

		Centre Number			
Candidate Numbe					
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General Certificate of Secondary Education 2019

Further Mathematics

Unit 1 (With calculator)

Pure Mathematics





[**GFM11**]

GFM11

FRIDAY 14 JUNE, AFTERNOON

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page. You must answer the questions in the spaces provided.

Do not write outside the boxed area on each page.

Complete in black ink only. Do not write with a gel pen.

All working **must** be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.

Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise. Answer **all fourteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

You may use a calculator.

The Formula Sheet is on page 2.



Formula Sheet

PURE MATHEMATICS

Quadratic equations: If
$$ax^2 + bx + c = 0$$
 $(a \ne 0)$

then
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Differentiation: If
$$y = ax^n$$
 then $\frac{dy}{dx} = nax^{n-1}$

Integration:
$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c \qquad (n \neq -1)$$

Logarithms: If
$$a^x = n$$
 then $x = \log_a n$

$$\log\left(ab\right) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log a^n = n \log a$$

Matrices: If
$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

then
$$\det \mathbf{A} = ad - bc$$

and
$$\mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
 $(ad - bc \neq 0)$

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$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$$

Find the value of $\mathbf{B} - \mathbf{A}^2$

Answer _____ [4]

[Turn over

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2 A function f(x) is defined by

$$f(x) = x^2 - x + 4$$

(i) Use the method of completing the square to rewrite f(x) in the form

$$(x + a)^2 + b$$

where a and b are constants.

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(ii) Hence find the minimum value of f(x) and the value of x for which it occurs.

Answer Minimum value _____[1]

when
$$x =$$
 [1]



3 (a) Find $\frac{dy}{dx}$ if $y = \frac{3}{8}x^2 + \frac{5}{x^4} - 12x$

Answer _____[3]

(b) Find $\int \left(\frac{3}{4x^2} - 2x^3\right) dx$

Answer _____[3]

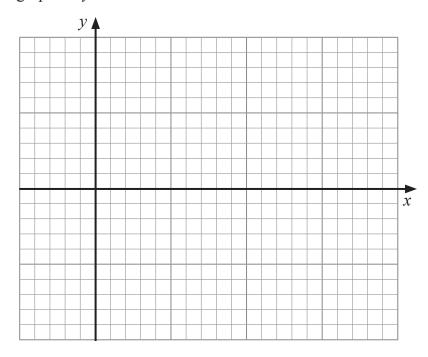
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4 (a) Sketch the graph of $y = \cos x$ for $-90^{\circ} \le x \le 360^{\circ}$



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[2]

(b) (i) Solve the equation

$$\cos x = -0.184$$

for
$$0^{\circ} \leq x \leq 360^{\circ}$$

Give your answers correct to 1 decimal place.

Answers _____[2]



(ii) Hence solve the equation

$$\cos\left(2\theta - 15^{\circ}\right) = -0.184$$

for
$$90^{\circ} \leq \theta \leq 180^{\circ}$$

Give your answer correct to 1 decimal place.

Answer [2]

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5 Matrices **P** and **Q** are defined by

$$\mathbf{P} = \begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix} \text{ and } \mathbf{Q} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

Using a matrix method, find the matrix X such that

$$PX = Q$$

Answer [4]

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$$x^2 - 4x - 5 < 0$$

You **must** show clearly each stage of your solution.

Answer _____[4]

Turn over



7 Solve the following set of simultaneous equations

$$2x + 3y + z = 5$$

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$$3x - 4y + 2z = -9$$

$$x + 5y - 3z = 6$$

You **must** show clearly each stage of your solution.

Answer
$$x =$$
, $y =$, $z =$ [8]



You may use this page for Question 7 if needed. (Questions continue overleaf) [Turn over 11900

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8	A curve is defined by the equation	y = x(3x - 5)(x + 1)

(i) Write down the **coordinates** of the points where the curve meets the x-axis.

Answer _____[3]

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(ii) Find the coordinates of the turning points of the curve.

Answer ______[6]



(iii) Using calculus, identify each turning point as either a maximum or a minimum point. You must show working to justify your answer. Answer _____ [2] (iv) Sketch the curve on the axes below. [2] [Turn over

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9 (a) If $2 \log y = 3 \log x$ write y in terms of x.

Answer _____ [2]

(b) (i) If $4 \times 2^x = 2^y$ show that

$$y = x + 2$$

[1]

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Com (ii) Hence or otherwise solve the equation $6^{3x-2} = 4 \times 2^x$

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Answer _____[4]

[Turn over

10 Rory has a set of objects, each with a circular base. He records the base radius, r cm, and the volume, $V \text{ cm}^3$, of 5 of these objects.

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The results are given in the table below.

Base radius r (cm)	Volume V (cm ³)	
3.4	98.0	
4.6	221.7	
5.2	308.7	
6.8	636.9	
7.5	829.8	

Rory believes that a relationship of the form

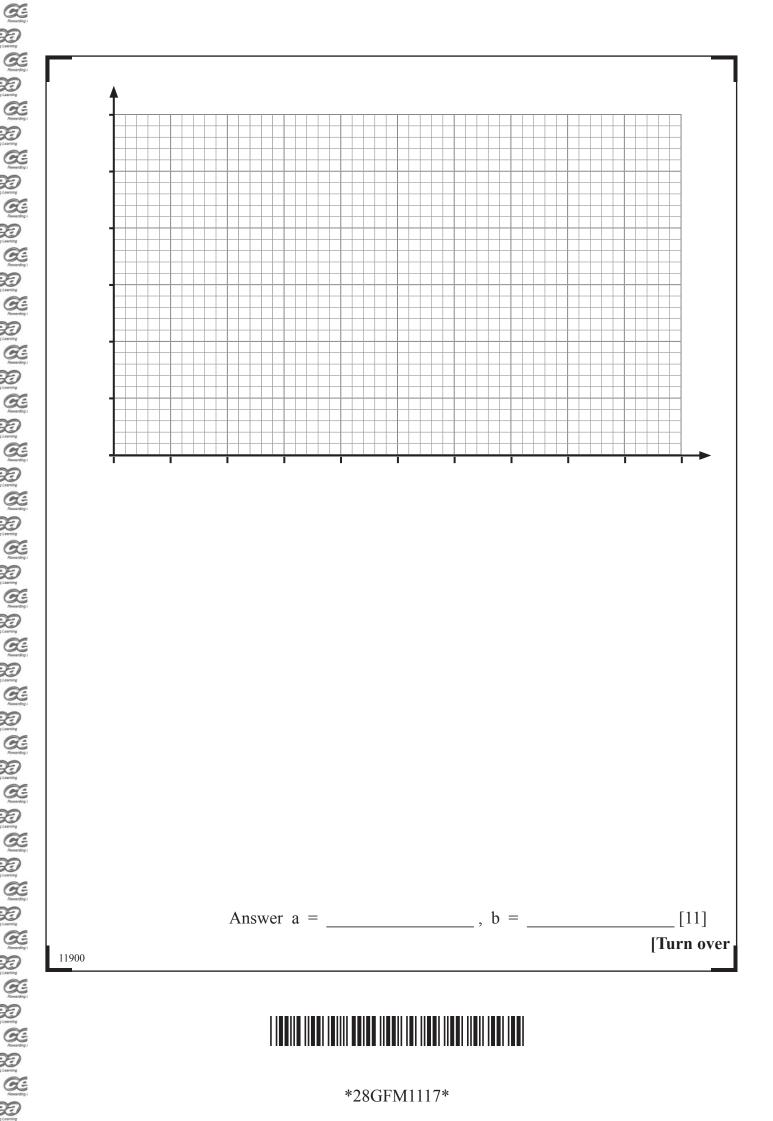
$$V = ar^b$$

exists, where a and b are constants.

(i) Verify that a relationship of the form $V = ar^b$ exists by drawing a suitable straight line graph on the grid opposite.

Show clearly the values used, correct to 3 decimal places, in the table above.

Hence find the values of a and b, correct to 1 decimal place.





Use the formula $V = ar^b$ with your values for a and b to calculate				
(ii) the volume of an object with radius 5.6 cm, giving your answer to the nearest				
	Answer	cm ³ [1		
	7 1115 W C1			
Give your answer correct to t make.	the nearest cm and state any assum	ption that you		
	Answer	cm [2		
Assumption				
		[1		

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11 Find the equation of the **normal** to the curve

$$y = 2 - \frac{3}{x}$$

at the point where the curve cuts the *x*-axis.

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Answer ______[5]

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12. ((i)	Expand	and	simplify	, the	expression
14 (1)	Expand	anu	Simping	/ uic	CAPICSSIOII

$$(x + 3)(x - 4)(2x + 5)$$

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Answer [3]



(ii) Hence simplify fully the expression

$$\frac{(x+3)(x-4)(2x+5) - 2x(x^2+2) - 18}{x^2 - 13x}$$

Answer _____ [4]

[Turn over

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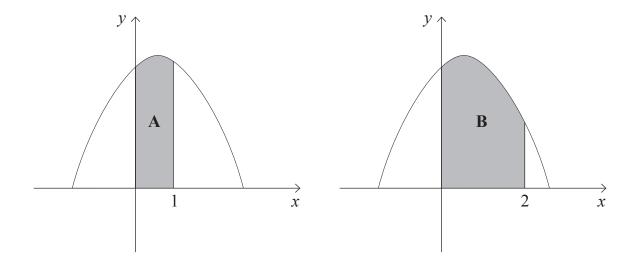
$$y = k + 2x - 3x^2$$

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where k is a constant.



(i) Find expressions in terms of k for the area of A and the area of B.



Answer Area of A	[3]
Area of B	[2]
(ii) Given that the area of A is $\frac{5}{9}$ of the area of B , find the variable.	lue of k .
Answer	[2]
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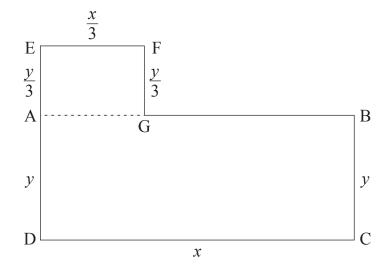
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14 The owners of a hotel wish to build a swimming pool in the hotel grounds. They plan to build a rectangular pool ABCD for adults, of length *x* m and width *y* m.

At one end of the pool they plan to add a children's pool AEFG, as shown in the diagram below.

The length and width of the children's pool are to be $\frac{1}{3}$ of the length and width of the adults' pool, respectively.



Write down, in terms of x and y,

(i) the total area of the two pools,

Answer _____ m² [1]

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-	(ii)	the total length	of the perimete	er round the oute	er edges of	the nools
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Answer m [1

The total length of the perimeter round the outer edges of the pools is 96 m.

(iii) Show that

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$$y = 36 - \frac{3}{4}x$$

[2]

[Turn over



(iv)	Find the dimensions of the pools wh that it is a maximum.	ich will give a maximum total area,	showing
	Answer Adults' pool	m by	m
		m by	
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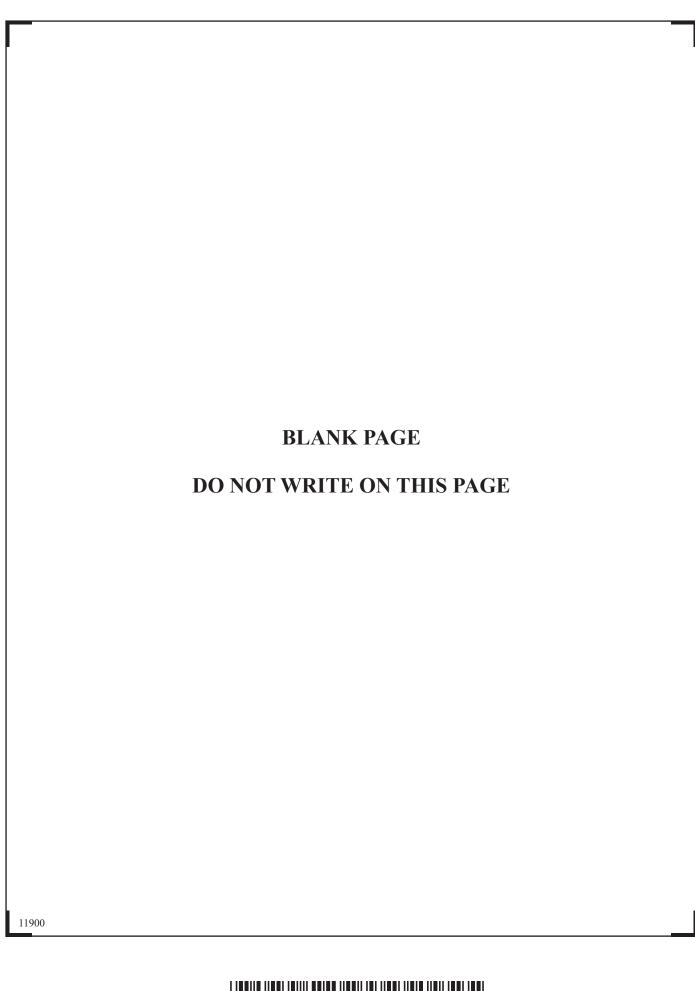
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