of a right-angled triangle in terms of P and w.

.... [1]

(c)

Pythagoras' Theorem
$$w^2 = b^2 + h^2$$

NOT TO
SCALE

Use your expression from **part (b)** to find the area of this triangle.

.... [4]

(d)
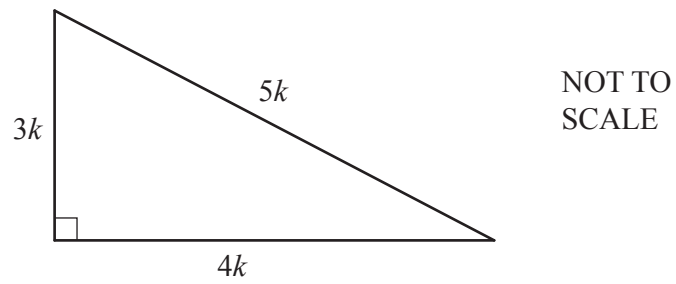
Show that your expression from **part (b)** works for right-angled triangles with sides $3k$, $4k$ and $5k$.

[2]

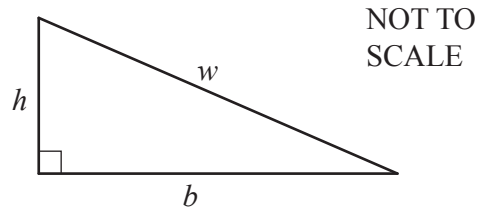**4** **(a)** An isosceles right-angled triangle has sides x, x and 10.

    (i) Use **Question 3(b)** to find an expression for the area of this triangle.
Give your answer in its simplest form.

.... [2]

    (ii) Use your answer to **part (i)** and the formula for the area of a triangle, to find the exact value of x.

.... [2]

(b)

NOT TO
SCALE

(i) By writing $u = b + h$ and using your expression from **Question 3(b)**, find an expression, in terms of u and w, for the area of any right-angled triangle.

[3]

(ii) Use Pythagoras' theorem to show that your expression from **part (i)** gives $\frac{1}{2}bh$ for all right-angled triangles.

[1]

B   MODELLING (QUESTIONS 5 to 9)

**HOT AIR BALLOON FLIGHT   (30 marks)**

You are advised to spend no more than 50 minutes on this part.

This task is about the flight of a hot air balloon.

A balloon travels in the direction of the wind.
The pilot can make the balloon rise or descend.

A journey is in four parts.

Part 1   Lift-off. The balloon leaves the ground and rises.

Part 2   The flight.

Part 3   The balloon descends quickly.
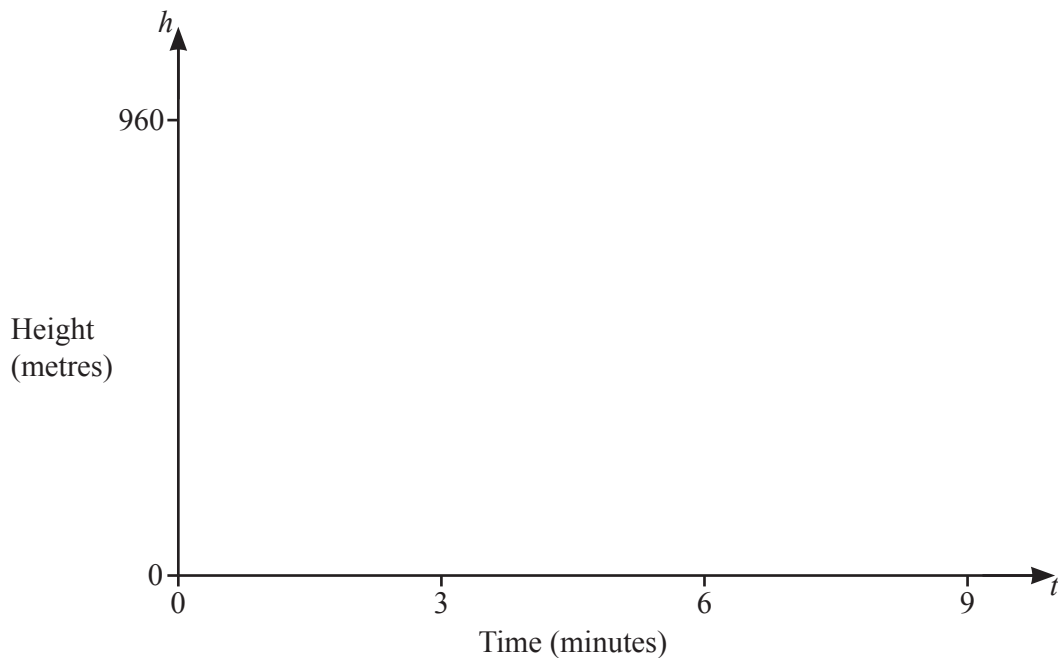
Part 4   The balloon descends slowly and lands.
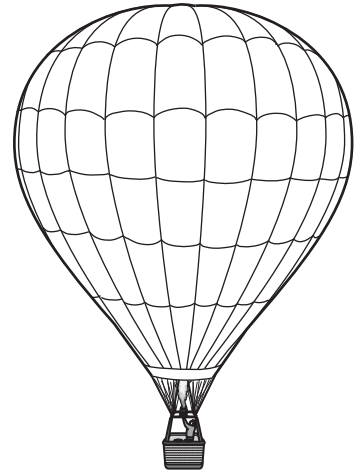
5   This journey is at sunrise.

For Part 1, a model for the height of the balloon above the ground (h metres), t minutes after lift-off, is

$$h = 480\left(1 - \cos(20t)^\circ\right) \quad \text{for } 0 \leqslant t \leqslant 9.$$

(a) On the diagram, sketch the graph of h for $0 \leqslant t \leqslant 9$.

[2]

(b) Find the height of the balloon 3 minutes after lift-off.

.... [1]

(c) Find the increase in height between 3 minutes and 6 minutes after lift-off.

.... [2]

(d) Find the average speed at which the balloon is rising between 3 minutes and 6 minutes after lift-off. Give your answer in metres per second.

.... [3]

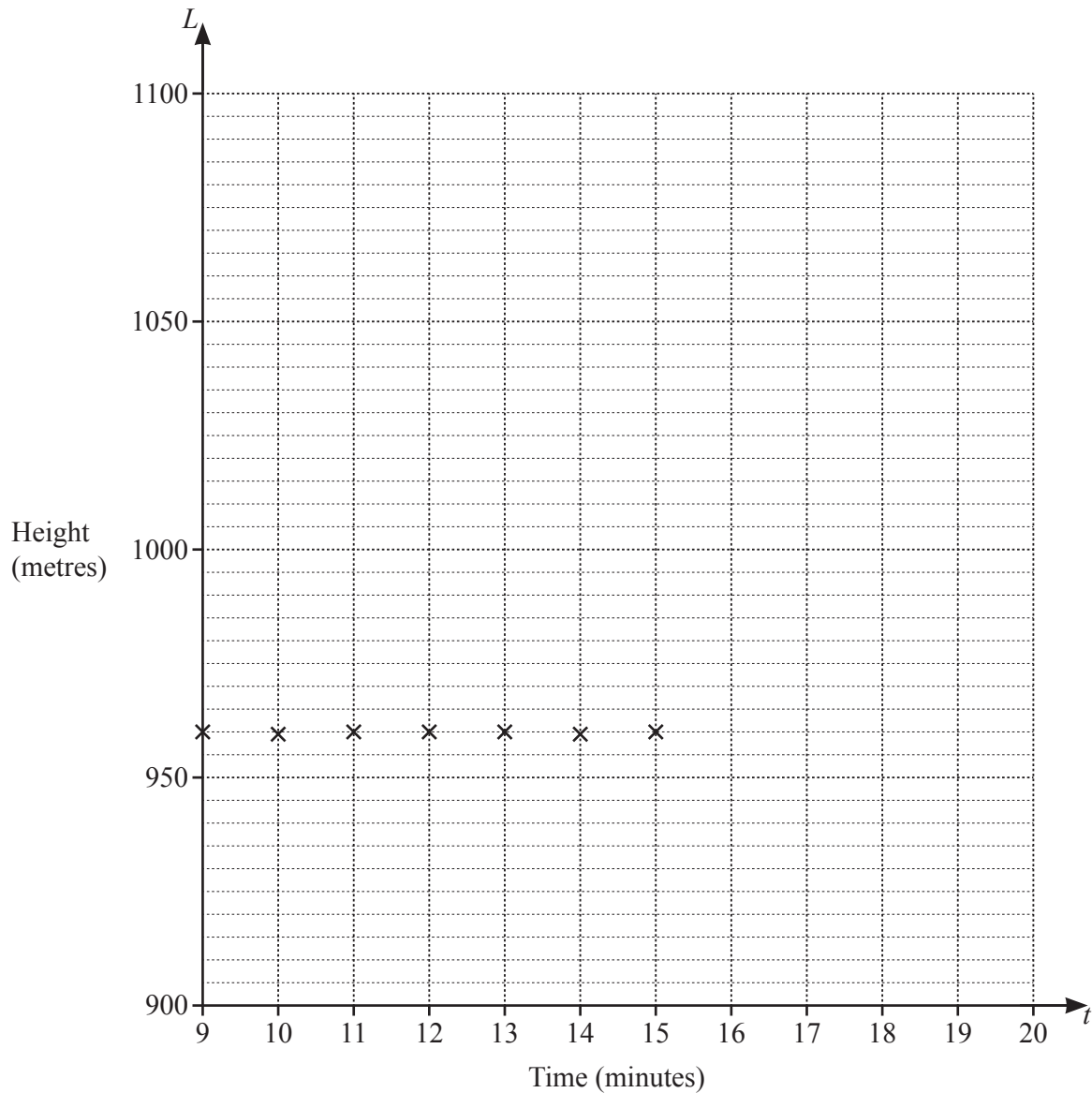**(e)** Part 1 is complete 9 minutes after lift-off.

Use the model for h in terms of t to show that the height of the balloon at this time is $960\,$m.

[1]

6   For Part 2, the table shows the height of the balloon above the ground (*L* metres), *t* minutes after lift-off.

Time (*t* minutes)	9	10	11	12	13	14	15	16	17	18	19	20
Height (*L* metres)	960	959	960	960	960	959	960	987	1014	1041	1068	1095

(a) On the grid, complete the scatter diagram for these results.
     The first seven points have been plotted for you.

[2]

(b) Between 15 minutes and 25 minutes after lift-off, the balloon rises at the same rate.
It then travels at a constant height for 10 minutes.

Complete the list of linear functions to model L for Part 2.

(i) For $9 < t \leqslant 15$ $L = $ ....

(ii) For $15 < t \leqslant 25$ $L = $ ....

(iii) For .... $< t \leqslant$ .... $L = $ ....

[5]

7 For Part 3, the balloon descends at a constant speed of 2.5 m/s until it is 180 m above the ground.

Find how many minutes it takes the balloon to travel from **lift-off** to the end of Part 3 of the journey.

.... [4]

8 For Part 4, a model for the height above the ground (d metres), t minutes after lift-off, is

$$d = \frac{450}{t - 40.125} - 60.$$

(a) Find how many minutes after lift-off the balloon lands.

.... [3]

(b) Find the average speed of the balloon during Part 4 of the journey.
Give your answer in metres per minute.

.... [2]

Question 9 is printed on the next page.

9 Another journey is at sunset.

(a) The balloon completes Part 1 of the journey in 7.5 minutes.
At the end of Part 1, the height of the balloon above the ground is 960 m.
A model for Part 1 is $h = 480\left(1 - \cos(kt)^\circ\right)$ for $0 \leqslant t \leqslant 7.5$.

Find the value of k.

.... [2]

(b) In Part 2, the first 6 minutes of the journey are at a constant height of 960 m.
Then, the balloon rises 2 times as fast as in **Question 6(b)(ii)**.

Change the model in **Question 6(b)(ii)** so that it models this part of the journey.

.... [3]