UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

FURTHER MATHEMATICS **9795/02**

Paper 2 Further Applications of Mathematics **October/November 2013**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\text{m}\,\text{s}^{-2}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

This document consists of **5** printed pages and **3** blank pages.

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UNIVERSITY *of* CAMBRIDGE International Examinations

Section A: Probability (60 marks)

1 The lifetime, T years, of a mortgage may be modelled by the random variable T with probability density function $\mathrm{f}(t)$, where

$$\mathrm{f}(t) = \begin{cases} k\sin\left(\frac{3}{32}t\right) & 0 \leqslant t \leqslant 8\pi, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $k = \frac{3}{32}\left(2 - \sqrt{2}\right)$. [4]

(ii) Sketch the graph of $\mathrm{f}(t)$ and state the mode. [3]

2 **(i)** The statistic T is derived from a random sample taken from a population which has an unknown parameter θ. T is an unbiased estimator of θ. What does the statement 'T is an unbiased estimator of θ' imply? [1]

(ii) A random sample of size n is taken from each of two independent populations. The first population has a non-zero mean μ and variance σ^2 and $\overline{X}_1$ denotes the sample mean. The second population has mean $\frac{1}{2}\mu$ and variance $b\sigma^2$, where b is a positive constant, and $\overline{X}_2$ denotes the sample mean. Two unbiased estimators for μ are defined by

$$T_1 = 3\overline{X}_1 - a\overline{X}_2 \qquad \text{and} \qquad T_2 = \tfrac{1}{5}(4\overline{X}_1 + 2\overline{X}_2).$$

(a) Determine the value of a. [3]

(b) Show that $\mathrm{Var}(T_1) = \dfrac{\sigma^2}{n}(9 + 16b)$ and find a similar expression for $\mathrm{Var}(T_2)$. [3]

(c) The estimator with the smaller variance is preferred. State which of T_1 and T_2 is the preferred estimator of μ. [1]

3 The number of signal failures in a certain region of the railway network averages 10 every 3 weeks. Assume that signal failures occur independently, randomly and at a constant mean rate.

(i) Find the probability that

(a) there are between 7 and 12 (inclusive) signal failures in a three-week period, [2]

(b) there are more than 4 signal failures in a **one**-week period. [3]

(ii) It has been calculated, using a suitable distributional approximation, that the probability of more than 62 signal failures in a period of n weeks is 0.0385. Find the value of n. [6]

4 It is given that X and Y are independent random variables with distributions $N(\mu_x, \sigma_x^2)$ and $N(\mu_y, \sigma_y^2)$ respectively, and that W is a random variable such that $W = X + Y$.

(i) Use moment generating functions to show that the distribution of W is $N(\mu_x + \mu_y, \sigma_x^2 + \sigma_y^2)$. [2]

(ii) State the distribution of $X - Y$. [1]

The diameters of the central poles of one brand of rotary clothes lines are normally distributed with mean 3.75 cm and variance 0.000 125 cm^2. The diameters of the cylindrical tubes, into which the central poles fit, are normally distributed with mean 3.85 cm and variance 0.0001 cm^2. Poles and tubes are chosen at random. The 'clearance' between a tube and a pole is the diameter of the tube minus the diameter of the pole.

(iii) Find the probability that a pole and tube have a clearance between 0.08 cm and 0.13 cm. [4]

(iv) Given that a pole and tube have a clearance between 0.08 cm and 0.13 cm, find the probability that the clearance is between 0.11 cm and 0.125 cm. [4]

5 The random variable X has a binomial distribution with parameters n and p, where $p > 0.5$. A random sample of $4n$ observations of X is taken and $\overline{X}$ denotes the sample mean. It is given that $E(\overline{X}) = 180$ and $Var(\overline{X}) = 0.0225$.

(i) Find

(a) the values of p and n, [6]

(b) $P(\overline{X} < 179.8)$, [3]

(c) the value of a for which $P(180 - a < \overline{X} < 180 + a) = 0.99$, giving your answer correct to 2 decimal places. [2]

(ii) State how you have used the Central Limit Theorem in part **(i)**. [1]

6 **(i)** Verify that $(1 - t^6) = (1 - t)(1 + t + t^2 + t^3 + t^4 + t^5)$. [1]

(ii) An unbiased six-faced die is rolled r times. Show that the probability generating function for the total score is

$$\left[\frac{t(1-t^6)}{6(1-t)}\right]^r.$$ [4]

(iii) Hence show that the probability of the total score being $(r + 3)$ is

$$\left(\tfrac{1}{6}\right)^{r+1} r(r+1)(r+2).$$ [6]

Section B: Mechanics (60 marks)

7 At a given instant two stunt cars, X and Y, are at distances 500 m and 800 m respectively from the point of intersection, O, of two straight roads crossing at right angles. The stunt cars are approaching O at uniform speeds of $15\,\mathrm{m\,s^{-1}}$ and $30\,\mathrm{m\,s^{-1}}$ respectively, one on each road. Find, in either order,

(i) the time taken to reach the point of closest approach,

(ii) the shortest distance between the stunt cars. [8]

8 A car of mass 1 tonne reaches the foot of an incline travelling at $30\,\mathrm{m\,s^{-1}}$. It reaches the top of the incline 50 seconds later travelling at $10\,\mathrm{m\,s^{-1}}$. The length of the incline is 1200 m and the angle made with the horizontal is $\sin^{-1}\left(\frac{1}{8}\right)$. The constant resistance to motion is 400 N. Find the average power developed by the engine of the car. [8]

9 A light string, of natural length 0.5 m and modulus of elasticity 4 N, has one end attached to the ceiling of a room. A particle of mass 0.2 kg is attached to the free end of the string and hangs in equilibrium.

(i) Find the extension of the string when the particle is in the equilibrium position. [2]

The particle is pulled down a further 0.5 m from the equilibrium position and released from rest. At time t seconds the displacement of the particle from the equilibrium position is x m.

(ii) Show that, while the string is taut, the equation of motion is $\ddot{x} = -40x$. [3]

(iii) Find the time taken for the string to become slack for the first time. [3]

(iv) Show that the particle comes to instantaneous rest 0.125 m below the ceiling. [3]

10 One end of a light inextensible string of length a is attached to a fixed point O. A particle of mass m is attached to the free end of the string and the particle hangs at rest vertically below O. The particle is projected horizontally with speed u.

(i) Find the tension in the string when it makes an angle θ with the downward vertical, whilst the string remains taut. [6]

(ii) Deduce that the particle will perform complete circles provided that $u^2 \geqslant 5ag$. [1]

(iii) It is given that $u^2 = 4ag$. Find

(a) the tension in the string when $\theta = 60°$, [1]

(b) the value of θ, to the nearest degree, at the instant when the string becomes slack. [2]

11 A smooth sphere of mass 2 kg has velocity $(24\mathbf{i} - 7\mathbf{j})\,\mathrm{m\,s^{-1}}$ and is travelling on a horizontal plane, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in the horizontal plane. The sphere strikes a vertical wall. The line of intersection of the wall and the plane is in the direction $(4\mathbf{i} + 3\mathbf{j})$.

(i) Show that the acute angle between the path of the sphere before the impact and the direction of the wall is $\tan^{-1}\left(\frac{4}{3}\right)$. [2]

(ii) After the impact, the velocity of the sphere is $(7.2\mathbf{i} + 15.4\mathbf{j})\,\mathrm{m\,s^{-1}}$. Find

(a) the coefficient of restitution between the sphere and the wall, [7]

(b) the magnitude of the impulse exerted by the sphere on the wall. [2]

12 A bullet of mass 0.0025 kg is fired vertically upwards from a point O. At time t s after projection the speed of the bullet is $v\,\mathrm{m\,s^{-1}}$ and the resistance to motion has magnitude $0.000\,01v^2$ N.

(i) Show that, while the bullet is rising,

$$250\frac{\mathrm{d}v}{\mathrm{d}t} = -2500 - v^2.$$ [2]

(ii) It is given that the speed of projection is $350\,\mathrm{m\,s^{-1}}$. Find

(a) the time taken after projection for the bullet to reach its greatest height above O, [5]

(b) the greatest height above O reached by the bullet. [5]

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