GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Mark Schemes for the Units

June 2007

3890-2/7890-2/MS/R/07

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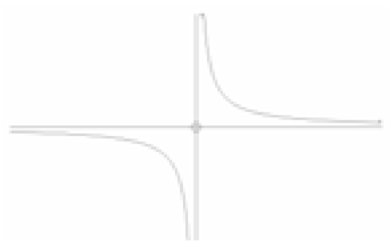
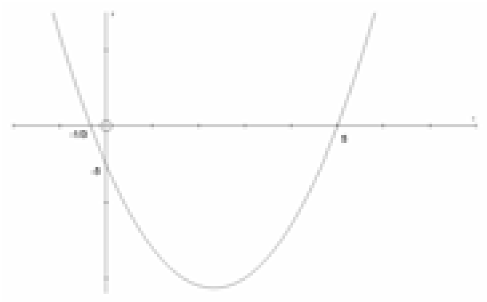
MARK SCHEME ON THE UNITS

Mark Scheme 4721
June 2007

1	$(4x^2 + 20x + 25) – (x^2 - 6x + 9)$ $= 3x^2 + 26x + 16$	M1 A1 A1 3	Square one bracket to give an expression of the form $ax^2 + bx + c$ $(a \neq 0, b \neq 0, c \neq 0)$ One squared bracket fully correct All 3 terms of final answer correct
	Alternative method using difference of two squares: $(2x + 5 + (x – 3))(2x + 5 – (x – 3))$ $= (3x + 2)(x + 8)$ $= 3x^2 + 26x + 16$	**3**	M1 2 brackets with same terms but different signs A1 One bracket correctly simplified A1 All 3 terms of final answer correct
2 (a)(i)		B1 B1 2	Excellent curve for $\frac{1}{x}$ in either quadrant Excellent curve for $\frac{1}{x}$ in other quadrant **SR B1** Reasonably correct curves in 1st and 3rd quadrants
(ii)		B1 1	Correct graph, minimum point at origin, symmetrical
(b)	Stretch Scale factor 8 in y direction **or** scale factor ½ in x direction	B1 B1 2 **5**	
3 (i)	$3\sqrt{20}$ or $3\sqrt{2}\ \sqrt{5} \times \sqrt{2}$ or $\sqrt{180}$ or $\sqrt{90} \times \sqrt{2}$ $= 6\sqrt{5}$	M1 A1 2	 Correctly simplified answer
(ii)	$10\sqrt{5} + 5\sqrt{5}$ $= 15\sqrt{5}$	M1 B1 A1 3 **5**	Attempt to change both surds to $\sqrt{5}$ One part correct and fully simplified cao

4 (i)	$(-4)^2 - 4 \times k \times k$ $= 16 - 4k^2$	M1 A1 2	Uses $b^2 - 4ac$ (involving k) $16 - 4k^2$
(ii)	$16 - 4k^2 = 0$ $k^2 = 4$ $k = 2$ or $k = -2$	M1 B1 B1 3 **5**	Attempts $b^2 - 4ac = 0$ (involving k) or attempts to complete square (involving k)
5 (i)	Length = 20 – 2x Area = x(20 – 2x) = $20x - 2x^2$	M1 A1 2	Expression for length of enclosure in terms of x Correctly shows that area = $20x - 2x^2$ **AG**
(ii)	$\frac{dA}{dx} = 20 - 4x$ For max, 20 – 4x = 0 x = 5 only Area = 50	M1 M1 A1 A1 4 **6**	Differentiates area expression Uses $\frac{dy}{dx} = 0$
6	Let $y = (x + 2)^2$ $y^2 + 5y - 6 = 0$ (y + 6)(y - 1) = 0 y = -6 or y = 1 $(x + 2)^2 = 1$ x = -1 or x = -3	B1 M1 A1 M1 A1 A1 6 **6**	Substitute for $(x + 2)^2$ to get $y^2 + 5y - 6$ (= 0) Correct method to find roots Both values for y correct Attempt to work out x One correct value Second correct value and no extra real values
7 (a)	$f(x) = x + 3x^{-1}$ $f'(x) = 1 - 3x^{-2}$	M1 A1 A1 A1 4	Attempt to differentiate First term correct x^{-2} soi **www** Fully correct answer
(b)	$\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}}$ When x = 4, $\frac{dy}{dx} = \frac{5}{2}\sqrt{4^3}$ = 20	M1 B1 B1 M1 A1 5 **9**	Use of differentiation to find gradient $\frac{5}{2}x^c$ $kx^{\frac{3}{2}}$ $\sqrt{4^3}$ soi **SR** If 0 scored for first 3 marks, award B1 if $\sqrt{4^n}$ correctly evaluated.

8 (i)	$(x + 4)^2 - 16 + 15$ $= (x + 4)^2 - 1$	B1 M1 A1 3	a = 4 15 – their a^2 cao in required form
(ii)	(-4, -1)	B1 ft B1 ft 2	Correct x coordinate Correct y coordinate
		M1 A1	Correct method to find roots -5, -3
(iii)	$x^2 + 8x + 15 > 0$ $(x + 5)(x + 3) > 0$	M1	Correct method to solve quadratic inequality eg +ve quadratic graph
	$x < -5, x > -3$	A1 4 **9**	$x < -5, x > -3$ (not wrapped, strict inequalities, no 'and')
9 (i)	$(x - 3)^2 - 9 + y^2 - k = 0$ $(x - 3)^2 + y^2 = 9 + k$ Centre (3, 0) $9 + k = 4^2$ $k = 7$	B1 B1 M1 A1 4	$(x-3)^2$ soi Correct centre Correct value for k (may be embedded) <u>Alternative method using expanded form:</u> Centre ($-g$, $-f$) M1 Centre (3, 0) A1 $4 = \sqrt{f^2 + g^2 - (-k)}$ M1 $k = 7$ A1
(ii)	$(3 - 3)^2 + y^2 = 16$ $y^2 = 16$ $y = 4$	M1 A1	Attempt to substitute x = 3 into original equation or their equation y = 4 (do not allow ± 4)
	Length of AB = $\sqrt{(-1-3)^2 + (0-4)^2}$ $= \sqrt{32}$ $= 4\sqrt{2}$	M1 A1 ft A1 5	Correct method to find line length using Pythagoras' theorem $\sqrt{32}$ or $\sqrt{16+a^2}$ cao
(iii)	Gradient of AB = 1 or $\frac{a}{4}$ $y - 0 = m(x + 1)$ or $y - 4 = m(x - 3)$ $y = x + 1$	B1 ft M1 A1 3 **12**	 Attempts equation of straight line through their A or B with their gradient Correct equation in any form with simplified constants

10 (i)	(3x + 1)(x – 5) = 0	M1	Correct method to find roots
	$x = \frac{-1}{3}$ or x = 5	A1	Correct brackets or formula
		A1 3	Both values correct
			SR B1 for x = 5 spotted **www**
(ii)		B1	Positive quadratic (must be reasonably symmetrical)
		B1	y intercept correct
		B1 ft 3	both x intercepts correct
(iii)	$\frac{dy}{dx} = 6x - 14$	M1*	Use of differentiation to find gradient of curve
	6x – 14 = 4	M1*	Equating their gradient expression to 4
	x = 3	A1	
	On curve, when x = 3, y = -20	A1 ft	Finding y co ordinate for their x value
	-20 = (4 x 3) + c	M1dep	N.B. dependent on both previous M marks
	c = -32	A1 6	
	<u>Alternative method:</u>		
	$3x^2 - 14x - 5 = 4x + c$	M1	Equate curve and line (or substitute for x)
	$3x^2 - 18x - 5 - c = 0$ has one solution	B1	Statement that only one solution for a tangent (may be implied by next line)
	$b^2 - 4ac = 0$	M1	Use of discriminant = 0
	$(-18)^2 - (4 \times 3 \times (-5 - c)) = 0$	M1	Attempt to use a, b, c from their equation
	c = -32	A1	Correct equation
		A1	c = -32
		12	

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1	**(i)** $u_2 = 12$	B1		State $u_2 = 12$
	$u_3 = 9.6$, $u_4 = 7.68$ (or any exact equivs)	B1√	2	Correct u_3 and u_4 from their u_2
	(ii) $S_{20} = \frac{15(1-0.8^{20})}{1-0.8}$	M1		Attempt use of $S_n = \frac{a(1-r^n)}{1-r}$, with n = 20 or 19
	$= 74.1$	A1		Obtain correct unsimplified expression
		A1	3	Obtain 74.1 or better
	OR			
		M1		List all 20 terms of GP
		A2		Obtain 74.1
			5	
2	$\left(x+\frac{2}{x}\right)^4 = x^4 + 4x^3\left(\frac{2}{x}\right) + 6x^2\left(\frac{2}{x}\right)^2 + 4x\left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$	M1*		Attempt expansion, using powers of x and $^2/_x$ (or the two terms in their bracket), to get at least 4 terms
		M1*		Use binomial coefficients of 1, 4, 6, 4, 1
		A1dep*		Obtain two correct, simplified, terms
		A1		Obtain a further one correct, simplified, term
	$= x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$ (or equiv)	A1	5	Obtain a fully correct, simplified, expansion
	OR			
		M1*		Attempt expansion using all four brackets
		M1*		Obtain expansion containing the correct 5 powers only (could be unsimplified powers eg $x^3 . x^{-1}$)
		A1dep*		Obtain two correct, simplified, terms
		A1		Obtain a further one correct, simplified, term
		A1		Obtain a fully correct, simplified, expansion
			5	
3	$\log 3^{(2x+1)} = \log 5^{200}$	M1		Introduce logarithms throughout
	$(2x+1)\log 3 = 200\log 5$	M1		Drop power on at least one side
		A1		Obtain correct linear equation (now containing no powers)
	$2x+1 = \frac{200\log 5}{\log 3}$	M1		Attempt solution of linear equation
	$x = 146$	A1	5	Obtain x = 146, or better
OR				
	$(2x + 1) = \log_3 5^{200}$	M1		Intoduce $\log_3$ on right-hand side
	$2x + 1 = 200\log_3 5$	M1		Drop power of 200
		A1		Obtain correct equation
		M1		Attempt solution of linear equation
		A1		Obtain x = 146, or better
			5	
4	**(i)** area $\approx \frac{1}{2}\times\frac{1}{2}\times\left\{\sqrt{5}+2\left(\sqrt{7}+\sqrt{9}+\sqrt{11}\right)+\sqrt{13}\right\}$	M1		Attempt y-values for at least 4 of the x = 1, 1.5, 2, 2.5, 3 only
		M1		Attempt to use correct trapezium rule
		A1		Obtain $\frac{1}{2}\times\frac{1}{2}\times\left\{\sqrt{5}+2\left(\sqrt{7}+\sqrt{9}+\sqrt{11}\right)+\sqrt{13}\right\}$, or decimal equiv
	$\approx 0.25\times 23.766...$			
	≈ 5.94	A1	4	Obtain 5.94 or better (answer only is 0/4)
	(ii) This is an underestimate……	*B1		State underestimate
	…as the tops of the trapezia are below the curve	B1dep*B1 **2**		Correct statement or sketch
			6	

5	**(i)** $3\left(1-\sin^2\theta\right)=\sin\theta+1$	M1		Use $\cos^2\theta=1-\sin^2\theta$
	$3-3\sin^2\theta=\sin\theta+1$			
	$3\sin^2\theta+\sin\theta-2=0$	A1	2	Show given equation correctly
	(ii) $(3\sin\theta-2)(\sin\theta+1)=0$	M1		Attempt to solve quadratic equation in $\sin\theta$
	$\sin\theta=\frac{2}{3}$ or -1	A1		Both values of $\sin\theta$ correct
	θ = 42°, 138°, 270°	A1		Correct answer of 270°
		A1		Correct answer of 42°
		A1√	5	For correct non-principal value answer, following their first value of θ in the required range (any extra values for θ in required range is max 4/5) (radians is max 4/5) SR: answer only (or no supporting method) is B1 for 42°, B1√ for 138°, B1 for 270°
			7	
6	**(a) (i)** $\int x^3-4x=\frac{1}{4}x^4-2x^2+c$	M1		Expand and attempt integration
		A1		Obtain ¼ x^4– $2x^2$ (A0 if ∫ or dx still present)
		B1	3	+ c (mark can be given in (b) if not gained here)
	(ii) $\left[\frac{1}{4}x^4-2x^2\right]_1^6$	M1		Use limits correctly in integration attempt (ie F(6) – F(1))
	=(324 – 72) – (¼ – 2)			
	= 253¾	A1	2	Obtain 253¾ (answer only is M0A0)
	(b) $\int 6x^{-3}\mathrm{d}x=-3x^{-2}+c$	B1		Use of $\frac{1}{x^3}=x^{-3}$
		M1		Obtain integral of the form kx^{-2}
		A1	3	Obtain correct -3x^{-2} (+ c) (A0 if ∫ or dx still present, but only penalise once in question)
			8	
7	**(a)** $S_{70}=\frac{70}{2}\left\{(2\times12)+(70-1)d\right\}$	M1		Attempt S_{70}
		A1		Obtain correct unsimplified expression
	35(24 + 69d) = 12915	M1		Equate attempt at S_{70} to 12915, and attempt to find d
	d = 5	A1	4	Obtain d = 5
OR				
	$\frac{70}{2}\left\{12+l\right\}=12915$	M1		Attempt to find d by first equating $^n/_2\,(a+l)$ to 12915
	l = 357	A1		Obtain l = 357
	12 + 69d = 357	M1		Equate u_{70} to l
	d = 5	A1		Obtain d = 5
	(b) ar = -4	B1		Correct statement for second term
	$\frac{a}{1-r}=9$	B1		Correct statement for sum to infinity
	$\frac{-4}{r}=9-9r$ or $a=9-\left(9\times\frac{-4}{a}\right)$	M1		Attempt to eliminate either a or r
	$9r^2-9r-4=0$ $\quad a^2-9a-36=0$	A1		Obtain correct equation (no algebraic denominators/brackets)
	$(3r-4)(3r+1)=0$ $\quad (a+3)(a-12)=0$	M1		Attempt solution of three term quadratic equation
	$r=\frac{4}{3}$, $r=-\frac{1}{3}$ $\quad a$ = -3, a = 12	A1		Obtain at least $r=-\frac{1}{3}$ (from correct working only)
	Hence $r=-\frac{1}{3}$	A1	7	Obtain $r=-\frac{1}{3}$ only (from correct working only) SR: answer only / T&I is B2 only
			11	

8 **(i)** $\frac{1}{2}\times AB^2\times 0.9=16.2$	M1	Use $\left(\frac{1}{2}\right)r^2\theta=16.2$
$AB^2=36\Rightarrow AB=6$	A1 **2**	Confirm AB = 6 cm (or verify ½ x 6^2 x 0.9 = 16.2)
(ii) $\frac{1}{2}\times 6\times AC\times\sin 0.9=32.4$	M1*	Use $\Delta=\frac{1}{2}bc\sin A$, or equiv
	M1dep*	Equate attempt at area to 32.4
AC = 13.8 cm	A1 **3**	Obtain AC = 13.8 cm, or better
(iii) $BC^2=6^2+13.8^2-2\times 6\times 13.8\times\cos 0.9$	M1	Attempt use of correct cosine formula in ΔABC
	A1√	Correct unsimplified equation, from their AC
Hence BC = 11.1 cm	A1	Obtain BC = 11.1 cm, or anything that rounds to this
$BD=6\times 0.9=5.4$ cm	B1	State BD = 5.4 cm (seen anywhere in question)
Hence perimeter = 11.1 + 5.4 + (13.8 – 6)	M1	Attempt perimeter of region BCD
= 24.3 cm	A1 **6**	Obtain 24.3 cm, or anything that rounds to this
	11	
9 **(i)** **(a)** f (-1) = – 1 + 6 – 1 – 4 = 0	B1 **1**	Confirm f(-1) = 0, through any method
(b) x = -1	B1	State x = -1 at any point
$f(x)=(x+1)(x^2+5x-4)$	M1	Attempt complete division by (x + 1), or equiv
	A1	Obtain x^2+5x+k
	A1	Obtain completely correct quotient
$x=\dfrac{-5\pm\sqrt{25+16}}{2}$	M1	Attempt use of quadratic formula, or equiv, find roots
$x=\frac{1}{2}\left(-5\pm\sqrt{41}\right)$	A1 **6**	Obtain $\frac{1}{2}\left(-5\pm\sqrt{41}\right)$
(ii) **(a)** $\log_2(x+3)^2+\log_2 x-\log_2(4x+2)=1$	B1	State or imply that 2log (x + 3) = log $(x+3)^2$
	M1	Add or subtract two, or more, of their algebraic logs correctly
$\log_2\left(\frac{(x+3)^2 x}{4x+2}\right)=1$	A1	Obtain correct equation (or any equivalent, with single term on each side)
$\frac{(x+3)^2 x}{4x+2}=2$	B1	Use $\log_2 a=1\Rightarrow a=2$ at any point
$(x^2+6x+9)x=8x+4$		
$x^3+6x^2+x-4=0$	A1 **5**	Confirm given equation correctly
(b) $x>0$, otherwise $\log_2 x$ is undefined	B1*	State or imply that log x only defined for $x>0$
$x=\frac{1}{2}\left(-5+\sqrt{41}\right)$	B1√dep*	State $x=\frac{1}{2}\left(-5+\sqrt{41}\right)$ (or x = 0.7) only, following their single positive root in (i)(b)
	2	
	14	

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June 2007

1	(i)	Attempt use of product rule	M1		
		Obtain $3x^2(x+1)^5+5x^3(x+1)^4$	A1	2	or equiv
		[Or: (following complete expansion and differentiation term by term)			
		Obtain $8x^7+35x^6+60x^5+50x^4+20x^3+3x^2$	B2		allow B1 if one term incorrect]
	(ii)	Obtain derivative of form $kx^3(3x^4+1)^n$	M1		any constants k and n
		Obtain derivative of form $kx^3(3x^4+1)^{-\frac{1}{2}}$	M1		
		Obtain correct $6x^3(3x^4+1)^{-\frac{1}{2}}$	A1	3	or (unsimplified) equiv
2		Identify critical value $x = 2$	B1		
		Attempt process for determining both critical values	M1		
		Obtain $\frac{1}{3}$ and 2	A1		
		Attempt process for solving inequality	M1		table, sketch …; implied by plausible answer
		Obtain $\frac{1}{3}<x<2$	A1	5	
3	(i)	Attempt correct process for composition	M1		numerical or algebraic
		Obtain (16 and hence) 7	A1	2	
	(ii)	Attempt correct process for finding inverse	M1		maybe in terms of y so far
		Obtain $(x-3)^2$	A1	2	or equiv; in terms of x, not y
	(iii)	Sketch (more or less) correct $y = \mathrm{f}(x)$	B1		with 3 indicated or clearly implied on y-axis, correct curvature, no maximum point
		Sketch (more or less) correct $y = \mathrm{f}^{-1}(x)$	B1		right hand half of parabola only
		State reflection in line $y = x$	B1	3	or (explicit) equiv; independent of earlier marks
4	(i)	Obtain integral of form $k(2x+1)^{\frac{4}{3}}$	M1		or equiv using substitution; any constant k
		Obtain correct $\frac{3}{8}(2x+1)^{\frac{4}{3}}$	A1		or equiv
		Substitute limits in expression of form $(2x+1)^n$ and subtract the correct way round	M1		using adjusted limits if subn used
		Obtain 30	A1	4	
	(ii)	Attempt evaluation of $k(y_0+4y_1+y_2)$	M1		any constant k
		Identify k as $\frac{1}{3}\times 6.5$	A1		
		Obtain 29.6	A1	3	or greater accuracy (29.554566…)
		[SR: (using Simpson's rule with 4 strips)			
		Obtain $\frac{1}{3}\times 3.25(1+4\times\sqrt[3]{7.5}+2\times\sqrt[3]{14}+4\times\sqrt[3]{20.5}+3)$ and hence 29.9	B1		or greater accuracy (29.897…)]

5	**(i)**	State $e^{-0.04t} = 0.5$	B1		or equiv
		Attempt solution of equation of form $e^{-0.04t} = k$	M1		using sound process; maybe implied
		Obtain 17	A1	**3**	or greater accuracy (17.328…)
	(ii)	Differentiate to obtain form $k\,e^{-0.04t}$	*M1		constant k different from 240
		Obtain $(\pm)\ 9.6e^{-0.04t}$	A1		or (unsimplified) equiv
		Equate attempt at first derivative to (±) 2.1 and attempt solution	M1		dep *M; method maybe implied
		Obtain 38	A1	**4**	or greater accuracy (37.9956…)
6	**(i)**	Obtain integral of form $k_1 e^{2x} + k_2 x^2$	M1		any non-zero constants k_1, k_2
		Obtain correct $3e^{2x} + \frac{1}{2}x^2$	A1		
		Obtain $3e^{2a} + \frac{1}{2}a^2 - 3$	A1		
		Equate definite integral to 42 and attempt rearrangement	M1		using sound processes
		Confirm $a = \frac{1}{2}\ln(15 - \frac{1}{6}a^2)$	A1	**5**	AG; necessary detail required
	(ii)	Obtain correct first iterate 1.348…	B1		
		Attempt correct process to find at least 2 iterates	M1		
		Obtain at least 3 correct iterates	A1		
		Obtain 1.344	A1	**4**	answer required to exactly 3 d.p.; allow recovery after error
		[1 → 1.34844 → 1.34382 → 1.34389]			
7	**(i)**	Show correct general shape (alternating above and below x-axis)	M1		with no branch reaching x-axis
		Draw (more or less) correct sketch	A1	**2**	with at least one of 1 and –1 indicated or clearly implied
	(ii)	Attempt solution of $\cos x = \frac{1}{3}$	M1		maybe implied; or equiv
		Obtain 1.23 or 0.392π	A1		or greater accuracy
		Obtain 5.05 or 1.61π	A1	**3**	or greater accuracy and no others within $0 \le x \le 2\pi$; penalise answer(s) to 2sf only once
	(iii)	Either: Obtain equation of form $\tan\theta = k$	M1		any constant k; maybe implied
		Obtain $\tan\theta = 5$	A1		
		Obtain two values only of form $\theta,\ \theta + \pi$	M1		within $0 \le x \le 2\pi$; allow degrees at this stage
		Obtain 1.37 and 4.51 (or 0.437π and 1.44π)	A1	**4**	allow ±1 in third sig fig; or greater accuracy
		Or: (for methods which involve squaring,etc.) Attempt to obtain eqn in one trig ratio	M1		
		Obtain correct value	A1		$\tan^2\theta = 25, \cos^2\theta = \frac{1}{26}$, …
		Attempt solution at least to find one value in first quadrant and one value in third	M1		
		Obtain 1.37 and 4.51 (or equivs as above)	A1		ignoring values in second and fourth quadrants

8	(i)	Attempt use of quotient rule	M1		allow for numerator 'wrong way round'; or equiv
		Obtain $\dfrac{(4\ln x+3)\frac{4}{x}-(4\ln x-3)\frac{4}{x}}{(4\ln x+3)^2}$	A1		or equiv
		Confirm $\dfrac{24}{x(4\ln x+3)^2}$	A1	3	AG; necessary detail required
	(ii)	Identify $\ln x=\frac{3}{4}$	B1		or equiv
		State or imply $x=e^{\frac{3}{4}}$	B1		
		Substitute e^k completely in expression for derivative	M1		and deal with $\ln e^k$ term
		Obtain $\frac{2}{3}e^{-\frac{3}{4}}$	A1	4	or exact (single term) equiv
	(iii)	State or imply $\int \dfrac{4\pi}{x(4\ln x+3)^2}\,dx$	B1		
		Obtain integral of form $k\dfrac{4\ln x-3}{4\ln x+3}$ or $k(4\ln x+3)^{-1}$	*M1		any constant k
		Substitute both limits and subtract right way round	M1		dep *M
		Obtain $\frac{4}{21}\pi$	A1	4	or exact equiv
9	(i)	Attempt use of either of $\tan(A\pm B)$ identities	M1		
		Substitute $\tan 60°=\sqrt{3}$ or $\tan^2 60°=3$	B1		
		Obtain $\dfrac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}\times\dfrac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta}$	A1		or equiv (perhaps with tan 60° still involved)
		Obtain $\dfrac{\tan^2\theta-3}{1-3\tan^2\theta}$	A1	4	AG
	(ii)	Use $\sec^2\theta=1+\tan^2\theta$	B1		
		Attempt rearrangement and simplification of equation involving $\tan^2\theta$	M1		or equiv involving $\sec\theta$
		Obtain $\tan^4\theta=\frac{1}{3}$	A1		or equiv $\sec^2\theta=1.57735...$
		Obtain 37.2	A1		or greater accuracy
		Obtain 142.8	A1	5	or greater accuracy; and no others between 0 and 180
	(iii)	Attempt rearrangement of $\dfrac{\tan^2\theta-3}{1-3\tan^2\theta}=k^2$ to form $\tan^2\theta=\dfrac{f(k)}{g(k)}$	M1		
		Obtain $\tan^2\theta=\dfrac{k^2+3}{1+3k^2}$	A1		
		Observe that RHS is positive for all k, giving one value in each quadrant	A1	3	or convincing equiv

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1	(i) Correct format $\frac{A}{x+2}+\frac{B}{x-3}$	M1		s.o.i. in answer
	$A=1$ and $B=2$	A1	**2**	for both
	(ii) $-A(x+2)^{-2}-B(x-3)^{-2}$ f.t.	√A1		
	Convincing statement that each denom > 0	B1		accept ≥ 0. Do not accept $x^2>0$.
	State whole exp < 0 **AG**	B1	**3**	Dep on previous 4 marks.
				5
2	Use parts with $u=x^2, \mathrm{d}v=\mathrm{e}^x$	*M1		obtaining a result $\mathrm{f}(x)+/-\int \mathrm{g}(x)(\mathrm{d}x)$
	Obtain $x^2\mathrm{e}^x-\int 2x\mathrm{e}^x\,(\mathrm{d}x)$	A1		
	Attempt parts again with $u=(-)(2)x, \mathrm{d}v=\mathrm{e}^x$	M1		
	Final = $(x^2-2x+2)\mathrm{e}^x$ AEF incl brackets	A1		s.o.i. eg $\mathrm{e}+(-2x+2)\mathrm{e}^x$
	Use limits correctly throughout	dep*M1		Tolerate (their value for $x=1$) (-0)
	$\mathrm{e}^{(1)}-2$ ISW Exact answer only	A1	**6**	Allow 0.718 → M1
				6
3	Volume = $(k)\int_0^{\pi}\sin^2 x\,(\mathrm{d}x)$	B1		where $k=\pi, 2\pi$ or 1; limits necessary
	Suitable method for integrating $\sin^2 x$	*M1		eg $\int +/-1+/-\cos 2x\,(\mathrm{d}x)$ or single integ by parts & connect to $\int \sin^2 x\,(\mathrm{d}x)$
	$\int \sin^2 x\,(\mathrm{d}x)=\frac{1}{2}\int 1-\cos 2x\,(\mathrm{d}x)$	A1		or $-\sin x\cos x+\int \cos^2 x(\mathrm{d}x)$
	$\int \cos 2x\,(\mathrm{d}x)=\frac{1}{2}\sin 2x$	A1		or $-\sin x\cos x+\int 1-\sin^2 x\,(\mathrm{d}x)$
	Use limits correctly	dep*M1		
	Volume = $\frac{1}{2}\pi^2$ WWW Exact answer	A1	**6**	**<u>Beware</u>**: wrong working leading to $\frac{1}{2}\pi^2$
				6
4	(i) $\left(1+\frac{x}{2}\right)^{-2}$ $=1+(-2)\left(\frac{x}{2}\right)+\frac{-2.-3}{2}\left(\frac{x}{2}\right)^2+\frac{-2.-3.-4}{3!}\left(\frac{x}{2}\right)^3$	M1		Clear indication of method of ≥ 3 terms
	$=1-x$	B1		First two terms, not dependent on M1
	$+\frac{3}{4}x^2-\frac{1}{2}x^3$	A1		For both third and fourth terms
	$(2+x)^{-2}=\frac{1}{4}\left(\text{their exp of }(1+ax)^{-2}\right)$ mult out	√B1		Correct: $\frac{1}{4}-\frac{1}{4}x+\frac{3}{16}x^2-\frac{1}{8}x^3$
	$\lvert x\rvert<2$ or $-2<x<2$ (but not $\lvert\frac{1}{2}x\rvert<1$)	B1	**5**	
	(ii) If (i) is $a+bx+cx^2+dx^3$ evaluate $b+d$	M1		
	$-\frac{3}{8}\ (x^3)$	√A1	**2**	Follow-through from $b+d$
				7

5(i) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1			
$= \frac{-4\sin 2t}{-\sin t}$	A1		Accept $\frac{4\sin 2t}{\sin t}$ WWW	
$= 8\cos t$	A1			
≤ 8 **AG**	A1	4	with brief explanation eg $\cos t \le 1$	
(ii) Use $\cos 2t = 2\cos^2 t +/- 1$ or $1 - 2\cos^2 t$	M1		<u>If starting with</u> $y = 4x^2 + 1$, then	
Use correct version $\cos 2t = 2\cos^2 t - 1$	A1		Subst $x = \cos t, y = 3 + 2\cos 2t$	M1
Produce WWW $y = 4x^2 + 1$ **AG**	A1	3	<u>Either</u> substitute <u>a</u> formula for cos 2*t*	M1
			Obtain 0=0 or $4\cos^2 t + 1 = 4\cos^2 t + 1$	A1
			<u>Or</u> Manip to give formula for cos 2*t*	M1
			Obtain corr formula & say it's correct	A1
(iii) U-shaped parabola abve *x*-axis, sym abt *y*-axis	B1		Any labelling must be correct	
Portion between $(-1, 5)$ and $(1, 5)$	B1	2	either $x = \pm 1$ or $y = 5$ must be marked	
N.B. If (ii) answered or quoted before (i) attempted,	allow in part		(i) B2 for $\frac{dy}{dx} = 8x$ +B1,B1 if earned.	**9**
6 (i) $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1			
Using $d(uv) = u\,dv + v\,du$ for the $(3)xy$ term	M1			
$\frac{d}{dx}(x^2 + 3xy + 4y^2) = 2x + 3x\frac{dy}{dx} + 3y + 8y\frac{dy}{dx}$	A1			
Solve for $\frac{dy}{dx}$ & subst (x, y) = (2,3)	M1		or v.v. Subst now or at normal eqn stage;	
			(M1 dep on either/both B1 M1 earned)	
$\frac{dy}{dx} = -\frac{13}{30}$	A1		Implied if grad normal = $\frac{30}{13}$	
Grad normal = $\frac{30}{13}$ follow-through	√B1		This f.t. mark awarded only if numerical	
Find equ <u>any</u> line thro (2,3) with <u>any</u> num grad	M1			
$30x - 13y - 21 = 0$ AEF	A1	8	No fractions in final answer	**8**
7 (i) Leading term in quotient = $2x$	B1			
<u>Suff evidence</u> of division or identity process	M1			
Quotient = $2x + 3$	A1		Stated or in relevant position in division	
Remainder = *x*	A1	4	Accept $\frac{x}{x^2+4}$ as remainder	
(ii) their quotient + $\frac{\text{their remainder}}{x^2+4}$	√B1	1	$2x + 3 + \frac{x}{x^2+4}$	
(iii) <u>Working with their expression in part (ii)</u>				
their *Ax* + *B* integrated as $\frac{1}{2}Ax^2 + Bx$	√B1			
their $\frac{Cx}{x^2+4}$ integrated as $k\ln(x^2+4)$	M1		Ignore any integration of $\frac{D}{x^2+4}$	
$k = \frac{1}{2}C$	√A1			
Limits used correctly throughout	M1			
$14 + \frac{1}{2}\ln\frac{13}{5}$	A1	5	logs need not be combined.	
				10

8	(i) Sep variables eg $\int \frac{1}{6-h}(\mathrm{d}h) = \int \frac{1}{20}(\mathrm{d}t)$	*M1		s.o.i. Or $\frac{\mathrm{d}t}{\mathrm{d}h} = \frac{20}{6-h}$ $\rightarrow$ M1
	LHS = $-\ln(6-h)$	A1		& then $t = -20\ln(6-h)$ (+c) $\rightarrow$ A1+A1
	RHS = $\frac{1}{20}t$ (+c)	A1		
	Subst $t = 0, h = 1$ into equation containing 'c'	dep*M1		
	Correct value of their c = $-(20)\ln 5$ **WWW**	A1		or $(20)\ln 5$ if on LHS
	Produce $t = 20\ln\frac{5}{6-h}$ **WWW AG**	A1	**6**	Must see $\ln 5 - \ln(6-h)$
	(ii) When *h* = 2, *t* = $20\ln\frac{5}{4}$ =4.46(2871)	B1	**1**	Accept 4.5, $4\frac{1}{2}$
	(iii) Solve $10 = 20\ln\frac{5}{6-h}$ to $\frac{5}{6-h} = e^{0.5}$	M1		or $\frac{6-h}{5} = e^{-0.5}$ or suitable $\frac{1}{2}$-way stage
	h = 2.97(2.9673467…)	A1	**2**	$6 - 5e^{-0.5}$ or $6 - e^{1.109}$
	[In (ii),(iii) accept non-decimal (exact) answers but -1 once.**]** Accept truncated values in (ii),(iii).			
	(iv) Any indication of (approximately) 6 (m)	B1	**1**	
				10
9	(i) Use -6**i** + 8**j** -2**k** and **i** + 3**j** + 2**k** only	M1		
	Correct method for scalar product	M1		of <u>any</u> two vectors ($-6+24-4=14$)
	Correct method for magnitude	M1		of <u>any</u> vector ($\sqrt{36+64+4} = \sqrt{104}$ or $\sqrt{1+9+4} = \sqrt{14}$)
	68 or 68.5 (68.47546); 1.2(0) (1.1951222) rad	A1	**4**	
	[N.B. 61 (60.562) will probably have been generated by 5**i** + **j** -2**k** and 3**i** – 8**j**]			
	(ii) Indication that relevant vectors are parallel	M1		-6**i** + 8**j** -2**k** & 3**i** + c**j** + **k** with some indic of method of attack eg -6**i** + 8**j** -2**k** =λ(3**i** + c**j** + **k**)
	c = -4	A1	**2**	$c = -4$ WW $\rightarrow$ B2
	(iii) Produce 2/3 equations containing *t,u* (& c)	M1		eg $3+t = 2+3u, -8+3t = 1+cu$ and $2t = 3+u$
	Solve the 2 equations not containing 'c'	M1		
	t = 2, *u* = 1	A1		
	Subst their (*t,u*) into equation containing c	M1		
	c = -3	A1	**5**	
	<u>Alternative method for final 4 marks</u> Solve two equations, one with 'c', for *t* and *u* in terms of c, and substitute into third equation	(M2)		
	$c = -3$	(A2)		**11**

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1	*EITHER* $a = 2$ $b = 2\sqrt{3}$, *OR* $a = 2 \quad b = 2\sqrt{3}$	M1 A1 M1 A1 M1 M1 A1 A1	 4 **4**	Use trig to find an expression for *a* (or *b*) Obtain correct answer Attempt to find other value Obtain correct answer a.e.f. (Allow 3.46) State 2 equations for *a* and *b* Attempt to solve these equations Obtain correct answers a.e.f. SR $\pm$ scores A1 only
2	$(1^3 =)\frac{1}{4} \times 1^2 \times 2^2$ $\frac{1}{4}n^2(n+1)^2 + (n+1)^3$ $\frac{1}{4}(n+1)^2(n+2)^2$	B1 M1 M1(indep) A1 A1	 5 **5**	Show result true for $n = 1$ Add next term to given sum formula Attempt to factorise and simplify Correct expression obtained convincingly Specific statement of induction conclusion
3	$3\Sigma r^2 - 3\Sigma r + \Sigma 1$ $3\Sigma r^2 = \frac{1}{2}n(n+1)(2n+1)$ $3\Sigma r = \frac{3}{2}n(n+1)$ $\Sigma 1 = n$ n^3	M1 A1 A1 A1 M1 A1	 6 **6**	Consider the sum of three separate terms Correct formula stated Correct formula stated Correct term seen Attempt to simplify Obtain given answer correctly
4	(i) $\frac{1}{2}\begin{pmatrix} 5 & -1 \\ -3 & 1 \end{pmatrix}$ (ii) $\frac{1}{2}\begin{pmatrix} 2 & 0 \\ 23 & -5 \end{pmatrix}$	B1 B1 M1 M1(indep) A1ft A1ft	 2 4 **6**	Transpose leading diagonal and negate other diagonal or solve sim. eqns. to get 1st column Divide by the determinant or solve 2nd pair to get 2nd column Attempt to use $B^{-1}A^{-1}$ or find B Attempt at matrix multiplication One element correct, a.e.f, All elements correct, a.e.f. NB ft consistent with their (i)

5	(i) $\frac{1}{r(r+1)}$ (ii) $1 - \frac{1}{n+1}$ (iii) $S_\infty = 1$ $\frac{1}{n+1}$	B1 M1 M1 A1 B1ft M1 A1 c.a.o.	1 3 3 **7**	Show correct process to obtain given result Express terms as differences using (i) Show that terms cancel Obtain correct answer, must be *n* not any other letter State correct value of sum to infinity Ft their (ii) Use sum to infinity – their (ii) Obtain correct answer a.e.f.
6	(i) (a) $\alpha+\beta+\gamma=3, \alpha\beta+\beta\gamma+\gamma\alpha=2$ (b) $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$ $=9-4=5$ (ii) (a) $\frac{3}{u^3}-\frac{9}{u^2}+\frac{6}{u}+2=0$ $2u^3+6u^2-9u+3=0$ (b) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=-3$	B1 B1 M1 A1 ft M1 A1 M1 A1ft	2 2 2 2 **8**	State correct values State or imply the result and use their values Obtain correct answer Use given substitution to obtain an equation Obtain correct answer Required expression is related to new cubic stated or implied -(their "b" / their "a")

7	(i) $a(a - 12) + 32$ (ii) det **M** = 12 non-singular (iii) *EITHER* *OR*	M1 M1 A1 M1 A1ft B1 M1 A1 M1 A1 A1	 3 2 3 **8**	Show correct expansion process Show evaluation of a 2 x 2 determinant Obtain correct answer a.e.f. Substitute $a = 2$ in their determinant Obtain correct answer and state a consistent conclusion det M = 0 so non-unique solutions Attempt to solve and obtain 2 inconsistent equations Deduce that there are no solutions Substitute $a = 4$ and attempt to solve Obtain 2 correct inconsistent equations Deduce no solutions
8	(i) Circle, centre (3, 0), y-axis a tangent at origin Straight line, through (1, 0) with +ve slope In 1st quadrant only (ii) Inside circle, below line, above x-axis	B1B1 B1 B1 B1 B1 B2ft	 6 2 **8**	Sketch showing correct features N.B. treat 2 diagrams asa MR Sketch showing correct region SR: B1ft for any 2 correct features

9	(i) $\begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$	B1	1	Correct matrix
	(ii) Rotation (centre O), 45^0, clockwise	B1B1B1	3	Sensible alternatives OK, must be a single transformation
	(iii)	B1	1	Matrix multiplication or combination of transformations
	(iv) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	M1 A1	2	For at least two correct images For correct diagram
	(v) det **C** = 2	B1		State correct value
	area of square has been doubled	B1	2	State correct relation a.e.f.
			9	
10	(i)	M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ and 16+30i
	$x^2 - y^2 = 16$ and $xy = 15$	A1A1		Obtain each result
		M1		Eliminate to obtain a quadratic in x^2 or y^2
	$\pm(5 + 3i)$	M1		Solve to obtain $x = (\pm)\ 5$ or $y = (\pm)\ 3$
	(ii)	A1	6	Obtain correct answers as complex numbers
	$z = 1 \pm \sqrt{16 + 30i}$	M1*		Use quadratic formula or complete the square
	6 + 3i, -4 - 3i	A1 *M1dep A1 A1ft	5	Simplify to this stage Use answers from (i) Obtain correct answers
			11	

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1		Correct formula with correct r	M1	Allow $r^2 = 2\sin^2 3\theta$
		Rewrite as $a + b\cos 6\theta$	M1	$a, b \neq 0$
		Integrate their expression correctly	A1√	From $a + b\cos 6\theta$
		Get $\frac{1}{3}\pi$	A1	cao
2	(i)	Expand to $\sin 2x \cos\frac{1}{4}\pi + \cos 2x \sin\frac{1}{4}\pi$	B1	
		Clearly replace $\cos\frac{1}{4}\pi$, $\sin\frac{1}{4}\pi$ to A.G.	B1	
	(ii)	Attempt to expand $\cos 2x$	M1	Allow $1 - 2x^2/2$
		Attempt to expand $\sin 2x$	M1	Allow $2x - 2x^3/3$
		Get $\frac{1}{2}\sqrt{2}\,(1 + 2x - 2x^2 - 4x^3/3)$	A1	Four correct unsimplified terms in any order; allow bracket; AEEF SR Reasonable attempt at $f^n(0)$ for $n = 0$ to 3 M1 Attempt to replace their values in Maclaurin M1 Get correct answer only A1
3	(i)	Express as $A/(x-1) + (Bx+C)/(x^2+9)$	M1	Allow C=0 here
		Equate (x^2+9x) to $A(x^2+9) + (Bx+C)(x-1)$	M1√	May imply above line; on their P.F.
		Sub. for x or equate coeff.	M1	Must lead to at least 3 coeff.; allow cover-up method for A
		Get A=1, B=0, C=9	A1	cao from correct method
	(ii)	Get $A\ln(x-1)$	B1√	On their A
		Get $C/3 \tan^{-1}(x/3)$	B1√	On their C; condone no constant; ignore any $B \neq 0$
4	(i)	Reasonable attempt at product rule	M1	Two terms seen
		Derive or quote diff. of $\cos^{-1}x$	M1	Allow +
		Get $-x^2(1 - x^2)^{-\frac{1}{2}} + (1 - x^2)^{\frac{1}{2}} + (1 - x^2)^{-\frac{1}{2}}$	A1	
		Tidy to $2(1 - x^2)^{\frac{1}{2}}$	A1	cao
	(ii)	Write down integral from (i)	B1	On any $k\sqrt{(1-x^2)}$
		Use limits correctly	M1	In any reasonable integral
		Tidy to $\frac{1}{2}\pi$	A1	
			SR	Reasonable sub. B1 Replace for new variable and attempt to integrate (ignore limits) M1 Clearly get $\frac{1}{2}\pi$ A1

5	(i)	Attempt at parts on $\int 1 (\ln x)^n \, dx$	M1	Two terms seen
		Get x (ln x)^n - $\int$ ^{n} (ln x)$^{n-1}$ dx	A1	
		Put in limits correctly in line above	M1	
		Clearly get A.G.	A1	ln e =1, ln1 =0 seen or implied
	(ii)	Attempt I_3 to I_2 as I_3 = e – 3I_2	M1	
		Continue sequence in terms of In	A1	I_2 = e-2I_1 and/or I_1 = e–I_0
		Attempt I_0 or I_1	M1	(I_0 =e-1, I_1=1)
		Get 6 – 2e	A1	cao
6	(i)	Area under graph (= $\int 1/x^2 \, dx$, 1 to n+1)		
		< Sum of rectangles (from 1 to n)	B1	Sum (total) seen or implied eg diagram; accept areas (of rectangles)
		Area of each rectangle = Width x Height = 1 x 1/x^2	B1	Some evidence of area worked out – seen or implied
	(ii)	Indication of new set of rectangles	B1	
		Similarly, area under graph from 1 to n > sum of areas of rectangles from 2 to n	B1	Sum (total) seen or implied
		Clear explanation of A.G.	B1	Diagram; use of left-shift of previous areas
	(iii)	Show complete integrations of RHS, using correct, different limits	M1	Reasonable attempt at $\int x^{-2}$ dx
		Correct answer, using limits, to one integral	A1	
		Add 1 to their second integral to get complete series	M1	
		Clearly arrive at A.G.	A1	
	(iv)	Get one limit	B1	Quotable
		Get both 1 and 2	B1	Quotable; limits only required

7	(i)	Use correct definition of cosh or sinh x	B1	Seen anywhere in (i)
		Attempt to mult. their cosh/sinh	M1	
		Correctly mult. out and tidy	A1√	
		Clearly arrive at A.G.	A1	Accept e^{x-y} and e^{y-x}
	(ii)	Get $\cosh(x-y) = 1$	M1	
		Get or imply $(x-y) = 0$ to A.G.	A1	
	(iii)	Use $\cosh^2 x = 9$ or $\sinh^2 x = 8$	B1	
		Attempt to solve cosh x = 3 (not –3) or sinh $x = \pm\sqrt{8}$ (allow $+\sqrt{8}$ or $-\sqrt{8}$ only)	M1	x = ln(3 + √8) from formulae book or from basic cosh definition
		Get at least one x solution correct	A1	
		Get both solutions correct, x and y	A1	x, y = ln(3 ±2√2); AEEF
				SR Attempt tanh = sinh/cosh B1
				Get tanh x = ±√8/3 (+ or -) M1
				Get at least one sol. correct A1
				Get both solutions correct A1
				SR Use exponential definition B1
				Get quadratic in e^x or e^{2x} M1
				Solve for one correct x A1
				Get both solutions, x and y A1
8	(i)	$x_2 = 0.1890$	B1	
		$x_3 = 0.2087$	B1√	From their x_1 (or any other correct)
		$x_4 = 0.2050$	B1√	Get at least two others correct, all to a minimum of 4 d.p.
		$x_5 = 0.2057$		
		$x_6 = 0.2055$		
		$x_7 (= x_8) = 0.2056$ (to x_7 minimum)		
		$\alpha = 0.2056$	B1	cao; answer may be retrieved despite some errors
	(ii)	Attempt to diff. f(x)	M1	$k/(2+x)^3$
		Use α to show $f'(\alpha) \neq 0$	A1√	Clearly seen, or explain $k/(2+x)^3 \neq 0$ as $k \neq 0$; allow ± 0.1864
			SR	Translate $y=1/x^2$ M1
				State/show $y=1/x^2$ has no TP A1
	(iii)	$\delta_3 = -0.0037$ (allow –0.004)	B1√	Allow ±, from their x_4 and x_3
	(iv)	Develop from $\delta_{10} = f'(\alpha)\,\delta_9$ etc. to get δ_i or quote $\delta_{10} = \delta_3\, f'(\alpha)^7$	M1	Or any δ_i eg use $\delta_9 = x_{10} - x_9$
		Use their δ_i and $f'(\alpha)$	M1	
		Get 0.000000028	A1	Or answer that rounds to ± 0.00000003

9	(i)	Quote $x = a$	B1	
		Attempt to divide out	M1	Allow M1 for y=x here; allow
			A1	$(x-a) + k/(x-a)$ seen or implied
		Get $y = x - a$	A1	Must be equations
	(ii)	Attempt at quad. in x (=0)	M1	
		Use $b^2 - 4ac \geq 0$ for real x	M1	Allow >
		Get $y^2 + 4a^2 \geq 0$	A1	
		State/show their quad. is always >0	B1	Allow ≥
	(iii)		B1√	Two asymptotes from (i) (need not be labelled)
			B1	Both crossing points
			B1√	Approaches – correct shape
			SR	Attempt diff. by quotient/product rule M1
				Get quadratic in x for $dy/dx = 0$ and note $b^2 - 4ac < 0$ A1
				Consider horizontal asymptotes B1
				Fully justify answer B1

Mark Scheme 4727
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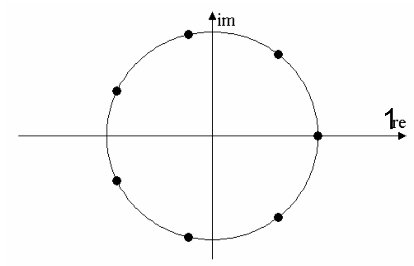
1 **(i)** $zz^* = re^{i\theta}.re^{-i\theta} = r^2 = \|z\|^2$	B1 **1**	For verifying result **AG**
(ii) Circle Centre $0\,(+0\mathrm{i})$ *OR* (0, 0) *OR* *O*, radius 3	B1 B1 **2** **3**	For stating circle For stating correct centre and radius
2 *EITHER*: $(\mathbf{r} =)\,[3+t, 1+4t, -2+2t]$ $8(3+t)-7(1+4t)+10(-2+2t)=7$ $\Rightarrow (0t)+(-3)=7 \Rightarrow$ contradiction *l* is parallel to Π, no intersection	M1 M1 A1 A1 B1 **5**	For parametric form of *l* seen or implied For substituting into plane equation For obtaining a contradiction For conclusion from correct working
OR: $[1, 4, 2]\,.\,[8, -7, 10]=0$ $\Rightarrow$ *l* is parallel to Π $(3, 1, -2)$ into Π $\Rightarrow\ 24-7-20 \neq 7$ *l* is parallel to Π, no intersection	M1 A1 M1 A1 B1	For finding scalar product of direction vectors For correct conclusion For substituting point into plane equation For obtaining a contradiction For conclusion from correct working
OR:Solve $\frac{x-3}{1}=\frac{y-1}{4}=\frac{z+2}{2}$ and $8x-7y+10z=7$ eg $y-2z=3$, $2y-2=4z+8$ eg $4z+4=4z+8$ *l* is parallel to Π, no intersection	 M1 A1 M1 A1 B1 **5**	 For eliminating one variable For eliminating another variable For obtaining a contradiction For conclusion from correct working
3 Aux. equation $m^2-6m+8\ (=0)$ $m=2, 4$ CF $(y=)\ Ae^{2x}+Be^{4x}$ PI $(y=)\ Ce^{3x}$ $9C-18C+8C=1 \Rightarrow C=-1$ GS $y=Ae^{2x}+Be^{4x}-e^{3x}$	M1 A1 A1√ M1 A1 B1√ **6** **6**	For auxiliary equation seen For correct roots For correct CF. f.t. from their *m* For stating and substituting PI of correct form For correct value of *C* For GS. f.t. from their CF + PI with 2 arbitrary constants in CF and none in PI

4 **(i)** $q(st) = qp = s$	B1		For obtaining s
$(qs)t = tt = s$	B1	**2**	For obtaining s
(ii) METHOD 1			
Closed: see table	B1		For stating closure with reason
Identity = r	B1		For stating identity r
Inverses: $p^{-1} = s,\ q^{-1} = t,\ (r^{-1} = r),$	M1		For checking for inverses
$s^{-1} = p,\ t^{-1} = q$	A1	**4**	For stating inverses *OR* For giving sufficient explanation to justify each element has an inverse eg r occurs once in each row and/or column
METHOD 2			
Identity = r	B1		For stating identity r
	M1		For attempting to establish a generator $\neq r$
eg $p^2 = t$, $p^3 = q$, $p^4 = s$	A1		For showing powers of p (*OR* q, s or t) are different elements of the set
$\Rightarrow p^5 = r$, so p is a generator	A1		For concluding p^5 (*OR* q^5 , s^5 or t^5) = r
(iii) e, d, d^2, d^3, d^4	B2	**2**	For stating all elements **AEF** eg d^{-1}, d^{-2} , dd
		8	

5 **(i)** $(\cos 6\theta =)\ \mathrm{Re}\left(c + \mathrm{i}\,s\right)^6$	M1		For expanding (real part of) $\left(c + \mathrm{i}\,s\right)^6$ at least 4 terms and 1 evaluated binomial coefficient needed
$(\cos 6\theta =)\ c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	A1		For correct expansion
$(\cos 6\theta =)$ $c^6 - 15c^4\left(1-c^2\right) + 15c^2\left(1-c^2\right)^2 - \left(1-c^2\right)^3$	M1		For using $s^2 = 1 - c^2$
$(\cos 6\theta =)\ 32c^6 - 48c^4 + 18c^2 - 1$	A1	**4**	For correct result **AG**
(ii) $64x^6 - 96x^4 + 36x^2 - 3 = 0 \Rightarrow \cos 6\theta = \frac{1}{2}$	M1		For obtaining a numerical value of $\cos 6\theta$
$\Rightarrow (\theta =)\ \frac{1}{18}\pi, \frac{5}{18}\pi, \frac{7}{18}\pi$ etc.	A1		For any correct solution of $\cos 6\theta = \frac{1}{2}$
$\cos 6\theta = \frac{1}{2}$ has multiple roots	M1		For stating or implying at least 2 values of θ
largest x requires smallest θ $\Rightarrow$ largest positive root is $\cos\frac{1}{18}\pi$	A1	**4**	For identifying $\cos\frac{1}{18}\pi$ **AEF** as the largest positive root from a list of 3 positive roots *OR* from general solution *OR* from consideration of the cosine function
		8	

6 **(i)** $\mathbf{n} = l_1 \times l_2$	B1	For stating or implying in **(i)** or **(ii)** that **n** is perpendicular to l_1 and l_2
$\mathbf{n} = [2, -1, 1] \times [4, 3, 2]$	M1*	For finding vector product of direction vectors
$\mathbf{n} = k[-1, 0, 2]$	A1	For correct vector (any k)
$[3, 4, -1] \,.\, k[-1, 0, 2] = -5k$	M1 (*dep)	For substituting a point of l_1 into $\mathbf{r}.\mathbf{n}$
$\mathbf{r}.[-1, 0, 2] = -5$	A1 **5**	For obtaining correct p. **AEF** in this form
(ii) $[5, 1, 1] \,.\, k[-1, 0, 2] = -3k$	M1	For using same **n** and substituting a point of l_2
$\mathbf{r}.[-1, 0, 2] = -3$	A1√ **2**	For obtaining correct p. **AEF** in this form f.t. on incorrect **n**
(iii) $d = \dfrac{\lvert -5+3 \rvert}{\sqrt{5}}$ *OR* $d = \dfrac{\lvert [2, -3, 2] \,.\, [-1, 0, 2] \rvert}{\sqrt{5}}$ *OR* d from $(5, 1, 1)$ to $\Pi_1 = \dfrac{\lvert 5(-1)+1(0)+1(2)+5 \rvert}{\sqrt{5}}$ *OR* d from $(3, 4, -1)$ to $\Pi_2 = \dfrac{\lvert 3(-1)+4(0)-1(2)+3 \rvert}{\sqrt{5}}$ *OR* $[3-t, 4, -1+2t] \,.\, [-1, 0, 2] = -3 \Rightarrow t = \frac{2}{5}$ *OR* $[5-t, 1, 1+2t] \,.\, [-1, 0, 2] = -5 \Rightarrow t = -\frac{2}{5}$	M1	For using a distance formula from their equations Allow omission of $\lvert\ \rvert$ *OR* For finding intersection of $\mathbf{n}_1$ and Π_2 or $\mathbf{n}_2$ and Π_1
$d = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5} = 0.894427...$	A1√ **2**	For correct distance **AEF** f.t. on incorrect **n**
(iv) d is the shortest *OR* perpendicular distance between l_1 and l_2	B1 **1** **10**	For correct statement
7 **(i)** $(z - e^{i\phi})(z - e^{-i\phi}) \equiv z^2 - (2)z\dfrac{(e^{i\phi} + e^{-i\phi})}{(2)} + 1$ $\equiv z^2 - (2\cos\phi)z + 1$	B1 **1**	For correct justification **AG**
(ii) $z = e^{\frac{2}{7}k\pi i}$	B1	For general form *OR* any one non-real root
for $k = 0, 1, 2, 3, 4, 5, 6$ *OR* $0, \pm 1, \pm 2, \pm 3$	B1	For other roots specified (k=0 may be seen in any form, eg 1, e^0, $e^{2\pi i}$) For answers in form $\cos\theta + i\sin\theta$ allow maximum B1 B0
	B1 B1 **4**	For any 7 points equally spaced round unit circle (circumference need not be shown) For 1 point on $+^{ve}$ real axis, and other points in correct quadrants
(iii) $\left(z^7 - 1 =\right)(z-1)(z - e^{\frac{2}{7}\pi i})(z - e^{\frac{4}{7}\pi i})$ $(z - e^{\frac{6}{7}\pi i})(z - e^{\frac{-2}{7}\pi i})(z - e^{\frac{-4}{7}\pi i})(z - e^{\frac{-6}{7}\pi i})$	M1	For using linear factors from **(ii),** seen or implied
$= (z - e^{\frac{2}{7}\pi i})(z - e^{\frac{-2}{7}\pi i}) \times (z - e^{\frac{4}{7}\pi i})(z - e^{\frac{-4}{7}\pi i})$ $(z - e^{\frac{6}{7}\pi i})(z - e^{\frac{-6}{7}\pi i}) \times$	M1	For identifying at least one pair of complex conjugate factors
$\times (z-1)$	B1	For linear factor seen
$= (z^2 - (2\cos\frac{2}{7}\pi)z + 1) \times$	A1	For any one quadratic factor seen
$(z^2 - (2\cos\frac{4}{7}\pi)z + 1) \times (z^2 - (2\cos\frac{6}{7}\pi)z + 1) \times$ $\times (z-1)$	A1 **5** **10**	For the other 2 quadratic factors and expression written as product of 4 factors

8 **(i)** Integrating factor $e^{\int \tan x\,(dx)}$	B1	For correct IF
$= e^{-\ln\cos x}$	M1	For integrating to ln form
$= (\cos x)^{-1}$ *OR* $\sec x$	A1	For correct simplified IF **AEF**
$\Rightarrow \frac{d}{dx}\left(y(\cos x)^{-1}\right) = \cos^2 x$	B1√	For $\frac{d}{dx}(y.\text{their IF}) = \cos^3 x$. their IF
$y(\cos x)^{-1} = \int \frac{1}{2}(1+\cos 2x)\,(dx)$	M1 M1	For integrating LHS For attempting to use $\cos 2x$ formula *OR* parts for $\int \cos^2 x\,dx$
$y(\cos x)^{-1} = \frac{1}{2}x + \frac{1}{4}\sin 2x\ (+c)$	A1	For correct integration both sides **AEF**
$y = \left(\frac{1}{2}x + \frac{1}{4}\sin 2x + c\right)\cos x$	A1 **8**	For correct general solution **AEF**
(ii) $2 = \left(\frac{1}{2}\pi + c\right).-1 \Rightarrow c = -2 - \frac{1}{2}\pi$	M1	For substituting $(\pi, 2)$ into their GS and solve for c
$y = \left(\frac{1}{2}x + \frac{1}{4}\sin 2x - 2 - \frac{1}{2}\pi\right)\cos x$	A1 **2** **10**	For correct solution **AEF**
9 **(i)** $3^n \times 3^m = 3^{n+m},\ n+m \in \mathbb{Z}$	B1	For showing closure
$\left(3^p \times 3^q\right) \times 3^r = \left(3^{p+q}\right) \times 3^r = 3^{p+q+r}$	M1	For considering 3 distinct elements, seen bracketed 2+1 or 1+2
$= 3^p \times \left(3^{q+r}\right) = 3^p \times \left(3^q \times 3^r\right) \Rightarrow$ associativity	A1	For correct justification of associativity
Identity is 3^0	B1	For stating identity. Allow 1
Inverse is 3^{-n}	B1	For stating inverse
$3^n \times 3^m = 3^{n+m} = 3^{m+n} = 3^m \times 3^n \Rightarrow$ commutativity	B1 **6**	For showing commutativity
(ii) (a) $3^{2n} \times 3^{2m} = 3^{2n+2m}\left(= 3^{2(n+m)}\right)$	B1*	For showing closure
Identity, inverse OK	B1 (*dep) **2**	For stating other two properties satisfied and hence a subgroup
(b) For 3^{-n},	M1	For considering inverse
$-n \notin$ subset	A1 **2**	For justification of not being a subgroup 3^{-n} must be seen here or in **(i)**
(c) *EITHER*: eg $3^{1^2} \times 3^{2^2} = 3^5$	M1	For attempting to find a specific counter-example of closure
$\neq 3^{r^2} \Rightarrow$ not a subgroup	A1 **2**	For a correct counter-example and statement that it is not a subgroup
OR: $3^{n^2} \times 3^{m^2} = 3^{n^2+m^2}$	M1	For considering closure in general
$\neq 3^{r^2}$ eg $1^2 + 2^2 = 5 \Rightarrow$ not a subgroup	A1 **12**	For explaining why $n^2 + m^2 \neq r^2$ in general and statement that it is not a subgroup

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1(i)	X = 5 Y = 12	B1 B1 [2]	X=-5 B0. Both may be seen/implied in (ii) No evidence for which value is X or Y available from (ii) award B1 for the pair of values 5 and 12 irrespective of order
(ii)	$R^2 = 5^2 + 12^2$ Magnitude is 13 N $\tan\theta$ = 12/5 Angle is 67.4°	M1 A1 M1 A1 [4]	For using $R^2 = X^2 + Y^2$ Allow 13 from X=-5 For using correct angle in a trig expression **SR:** p=14.9 and Q=11.4 giving R=13+/-0.1 B2, Angle = 67.5+/-0.5 B2

2(i)	250 + ½ (290 – 250) t = 270	M1 A1 [2]	Use of the ratio 12:12 (may be implied), or v = u+at
(ii)	 ½ x40x12+210x12+½x20x12–½x20x12 or ½ x40x12+210x12 or ½ x(210+250)x12etc Displacement is 2760m	M1 M1 A1 [3]	The idea that area represents displacement Correct <u>structure</u>, ie triangle1 + rectangle2 + triangle3 - |triangle4| with triangle3 = |triangle4|, triangle1 + rectangle2, trapezium1&2, etc
(iii)	appropriate <u>structure</u>, ie triangle + rectangle + triangle + |triangle|, triangle + rectangle + 2triangle, etc Distance is 3000m	M1 A1 [2]	All terms positive Treat candidate doing (ii) in (iii) and (iii) in (ii) as a mis-read.

3(i)	 R + Tsin72° = 50g	M1 A1 [2]	An equation with R, T and 50 in linear combination. R + 0.951T = 50g
(ii)	T = 50g/sin72° T = 515 (AG) T = mg m = 52.6	M1 A1 B1 B1 [4]	Using R = 0 (may be implied) and Tsin72° = 50(g) Or better Accept 52.5
(iii)	X = Tcos72° X = 159	B1 B1 [2]	Implied by correct answer Or better

4(i)	*In Q4 right to left may be used as the positive sense throughout.* 0.18 x 2 – 3m = 0 m = 0.12	M1 A1 A1 [3]	For using Momentum 'before' is zero 3 marks possible if g included consistently
(iia)	Momentum after = -0.18 x 1.5 + 1.5m 0.18 x 2 – 3m = -0.18 x 1.5 + 1.5m m = 0.14	B1 M1 A1 [3]	 For using conservation of momentum 3 marks possible if g included consistently
(iib)	0.18 x 2 – 3m = (0.18 + m)1.5 m = 0.02 0.18 x 2 – 3m= - (0.18 + m)1.5 m = 0.42	B1ft B1 B1ft B1 [4]	ft wrong momentum 'before' 0 marks if g included

5(i)		M1	Using $v^2 = u^2$ +/– 2gs with v = 0 or u = 0
	$8.4^2 - 2gs_{max} = 0$	A1	
	Height is 3.6m (AG)	A1	
		[3]	
(ii)		M1	Using u^2 =+/- 2g(ans(i) – 2)
	u = 5.6	A1	
		[2]	
(iii)	EITHER (time when at same height)	M1	Using s = ut + ½ at^2 for P *and* for Q, a = +/-g, expressions for s terms must differ
	s+/-2 = 8.4t – ½ gt^2 and		Or 8.4t (– ½ gt^2)=5.6t (– ½ gt^2)+/- 2
	(s+/-2) = 5.6t – ½ gt^2	A1	Correct sign for g, cv(5.6), +/-2 in only one equation
	t = 5/7 (0.714)	A1	cao
		M1	Using v = u +at for P *and* for Q, a = +/-g, cv(t)
	v_P =8.4 -0.714g and v_Q =5.6-0.714g	A1	Correct sign for g, cv(5.6), candidates answer for t (including sign)
	v_P = 1.4 and v_Q = -1.4	A1	cao
		[6]	
	OR (time when at same speed in opposite directions)	M1	Using v = u+at for P *and* for Q, a = +/-g
	v = 8.4 -gt and -v =5.6-gt	A1	Correct sign for g, cv(5.6)
	v = 1.4 {or t = 5/7 (0.714)}	A1	Only one correct answer is needed
	(with v = 1.4)	M1	Using $v^2 = u^2$ + 2as for P *and* for Q, a = +/-g, cv(v)
	$1.4^2 = 8.4^2 - 2gs_P$ and $(-1.4)^2 = 5.6^2 - 2gs_Q$	A1	Correct sign for g, cv(5.6), candidate's answer for v (including - for Q)
	s_P = 3.5 and s_Q = 1.5	A1	cao
	{(with t=5/7)	M1	Using s = ut + ½ at^2 for P *and* for Q, a = +/-g, cv(t)
	s = 8.4x0.714 – ½ gx0.714^2 and s = 5.6x0.714 – ½ gx0.714^2	A1	Correct sign for g, cv(5.6), candidate's answer for t (including sign of t if negative)
	s_P = 3.5 and s_Q = 1.5	A1	cao}
	OR (motion related to greatest height and verification)	M1	Using v = u+at t for P *and* for Q, a = +/-g
	0 = 8.4 -gt and 0 =5.6-gt		
	t = 6/7 and t = 4/7	A1	Both values correct
	v_P =8.4 -0.714g and v_Q =5.6-0.714g		mid-interval t (6/7+4/7)/2 = 0.714
	{0 = v_P - g/7 and v_Q = 0 +g/7}		{Or semi-interval = 6/7-4/7)/2=1/7}
	v_P = 1.4 and v_Q = -1.4	A1	cao
	s_P = 8.4x0.714 – ½ gx0.714^2 and s_Q = 5.6x0.714 – ½ gx0.714^2	M1	s = ut + ½ at^2 for P *and* for Q, correct sign for g, cv(5.6) and cv(t)
	{ s_P = 0/7 – ½(- g)x$(1/7)^2$ and s_Q = 0/7 + ½ gx$(1/7)^2$}	A1	{s = vt - ½ at^2 for P *and* s = ut + ½ at^2 for Q}
	s_P = 3.5 s_Q = 1.5		
	{ s_P = 0.1 s_Q = 0.1}	A1	cao
			continued

5(iii)	OR (without finding exactly where or when)	M1	Using $v^2 = u^2 + 2as$ for P *and* for Q, a = +/-g, cv(5.6), different expressions for s.
co*nt*	$v_P^2 = 8.4^2 - 2g(s+/-2)$ and		Correct sign for g, cv(5.6), (s+/-2) used only once
			cao. Verbal explanation essential
	$v_Q^2 = 5.6^2 - 2g[(s+/-2)]$	A1	Using v = u+at t for P *and* for Q, a = +/-g
	$v_P^2 = v_Q^2$ for all values of s so that the speeds are always the same at the same heights.	A1	Correct sign for g, correct choice for velocity of zero, cv(5.6)
		M1	
	0 = 8.4 -gt and 0 =5.6-gt	A1	
	$t_P = 6/7$ and $t_Q = 4/7$ means there is a time interval when Q has started to descend but P is still rising, and there will be a position where they have the same height but are moving in opposite directions.	A1	cao. Verbal explanation essential

6(i)		M1	For differentiating s
	$v = 0.004t^3 - 0.12t^2 + 1.2t$	A1	Condone the inclusion of +c
	$v(10) = 4 - 12 + 12 = 4ms^{-1}$ (AG)	A1	Correct formula for v (no +c) and t=10
		[3]	stated sufficient
(ii)		M1	For integrating a
	$v = 0.8t - 0.04t^2$ (+ C)	A1	
	8 - 4 + C = 4	M1*	Only for using v(10) = 4 to find C
	$v = 0.8x20 - 0.04x20^2$ (+ C)	M1	
	v(20) = 16 – 16 = 0 (AG)	DA1	Dependant on M1*
		[5]	
(iii)		M1	For integrating v
	$S = 0.4t^2 - 0.04t^3/3$ (+K)	A1	Accept $0.4t^2 - 0.013t^3$ (+ ct +K, must be linear)
	s(10) = 10 – 40 + 60 = 30	B1	
		M1	For using S(10) = 30 to find K
	40 – 40/3 + K = 30 ➔ K = 10/3	A1	Not if S includes ct term
	S(20) = 160 – 320/3 + 10/3 = 56.7m	B1	Accept 56.6 to 56.7, Adding 30 subsequently is not isw,
	OR	[6]	hence B0
	s(10) = 10 – 40 + 60 = 30	B1	
		M1	For integrating v
	$S = 0.4t^2 - 0.04t^3/3$	A1	Accept $0.4t^2 - 0.013t^3$ (+ ct +K, must be linear)
		M1	Using limits of 10 and 20 (limits 0, 10 M0A0B0)
	S(20) - S(10) = 26.6, 26.7	A1	For 53.3 - 26.7 or better (Note S(10) = 26.7 is fortuitously correct M0A0B0)
	displacement is 56.7m	B1	Accept 56.6 to 56.7

7(i)	$R = 1.5g\cos21^o$	B1	
		M1	For using $F = \mu R$
	Frictional force is 10.98N (AG)	A1 [3]	Note 1.2gcos21=10.98 fortuitously, B0M0A0
(ii)		M1	For obtaining an N2L equation relating to the block in which F, T, m and a are in linear combination or For obtaining an N2L equation relating to the object in which T, m and a are in linear combination
	$T + 1.5g\sin21^o - 10.98 = 1.5a$	A2	-A1 for each error to zero
	$1.2g - T = 1.2a$	A2	-A1 for each error to zero
		[5]	Error is a wrong/omitted term, failure to substitute a numerical value for a letter (excluding g), excess terms. Minimise error count.
(iii)	$T - 1.5a = 5.71$ and $1.2a + T = 11.76$	M1	For solving the simultaneous equations in T and a for a.
	a = 2.24 (AG)	A1 [2]	Evidence of solving needed
(iva)	$v^2 = 2 \times 2.24 \times 2$	M1	For using $v^2 = 2as$ with cv (a) or 2.24
	Speed of the block is $2.99ms^{-1}$	A1 [2]	Accept 3
(ivb)		M1	For using T = 0 to find a
	a = -3.81	A1	
	$v^2 = 2.99^2 + 2 \times (-3.81) \times 0.8$	M1	For using $v^2 = u^2 + 2as$ with cv(2.99) and s = 2.8 - 2 and any value for a
	Speed of the block is $1.69ms^{-1}$	A1 [4]	Accept art 1.7 from correct work

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1	40 cos35°	B1	
	WD = 40cos35° x 100	M1	
	3280 J	A1 **3**	ignore units **3**

2	$0 = 12\sin 27°t - 4.9t^2$ any correct.	M1	**or** $R = u^2\sin 2\theta/g$ (B2)
	t = 1.11 …..method for total time	A1	correct formula only
	R = 12cos27° x t	M1	12^2 x sin54° / 9.8 sub in values
	11.9	A1 **4**	11.9 **4**

3 (i)	WD = $½x250x150^2 – ½x250x100^2$	M1	
	1 560 000	A1	1 562 500
	450 000 = 1 560 000/t	M1	
	3.47	A1 **4**	
(ii)	F = 450 000/120	M1	
	3750	A1	
	3750 = 250a	M1	
	15 ms^{-2}	A1 **4**	**8**

4 (i)	$x = 7t$	B1	
	$y = 21t - 4.9t^2$	M1	**or** – g/2
		A1	
	$y = 21.x/7 - 4.9\, x^2/49$	M1	
	$y = 3x - x^2/10$	A1 **5**	**AG**
(ii)	$-25 = 3x - x^2/10$ (must be -25)	M1	**or** method for total time (5.26)
	solving quadratic	M1	**or** 7 x total time
	36.8 m	A1 **3**	**8**

5(i)	$½ . 70 .4^2$	M1	
	560 J	A1 **2**	
(ii)	70 x 9.8 x 6	M1	
	4120	A1 **2**	4116
(iii)	60d	B1	
	8000 = 560 + 4120 + 60d	M1	4 terms
		A1 √	√ their KE and PE
	55.4 m	A1 **4**	**8**

6 (i)	5cos30° = 0.3x9.8 + Scos60°	M1	res. vertically (3 parts with comps)
		A1	
	2.78 N	A1 **3**	
(ii)	r = 0.4sin30° = 0.2	B1	may be on diagram
	5sin30° + Ssin60° =0.3 x 0.2 x ω^2	M1	res. horizontally (3 parts with comps)
	9.04 rads^{-1}	A1 **3**	
(iii)	v = 0.2 x 9.04	M1	**or** previous v via mv^2/r
	KE = ½ x 0.3 x (0.2 x 9.04)^2	M1	
	0.491 J or 0.49	A1√ **3**	√ their ω^2 x 0.006 **9**

7 (i)	1.8 = –0.3 + 3m	M1	
	m = 0.7	A1 **2**	**AG**
(ii)	e = 4/6	M1	accept 2/6 for M1
	2/3	A1 **2**	accept 0.67
(iii)	± 3f	B1	
	1/3 ≤ f (≤ 1)	B1 **2**	
(iv)	I = 3f x 0.7 – – 3 x 0.7	M1	ok for only one minus sign for M1
		A1	
	I = 2.1 (f + 1)	A1 **3**	aef 2 marks only for -2.1(f + 1)
(v)	0.3 + 6.3/4 = 0.3*a* + 0.7*b*	M1	can be – 0.7*b*
	3*a* + 7*b* = 18.75	A1 *	aef
	2/3 = (*a* – *b*)/ 5/4	M1	allow e=3/4 or their e for M1
	3*a* – 3*b* = 5/2	A1 *	aef * means dependent.
	solve	M1	
	a = 2.5	A1	(2.46) allow ± (59/24)
	b = 1.6	A1 **7**	(1.625) allow ± (13/8) **16**

8 (i)	com of hemisphere 0.3 from *O*	B1	**or** 0.5 from base
	com of cylinder *h*/2 from *O*	B1	
	0.6x45 = 40x0.5 + (0.8+*h*/2) x 5 **or**	M1	**or** 40x0.3 – 5xh/2 = 45 x 0.2
	45(h+0.2) = 5h/2 +40(h+0.3)	A1	**or** 5(0.2 + h/2) = 40x0.1
	27 = 20 + (0.8+*h*/2) x 5	M1	solving
	h = 1.2	A1 **6**	**AG**
(ii)	1.2 T	B1	
	0.8 F	B1	
	0.8F = 1.2T	M1	
	F = 3T/2	A1 **4**	aef
(iii)	F + Tcos30°	B1	**or** 45 x 0.8 sin30°
	45sin30° must be involved in res.	B1	T x (1.2 + 0.8cos30°)
	resolving parallel to the slope	M1	mom. about point of contact
	F + Tcos30° = 45sin30° aef	A1	45.0.8sin30°=T(1.2+0.8cos30°)
	T = 9.51	A1	
	F = 14.3	A1 **6**	**16**
or	T + Fcos30° = Rsin30°	B1	res. horizontally
(iii)	Rcos30° + Fsin30° = 45	B1	res. vertically
	tan30°=(T+Fcos30°)/(45-Fsin30°)	M1	eliminating R

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1	(i) $[\omega = 2\pi/6.1 = 1.03]$	M1		For using $T = 2\pi/\omega$
		M1		For using $v_{max} = a\omega$
	Speed is $3.09ms^{-1}$	A1	3	
	(ii)	M1		For using $v^2 = \omega^2(A^2 - x^2)$ or for using $v = A\omega\cos\omega t$ and $x = A\sin\omega t$
	$2.5^2 = 1.03^2(3^2 - x^2)$ or $x = 3\sin(1.03x0.60996..)$	A1ft		ft incorrect ω
	Distance is 1.76m	A1	3	

2	[Magnitudes 0.6, 0.057 x 7, 0.057 x 10]	M1		For triangle with magnitudes shown
	For magnitudes of 2 sides correctly marked	A1		
	For magnitudes of all 3 sides correctly marked	A1		
		M1		For attempting to find angle (α) opposite to the side of magnitude 0.057 x 7
		M1		For correct use of the cosine rule or equivalent
	$0.399^2 = 0.57^2 + 0.6^2 - 2 \times 0.57 \times 0.6\cos\alpha$	A1ft		
	Angle is 140^o	A1	7	$(180 - 39.8)^o$

2	ALTERNATIVE METHOD			
		M1		For using I= Δmv parallel to the initial direction of motion or parallel to the impulse
	$-0.6\cos\alpha = 0.057 \times 7\cos\beta - 0.057 \times 10$ or $0.6 = 0.057x10\cos\alpha + 0.057x7\cos\gamma$	A1		
		M1		For using I= Δmv perpendicular to the initial direction of motion or perpendicular to the impulse
	$0.6\sin\alpha = 0.057 \times 7\sin\beta$ or $0.057x10\sin\alpha = 0.057x7\sin\gamma$	A1		
		M1		For eliminating β *or γ
	$0.399^2 = (0.57 - 0.6\cos\alpha)^2 + (0.6\sin\alpha)^2$ or $0.399^2 = (0.6 - 0.57\cos\alpha)^2 + (0.057\sin\alpha)^2$	A1ft		
	Angle is 140^o	A1	7	$(180 - 39.8)^o$

3	(i) $[0.2v\,dv/dx = -0.4v^2]$	M1		For using Newton's second law with a = v dv/dx
	$(1/v)\,dv/dx = -2$	A1	2	AG
	(ii) $[\int (1/v)dv = \int -2dx]$	M1		For separating variables and attempting to integrate
	ln v = -2x (+C)	A1		
	[ln v = -2x + ln u]	M1		For using v(0) = u
	$v = ue^{-2x}$	A1	4	AG
	(iii) $[\int e^{2x}dx = \int u dt]$	M1		For using v = dx/dt and separating variables
	$e^{2x}/2 = ut$ (+C)	A1		
	$[e^{2x}/2 = ut + ½]$	M1		For using x(0) = 0
	u = 6.70	A1	4	Accept $(e^4 – 1)/8$

	ALTERNATIVE METHOD FOR PART (iii)			
	$[\int \frac{1}{v^2}dv = -2\int dt$ ➔ -1/v = -2t + A, and A = -1/u]	M1		For using a = dv/dt, separating variables, attempting to integrate and using v(0) = u
		M1		For substituting $v = ue^{-2x}$
	$-e^{2x}/u = -2t – 1/u$	A1		
	u = 6.70	A1	4	Accept $(e^4 – 1)/8$

4	$y = 15\sin\alpha$ (=12)	B1		
	$[4(15\cos\alpha) – 3 \times 12 = 4a + 3b]$	M1		For using principle of conservation of momentum in the direction of l.o.c.
	Equation complete with not more than one error	A1		
	4a + 3b = 0	A1		
		M1		For using NEL in the direction of l.o.c.
	$0.5(15\cos\alpha + 12) = b - a$	A1		
	[a = -4.5, b = 6]	M1		For solving for a and b
	[Speed = $\sqrt{(-4.5)^2 + 12^2}$, Direction $\tan^{-1}(12/(-4.50))$]	M1		For correct method for speed or direction of A
	Speed of A is 12.8ms^{-1} and direction is 111° anticlockwise from '**i**' direction	A1		Direction may be stated in any form , including θ = 69° with θ clearly and appropriately indicated
	Speed of B is 6ms^{-1} to the right	A1	10	Depends on first three M marks

5	(i)	M1		For taking moments of forces on BC about B
	$80 \times 0.7\cos60^o = 1.4T$	A1		
	Tension is 20N	A1		
	[$X = 20\cos30^o$]	M1		For resolving forces horizontally
	Horizontal component is 17.3N	A1ft		ft $X = T\cos30^o$
	[$Y = 80 – 20\sin30^o$]	M1		For resolving forces vertically
	Vertical component is 70N	A1ft	7	ft $Y = 80 – T\sin30^o$
	(ii)	M1		For taking moments of forces on AB, or on ABC, about A
	$17.3 \times 1.4\sin\alpha = (80 \times 0.7 + 70 \times 1.4)\cos\alpha$ or $80 \times 0.7\cos\alpha + 80(1.4\cos\alpha + 0.7\cos60^o) = 20\cos60^o(1.4\cos\alpha + 1.4\cos60^o) + 20\sin60^o(1.4\sin\alpha + 14\sin60^o)$	A1ft		
	[$\tan\alpha = (\frac{1}{2}80 + 70)/17.3 = 11/\sqrt{3}$]	M1		For obtaining a numerical expression for $\tan\alpha$
	$\alpha = 81.1^o$	A1	4	

	ALTERNATIVE METHOD FOR PART (i)			
		M1		For taking moments of forces on BC about B
	$H \times 1.4\sin60^o + V \times 1.4\cos60^o = 80 \times 0.7\cos60^o$	A1		Where H and V are components of T
		M1		For using $H = V\sqrt{3}$ and solving simultaneous equations
	Tension is 20N	A1		
	Horizontal component is 17.3N	B1ft		ft value of H used to find T
	[$Y = 80 – V$]	M1		For resolving forces vertically
	Vertical component is 70N	A1ft	7	ft value of V used to find T

6	(i) [T = 2058x/5.25]	M1		For using $T = \lambda x/L$
	2058x/5.25 = 80 x 9.8 (x = 2)	A1		
	OP = 7.25m	A1	3	AG From 5.25 + 2
	(ii) Initial PE = (80 + 80)g(5) (= 7840) or (80 + 80)gX used in energy equation	B1		
	Initial KE = ½ (80 + 80)3.5^2 (= 980)	B1		
	[Initial EE = $2058x2^2/(2x5.25)$ (= 784), Final EE = $2058x7^2/(2x5.25)$ (= 9604), or $2058(X + 2)^2/(2x5.25)$]	M1		For using $EE = \lambda x^2/2L$
	[Initial energy = 7840 + 980 + 784, final energy = 9604 or 1568X + 980 + 784 = $196(X^2 + 4X + 4)$ ➔ $196X^2 - 784X - 980 = 0$]	M1		For attempting to verify compatibility with the principle of conservation of energy, or using the principle and solving for X
	Initial energy = final energy or X = 5 ➔P&Q just reach the net	A1	5	AG
	(iii) [PE gain = 80g(7.25 + 5)]	M1		For finding PE gain from net level to O
	PE gain = 9604	A1		
	PE gain = EE at net level ➔ P just reaches O	A1	3	AG
	(iv) For any one of 'light rope', 'no air resistance', 'no energy lost in rope'	B1		
	For any other of the above	B1	2	

FIRST ALTERNATIVE METHOD FOR PART (ii)			
[160g – 2058x/5.25 = 160v dv/dx]	M1		For using Newton's second law with a = v dv/dx, separating the variables and attempting to integrate
$v^2/2 = gx - 1.225x^2$ (+ C)	A1		Any correct form
	M1		For using v(2) = 3.5
C = -8.575	A1		
$[v(7)^2]/2$ = 68.6 – 60.025 – 8.575 = 0 ➔P&Q just reach the net	A1	5	AG

SECOND ALTERNATIVE METHOD FOR PART (ii)			
$\ddot{x} = g - 2.45x$ (= -2.45(x – 4))	B1		
	M1		For using $n^2 = 2.45$ and $v^2 = n^2(A^2 - (x-4)^2)$
$3.5^2 = 2.45(A^2 - (-2)^2)$ (A = 3)	A1		
[(4 – 2) + 3]	M1		For using 'distance travelled downwards by P and Q = distance to new equilibrium position + A
distance travelled downwards by P and Q = 5 ➔P&Q just reach the net	A1	5	AG

7	(i) $[a = 0.7^2/0.4]$	M1		For using $a = v^2/r$
	For not more than one error in $T - 0.8g\cos60^\circ = 0.8 \times 0.7^2/0.4$	A1		
	Above equation complete and correct	A1		
	Tension is 4.9N	A1	4	
	(ii)	M1		For using the principle of conservation of energy
	$\frac{1}{2}\,0.8v^2 = \frac{1}{2}\,0.8(0.7)^2 + 0.8g0.4 - 0.8g0.4\cos60^\circ$	A1		$(v = 2.1)$
	$(2.1 - 0)/7 = 2u$	M1		For using NEL
	Q's initial speed is 0.15ms^{-1}	A1	4	AG
	(iii)	M1		For using Newton's second law transversely
	$(m)0.4\ddot{\theta} = -(m)g\sin\theta$	A1		*Allow m = 0.8 (or any other numerical value)
	$[0.4\ddot{\theta} \approx -g\theta]$	M1		For using $\sin\theta \approx \theta$
	$[\frac{1}{2}m0.15^2 = mg0.4(1 - \cos\theta_{max})$ ➔ $\theta_{max} = 4.34^\circ$ (0.0758rad)]	M1		For using the principle of conservation of energy to find θ_{max}
	θ_{max} small justifies $0.4\ddot{\theta} \approx -g\theta$, and this implies SHM	A1	5	
	(iv) $[T = 2\pi/\sqrt{24.5} = 1.269..]$ $[\sqrt{24.5}\,t = \pi]$	M1		For using $T = 2\pi/n$ or for solving either sin nt = 0 (non-zero t) (considering displacement) or cos nt = -1 (considering velocity)
	Time interval is 0.635s	A1ft	2	From t = ½ T

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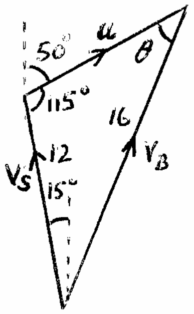
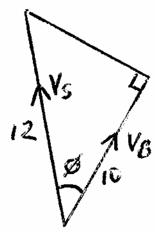
1 (i)	Using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, $56 = 0 + \frac{1}{2}\alpha \times 8^2$ $\alpha = 1.75 \text{ rad s}^{-2}$	M1 A1 **2**	
(ii)	Using $\omega_1^2 = \omega_0^2 + 2\alpha\theta$, $36^2 = 20^2 + 2 \times 1.75\theta$ $\theta = 256$ rad	M1 A1 ft **2**	ft is $448 \div \alpha$
2	Volume is $\int_0^a \pi(4a^2 - x^2)\,dx = \pi\left[4a^2x - \frac{1}{3}x^3\right]_0^a$ $= \frac{11}{3}\pi a^3$ $\int_0^a \pi x(4a^2 - x^2)\,dx$ $= \pi\left[2a^2x^2 - \frac{1}{4}x^4\right]_0^a$ $= \frac{7}{4}\pi a^4$ $\bar{x} = \dfrac{\frac{7}{4}\pi a^4}{\frac{11}{3}\pi a^3}$ $= \dfrac{21}{44}a$	M1 A1 M1 A1 A1 M1 A1 **7**	*π may be omitted throughout* (Limits not required) (Limits not required) for $\dfrac{\int xy^2\,dx}{\int y^2\,dx}$
3 (i)	$I = 6.2 + 2.8 = 9.0 \text{ kg m}^2$	B1 **1**	
(ii)	WD against frictional couple is $L \times \frac{1}{2}\pi$ Loss of PE is $6 \times 9.8 \times 1.3$ $(= 76.44)$ Gain of KE is $\frac{1}{2} \times 9.0 \times 2.4^2$ $(= 25.92)$ By work-energy principle, $L \times \frac{1}{2}\pi = 76.44 - 25.92$ $L = 32.2$ N m	B1 B1 B1 ft M1 A1 **5**	 Equation involving WD, KE and PE Accept 32.1 to 32.2
(iii)	$6 \times 9.8 \times 0.8 - L = I\alpha$ $\alpha = 1.65 \text{ rad s}^{-2}$	M1 A1 ft A1 **3**	Moments equation

4 (i)	MI of elemental disc about a diameter is $\frac{1}{4}\left(\frac{M}{3a}\delta x\right)a^2$	B1	$\frac{M}{3a}$ may be $\rho\pi a^2$ throughout *(condone use of $\rho = 1$)*
	MI of elemental disc about AB is $\frac{1}{4}\left(\frac{M}{3a}\delta x\right)a^2 + \left(\frac{M}{3a}\delta x\right)x^2$	M1 A1	Using parallel axes rule (can award A1 for $\frac{1}{4}ma^2 + mx^2$)
	$I = \frac{M}{3a}\int_0^{3a}(\frac{1}{4}a^2 + x^2)\,\mathrm{d}x$	M1 A1	Integrating MI of disc *about AB* Correct integral expression for I
	$= \frac{M}{3a}\left[\frac{1}{4}a^2x + \frac{1}{3}x^3\right]_0^{3a}$ $= \frac{M}{3a}(\frac{3}{4}a^3 + 9a^3)$ $= M(\frac{1}{4}a^2 + 3a^2)$ $= \frac{13}{4}Ma^2$	M1 A1 (ag) **7**	Obtaining an expression for I in terms of M and a *Dependent on previous M1*
(ii)	Period is $2\pi\sqrt{\frac{I}{Mgh}}$	M1	or $-Mgh\sin\theta = I\ddot{\theta}$
	$= 2\pi\sqrt{\frac{\frac{13}{4}Ma^2}{Mg\frac{3}{2}a}}$	A1	
	$= 2\pi\sqrt{\frac{13a}{6g}}$	A1 **3**	

5 (i)	 $\frac{\sin\theta}{12} = \frac{\sin 115}{16}$ $\theta = 42.8°$ Bearing of $\mathbf{v}_B$ is 007.2° $\frac{u}{\sin 22.2} = \frac{16}{\sin 115}$ $u = 6.66$ Time taken is $\frac{2400}{6.664} = 360$ s	M1 A1 M1 A1 M1 A1 M1*A1 ft **8**	Relative velocity on bearing 050 Correct velocity diagram; or $\begin{pmatrix} u\sin 50 \\ u\cos 50 \end{pmatrix} = \begin{pmatrix} 16\sin\alpha \\ 16\cos\alpha \end{pmatrix} - \begin{pmatrix} 12\sin 345 \\ 12\cos 345 \end{pmatrix}$ or eliminating u (or α) or obtaining equation for u (or α) *For equations in α and t* *M1*M1A1 for equations* *M1 for eliminating t (or α)* *A1 for $\alpha = 7.2$* *M1A1 ft for equation for t (or α)* *A1 cao for $t = 360$*
(ii)	 $\cos\phi = \frac{10}{12}$ $\phi = 33.6°$ Bearing of $\mathbf{v}_B$ is 018.6°	M1 A1 M1 A1 **4**	Relative velocity perpendicular to $\mathbf{v}_B$ Correct velocity diagram For alternative methods: M2 for a completely correct method A2 for 018.6 (give A1 for a correct relevant angle)

6 (i)	$I = \frac{1}{3}ma^2 + m(\frac{1}{3}a)^2$	M1	Using parallel axes rule
	$= \frac{4}{9}ma^2$	A1	
	$mg(\frac{1}{3}a\cos\theta) = I\alpha$	M1	
	$\alpha = \dfrac{\frac{1}{3}mga\cos\theta}{\frac{4}{9}ma^2} = \dfrac{3g\cos\theta}{4a}$	A1 (ag) **4**	
(ii)	By conservation of energy,	M1	
	$\frac{1}{2}I\omega^2 = mg(\frac{1}{3}a\sin\theta)$	A1 ft	
	$\frac{2}{9}ma^2\omega^2 = \frac{1}{3}mga\sin\theta$		
	$\omega = \sqrt{\dfrac{3g\sin\theta}{2a}}$	A1 **3**	*Condone* $\omega^2 = \dfrac{3g\sin\theta}{2a}$
	OR $\omega\dfrac{d\omega}{d\theta} = \dfrac{3g\cos\theta}{4a}$ M1		
	$\frac{1}{2}\omega^2 = \int \dfrac{3g\cos\theta}{4a}d\theta$		
	$= \dfrac{3g\sin\theta}{4a}$ $(+C)$ A1		
	$\omega = \sqrt{\dfrac{3g\sin\theta}{2a}}$ A1		
(iii)	Acceleration parallel to rod is $(\frac{1}{3}a)\omega^2$	B1	
	$F - mg\sin\theta = m(\frac{1}{3}a)\omega^2$	M1	Radial equation with 3 terms
	$F - mg\sin\theta = \frac{1}{2}mg\sin\theta$		
	$F = \frac{3}{2}mg\sin\theta$	A1	
	Acceleration perpendicular to rod is $(\frac{1}{3}a)\alpha$	B1 ft	ft is $r\alpha$ with r the same as before
	$mg\cos\theta - R = m(\frac{1}{3}a)\alpha$	M1	Transverse equation with 3 terms
	$mg\cos\theta - R = \frac{1}{4}mg\cos\theta$		
	$R = \frac{3}{4}mg\cos\theta$	A1 **6**	
	OR $R(\frac{1}{3}a) = I_G\alpha$ M1		Must use I_G
	$R(\frac{1}{3}a) = (\frac{1}{3}ma^2)\left(\dfrac{3g\cos\theta}{4a}\right)$ A1		
	$R = \frac{3}{4}mg\cos\theta$ A1		
(iv)	On the point of slipping, $F = \mu R$		
	$\frac{3}{2}mg\sin\theta = \mu(\frac{3}{4}mg\cos\theta)$	M1	
	$\tan\theta = \frac{1}{2}\mu$	A1 (ag) **2**	Correctly obtained *Dependent on 6 marks earned in (iii)*

7 (i)	GPE $= (-)\, mg(2a\cos\theta)\cos\theta$ EPE $= \dfrac{\frac{1}{2}mg}{2a}(AR-a)^2$ $= \dfrac{\frac{1}{2}mg}{2a}(2a\cos\theta - a)^2$ $V = \frac{1}{4}mga(2\cos\theta-1)^2 - 2mga\cos^2\theta$ $= mga(\cos^2\theta - \cos\theta + \frac{1}{4} - 2\cos^2\theta)$ $= mga(\frac{1}{4} - \cos\theta - \cos^2\theta)$	B1 M1 A1 A1 (ag) **4**	or $(-)\, mg(a + a\cos 2\theta)$
(ii)	$\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = mga(\sin\theta + 2\cos\theta\sin\theta)$ $= mga\sin\theta(1+2\cos\theta)$ Equilibrium when $\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = 0$ ie when $\theta = 0$	B1 M1 A1 (ag) **3**	
(iii)	KE is $\frac{1}{2}m(2a\dot{\theta})^2$ $2ma^2\dot{\theta}^2 + V = \text{constant}$ Differentiating with respect to t, $4ma^2\dot{\theta}\ddot{\theta} + \dfrac{\mathrm{d}V}{\mathrm{d}\theta}\dot{\theta} = 0$ $4ma^2\dot{\theta}\ddot{\theta} + mga\sin\theta(1+2\cos\theta)\dot{\theta} = 0$ $\ddot{\theta} = -\dfrac{g}{4a}\sin\theta(1+2\cos\theta)$	B1 M1 M1 A1 ft A1 (ag) **5**	(can award this M1 if no KE term) *SR* B2 (replacing the last 3 marks) for the given result correctly obtained by differentiating w.r.t. θ
(iv)	When θ is small, $\sin\theta \approx \theta$, $\cos\theta \approx 1$ $\ddot{\theta} \approx -\dfrac{g}{4a}\theta(1+2) = -\dfrac{3g}{4a}\theta$ Period is $2\pi\sqrt{\dfrac{4a}{3g}}$	M1 A1 A1 **3**	

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Note: "3 sfs" means an answer which is equal to, or rounds to, the given answer. If such an answer is seen and then later rounded, apply ISW.

1	$(0\times0.1) + 1\times0.2 + 2\times0.3 + 3\times0.4$ $= 2(.0)$ $(0^2\times0.1) + 1\times0.2 + 2^2\times0.3 + 3^2\times0.4$ (= 5) $- 2^2$ $= 1$	M1 A1 M1 M1 A1 5	≥ 2 non-zero terms correct eg $\div 4$: M0 ≥ 2 non-zero terms correct $\div 4$: M0 Indep, ft their μ. Dep +ve result $(-2)^2\times0.1+(-1)^2\times0.2+0^2\times0.3+1^2\times0.4$:M2 ≥ 2 non-0 correct: M1 $\div 4$: M0
Total		**5**	
2	UK Fr Ru Po Ca 1 2 3 4 5 or 5 4 3 2 1 4 3 1 5 2 2 3 5 1 4 Σd^2 (= 24) $r_s = 1 - \frac{6 \times \text{“24”}}{5 \times (5^2-1)}$ $= -\frac{1}{5}$ or -0.2	 M1 A1 M1 M1 A1 5	Consistent attempt rank other judge RCFUP 3 5 2 1 4 3 1 4 5 2 1 2 3 4 5 5 4 3 2 1 All 5 d^2 attempted & added. Dep ranks att'd Dep 2nd M1 $\frac{43 - 15^2/5}{\sqrt{((55-15^2/5)(55-15^2/5))}}$ Corr sub in ≥ 2 *S*'s M1 All correct: M1
Total		**5**	
3i	${}^{15}C_7$ or $^{15!}/_{7!8!}$ 6435	M1 A1 2	
ii	${}^6C_3 \times {}^9C_4$ or $^{6!}/_{3!3!} \times {}^{9!}/_{4!5!}$ 2520	M1 A1 2	Alone except allow $\div {}^{15}C_7$ Or ${}^6P_3 \times {}^9P_4$ or $^{6!}/_{3!} \times {}^{9!}/_{5!}$ Allow $\div {}^{15}P_7$ NB not $^{6!}/_{3!} \times {}^{9!}/_{4!}$ 362880
Total		**4**	
4ia	$^1/_3$ oe	B1 1	B↔W MR: max (a)B0(b)M1M1(c)B1M1
b	P(BB) + P(WB) attempted $= {}^4/_{10} \times {}^3/_9 + {}^6/_{10} \times {}^4/_9$ or $^2/_{15} + {}^4/_{15}$ $= {}^2/_5$ oe	M1 M1 A1 3	Or $^4/_{10} \times {}^3/_9$ OR $^6/_{10} \times {}^4/_9$ correct NB $^4/_{10} \times {}^4/_{10} + {}^6/_{10} \times {}^4/_{10} = {}^2/_5$: M1M0A0
c	Denoms 9 & 8 seen or implied $^3/_9 \times {}^2/_8 + {}^6/_9 \times {}^3/_8$ $= {}^1/_3$ oe	B1 M1 A1 3	Or $^2/_{15}$ as numerator Or $\frac{^2/_{15}}{^4/_{10}}$ Or $\frac{^4/_{10}\times^6/_9\times^3/_8 + {}^4/_{10}\times^3/_9\times^2/_8}{\text{above} + {}^6/_{10}\times^5/_9\times^4/_8 + {}^6/_{10}\times^4/_9\times^3/_8}$ May not see wking
ii	P(Blue) not constant or discs not indep, so no	B1 1	Prob changes as discs removed Limit to no. of discs. Fixed no. of discs Discs will run out Context essential: "disc" or "blue" NOT fixed no. of trials NOT because without repl Ignore extra
Total		**8**	

5i	1991 100 000 to 110 000	B1 ind B1 ind 2	Or fewer in 2001 Allow digits100 to 110
iia	Median = 29 to 29.9 Quartiles 33 to 34, 24.5 to 26 = 7.5 to 9.5 140 to 155 23 to 26.3%	B1 M1 A1 M1 A1 5	 Or one correct quartile and subtr NOT from incorrect wking ×1000, but allow without Rnded to 1 dp or integer 73.7 to 77% : SC1
b	Older Median (or ave) greater } % older mothers greater oe} % younger mothers less oe}	B1 B1 B1 3	Or 1991 younger Any two Or 1991 steeper so more younger: B2 NOT mean gter Ignore extra
Total		**10**	

6ia	Correct subst in ≥ two S formulae $\dfrac{767-\dfrac{60\times 72}{8}}{\sqrt{(1148-\dfrac{60^2}{8})(810-\dfrac{72^2}{8})}}$ or $\dfrac{227}{\sqrt{698}\sqrt{162}}$ = 0.675 (3 sfs)	M1 M1 A1 3	Any version All correct. Or $\dfrac{767\text{-}8\text{x}7.5\text{x}9}{\sqrt{((1148\text{-}8\text{x}7.5^2)(810\text{-}8\text{x}9^2))}}$ or correct substn in any correct formula for r
b	1 y always increases with x or ranks same oe	B1 B1 2	+ve grad thro'out. Increase in steps. Same order. Both ascending order Perfect RANK corr'n Ignore extra NOT Increasing proportionately
iia	Closer to 1, or increases because nearer to st line	B1 B1 2	Corr'n stronger. Fewer outliers. "They" are outliers Ignore extra
b	None, or remains at 1 Because y still increasing with x oe	B1 B1 2	Σd^2 still 0. Still same order. Ignore extra NOT differences still the same. NOT ft (i)(b)
iii	13.8 to 14.0	B1 1	
iv	(iii) or graph or diag or my est Takes account of curve	B1 B1 2	Must be clear which est. Can be implied. "This est" probably ⇒ using equn of line Straight line is not good fit. Not linear. Corr'n not strong.
Total		**12**	
7i	P(contains voucher) constant oe Packets indep oe	B1 B1 2	Context essential NOT vouchers indep
ii	0.9857 or 0.986 (3 sfs)	B2 2	B1 for 0.9456 or 0.946 or 0.997(2) or for 7 terms correct, allow one omit or extra NOT 1 – 0.9857 = 0.0143 (see (iii))
iii	(1 – 0.9857) = 0.014(3) (2 sfs)	B1ft 1	Allow 1- their (ii) correctly calc'd
iv	B(11, 0.25) or 6 in 11 wks stated or impl ${}^{11}C_6 \times 075^5 \times 0.25^6$ (= 0.0267663) P(6 from 11) × 0.25 = 0.00669 or 6.69×10^{-3} (3 sfs)	B1 M1 M1 A1 4	or $0.75^a \times 0.25^b$ ($a + b = 11$) or ${}^{11}C_6$ dep B1
Total		**9**	

8i	$\sqrt{0.04}$ (= 0.2) (1 – their $\sqrt{0.04}$)^2 = 0.64	M1 M1 A1 3	
ii	$1 - p$ seen M1 for either $2p(1-p) = 0.42$ or $p(1-p) = 0.21$ oe $2p^2 - 2p + 0.42 (= 0)$ or $p^2 - p + 0.21 (= 0)$ $\frac{2\pm\sqrt{((-2)^2 - 4\times0.42)}}{2\times2}$ or $\frac{1\pm\sqrt{((-1)^2 - 4\times0.21)}}{2\times1}$ or $(p-0.7)(p-0.3)=0$ or $(10p-7)(10p-3)=0$ $p = 0.7$ or 0.3	B1 M1 M1 M1 A1 5	$2pq = 0.42$ or $pq = 0.21$ Allow $pq = 0.42$ or opp signs, correct terms any order (= 0) oe Correct Dep B1M1M1 Any corr subst'n or fact'n Omit 2 in 2nd line: max B1M1M0M0A0 One corr ans with no or inadeq wking: SC1 eg 0.6 × 0.7 = 0.42 ⇒ p = 0.7 or 0.6 $p^2 + 2pq + q^2 = 1$ B1 $p^2 + q^2 = 0.58$ } $p = 0.21/q$ } $p^4 - 0.58p^2 + 0.0441 = 0$ M1 corr subst'n or fact'n M1 $1 - p$ seen B1 $2p(1-p) = 0.42$ or $p(1-p) = 0.21$ M1 $p^2 - p = -0.21$ $p^2 - p + 0.25 = -0.21 + 0.25$ oe } M1 OR $(p - 0.5)^2 - 0.25 = -0.21$ oe } $(p - 0.5)^2 = 0.04$ M1 $(p - 0.5) = \pm0.02$ $p = 0.3$ or 0.7 A1
Total		**8**	
9ia	$1 / {}^1/_5$ = 5	M1 A1 2	
b	$({}^4/_5)^3 \times {}^1/_5$ = ${}^{64}/_{625}$ or 0.102 (3 sfs)	M1 A1 2	
c	$({}^4/_5)^4$ = ${}^{256}/_{625}$ or a.r.t 0.410 (3 sfs) or 0.41	M1 A1 2	or $1 - ({}^1/_5 + {}^4/_5\times{}^1/_5 + ({}^4/_5)^2\times{}^1/_5 + ({}^4/_5)^3\times{}^1/_5)$ NOT $1 - ({}^4/_5)^4$
iia	P(Y=1) = p, P(Y=3) = q^2p, P(Y=5) = q^4p	B1 1	P(Y=1)+P(Y=3)+P(Y=5)= $p + q^2p + q^4p$ p, $p(1-p)^2$, $p(1-p)^4$ q^{1-1}, q^{3-1}, q^{5-1} or any of these with $1 - p$ instead of q "Always q to even power × p" Either associate each term with relevant prob Or give indication of how terms derived ≥ two terms
b	Recog that c.r. = q^2 or $(1-p)^2$ $S_\infty = \frac{p}{1-q^2}$ or $\frac{p}{1-(1-p)^2}$ P(odd) = $\frac{1-q}{1-q^2}$ $= \frac{1-q}{(1-q)(1+q)}$ Must see this step for A1 (= $\frac{1}{1+q}$ **AG**)	M1 M1 M1 A1 4	or eg $r = q^2p/p$ $\left(= \frac{p}{2p-p^2}\right) = \frac{p}{p(2-p)}$ $\left(= \frac{1}{2-p}\right) = \frac{1}{2-(1-q)}$

Total		**11**	

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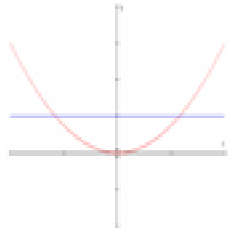
1	(i)	$\hat{\mu}$ = 4830.0/100 = 48.3 249509.16/100 – (their $\bar{x}^2$) × 100/99 = 163.84	B1 M1 M1 A1	 **4**	48.3 seen Biased estimate: 162.2016: can get B1M1M0 Multiply by $n/(n-1)$ Answer, 164 or 163.8 or 163.84
	(ii)	No, Central Limit theorem applies, so can assume distribution is normal	B2	**2**	"No" with statement showing CLT is understood (though CLT does not need to be mentioned) [SR: No with reason that is not wrong: B1]
2		B(130, 1/40) ≈ Po(3.25) $e^{-\lambda}\frac{\lambda^4}{4!}$ = 0.180	B1 M1 A1√ M1 A1	 **5**	B(130, 1/40) stated or implied Poisson, *or* correct N on their B(n, p) Parameter their np, *or* correct parameter(s)√ Correct formula, or interpolation Answer, 0.18 or a.r.t. 0.180 [SR: N(3.25, 3.17) or N(3.25, 3.25): B1M1A1]
3	(i)	Binomial	B1	**1**	Binomial stated or implied
	(ii)	Each element equally likely Choices independent	B1 B1	 **2**	All elements, or selections, equally likely stated Choices independent [not just "independent"] [can get B2 even if (i) is wrong]
4	(i)	*Two of:* Distribution symmetric No substantial truncation Unimodal/Increasingly unlikely further from μ, etc	B1 B1	 **2**	One property Another definitely different property Don't give both marks for just these two "Bell-shaped": B1 only unless "no truncation"
	(ii)	Variance $8^2/20$ $z = \frac{47.0-50.0}{\sqrt{8^2/20}} = -1.677$ Φ(1.677) = 0.9532	M1 A1 A1 A1	 **4**	Standardise, allow cc, don't need n Denominator (8 or 8^2 or √8) ÷ (20 or √20 or 20^2) z-value, a.r.t. –1.68 or +1.68 Answer, a.r.t. 0.953
5	(i)	H_1: $\lambda > 2.5$ or 15	B1	**1**	λ > 2.5 or 15, allow μ, don't need "H_1"
	(ii)	Use parameter 15 P(> 23) 1 – 0.9805 = 0.0195 or 1.95%	M1 M1 A1	 **3**	λ = 15 used [N(15, 15) gets this mark only] Find P(> 23 or ≥ 23), final answer < 0.5 eg 0.0327 or 0.0122 Answer, 1.95% or 2% or 0.0195 or 0.02 [SR: 2-tailed, 3.9% gets 3/3 here]
	(iii)	P(≤ 23 | λ = 17) = 0.9367 P(≤ 23 | λ = 18) = 0.8989 Parameter = 17 λ = 17/6 or 2.83	M1 A1 M1	 **3**	One of these, or their complement: .9367, .8989, 0.9047, 0.8551, .9317, .8933, .9907, .9805 Parameter 17 *[17.1076]*, needs P(≤ 23), cwo [SR: if insufficient evidence can give B1 for 17] Their parameter ÷ 6 *[2.85]* [SR: Solve (23.5 – λ)/√λ = 1.282 M1; 18.05 A0]
6	(i)	H_0: $p = 0.19$, H_1: $p < 0.19$ where p is population proportion $0.81^{20} + 20 \times 0.81^{19} \times 0.19$ = 0.0841 Compare 0.1	B2 M1 A1 A1 B1		Correct, B2. One error, B1, but x or $\bar{x}$ or r: B0 Binomial probabilities, allow 1 term only Correct expression [0.0148 + 0.0693] Probability, a.r.t. 0.084 Explicit comparison of "like with like"
	or	Add binomial probs until ans > 0.1 Critical region ≤ 1	A1 B1		[P(≤ 2) = 0.239]
		Reject H_0 Significant evidence that proportion of e's in language is less than 0.19	M1 A1√	 **8**	Correct deduction and method [needs P(≤ 1)] Correct conclusion in context [SR: N(3.8, 3.078): B2M1A0B1M0]
	(ii)	Letters not independent	B1	**1**	Correct modelling assumption, stated in context Allow "random", "depends on message", etc

Q	Part	Answer	Marks	Total	Guidance
7	(i)		B1 B1 B1	3	Horizontal straight line Positive parabola, symmetric about 0 Completely correct, including correct relationship between two Don't need vertical lines or horizontal lines outside range, but don't give last B1 if horizontal line continues past "±1"
	(ii)	*S* is equally likely to take any value in range, *T* is more likely at extremities	B2	2	Correct statement about distributions (*not* graphs) *[Partial statement, or correct description for one only: B1]*
	(iii)	$\int_t^1 \frac{3}{2}x^2 dx = \left[\frac{x^3}{2}\right]_t^1$ ½(1 – t^3) = 0.2 or ½(t^3 + 1) = 0.8 $t^3 = 0.6$ $t = 0.8434$	M1 B1 M1 M1 A1	5	Integrate f(x) with limits (–1, t) or (t, 1) [recoverable if t used later] Correct indefinite integral Equate to 0.2, or 0.8 if [–1, t] used Solve cubic equation to find t Answer, in range [0.843, 0.844]
8	(i)	$\frac{64.2-63}{\sqrt{12.25/23}} = 1.644$ P(z > 1.644) = 0.05	M1dep A1 dep M1 A1	4	Standardise 64.2 with √n z = 1.644 or 1.645, must be + Find Φ(z), answer < 0.5 Answer, a.r.t. 0.05 or 5.0%
	(ii) (a)	$63 + 1.645 \times \frac{3.5}{\sqrt{50}}$ ≥ 63.81	M1 B1 A1	3	63 + 3.5 × k / √50, k from Φ^{-1}, *not* – k = 1.645 (allow 1.64, 1.65) Answer, a.r.t. 63.8, allow >, ≥, =, c.w.o.
	(b)	P(< 63.8 | μ = 65) $\frac{63.8-65}{3.5/\sqrt{50}} = -2.3956$ 0.0083	M1 M1 A1 A1	4	Use of correct meaning of Type II Standardise their c with √50 z = (±) 2.40 [or –2.424 or – 2.404 etc] Answer, a.r.t. 0.008 [eg, 0.00767]
	(iii)	B better: Type II error smaller (and same Type I error)	B2√	2	This answer: B2. "B because sample bigger": B1. *[SR: Partial answer: B1]*
9	(a)	$np > 5$ and $nq > 5$ $0.75n > 5$ is relevant $n > 20$	M2 A1	3	Use either $nq > 5$ or $npq > 5$ [SR: If M0, use $np > 5$, or "$n = 20$" seen: M1] Final answer $n > 20$ or $n \geq 20$ only
	(b) (i)	70.5 – μ = 1.75σ μ – 46.5 = 2.25σ Solve simultaneously μ = 60 σ = 6	M1 A1 B1 M1 A1√ A1√	6	Standardise once, and equate to Φ^{-1}, ± cc Standardise twice, signs correct, cc correct Both 1.75 and 2.25 Correct solution method to get one variable μ, a.r.t. 60.0 or ± 154.5 σ, a.r.t. 6.00 [Wrong cc (below): A1 both] [SR: σ^2: M1A0B1M1A1A0]
	(ii)	$np = 60$, $npq = 36$ $q = 36/60 = 0.6$ $p = 0.4$ $n = 150$	M1dep depM1 A1√ A1√	4	$np = 60$ and $npq = 6^2$ or 6 Solve to get q or p or n $p = 0.4$ √ on wrong cc or z $n = 150$ √ on wrong cc or z

		σ	μ	q	p (±0.01)	n
70.5	46.5	6	60	0.6	0.4	150
71	46	6.25	60.0625	0.6504	0.3496	171.8
71.5	46.5	6.25	60.5625	0.6450	0.3550	170.6
70.5	45.5	6.25	59.5625	0.6558	0.3442	173.0
71.5	45.5	6.5	60.125	0.7027	0.2973	202.2
70	46	6	59.5	0.6050	0.3950	150.6

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1	$\int_0^1 a\,dx + \int_1^\infty \frac{a}{x^2}\,dx = 1$	M1		For sum of integrals =1
	$[ax]_0^1 + \left[-\frac{a}{x^3}\right]_1^\infty = 1$	A1		For second integral.
	$a + a = 1$	A1		For second a
	$a = ½$	A1	**4**	Or from F(x) M1A1 then F(∞)=1 M1, $a = {}^1/_2$ A1
2	(i) $\bar{X}_I \sim N(5, \frac{0.7^2}{20})$	B1		If no parameters allow in (ii)
	$\bar{X}_E \sim N(4.5, \frac{0.5^2}{25})$	B1	**2**	If 0.7/20, 0.5/25 then B1 for both, with means in (ii)
	(ii) Use $\bar{X}_I - \bar{X}_E \sim N(0.5, \sigma^2)$	M1A1		OR $\bar{X}_I - \bar{X}_E - 1 \sim N(-0.5, \sigma^2)$
	$\sigma^2 = 0.49/20 + 0.25/25$	B1		cao
	$1 - \Phi([1-0.5]/\sigma)$	M1		RH probability implied. If 0.7, 0.5 in σ^2, M1A1B0M1A1 for 0.165
	= 0.0036 or 0.0035	A1	**5**	
3	Assumes differences form a random sample from a normal distribution.		B1	
	$H_0: \mu = 0$, $H_1: \mu > 0$	B1		Other letters if defined; or in words
	$\bar{x} = 17.2/12$; $s^2 = 10.155$ AEF	B1B1		Or $(12/11)(136.36/12 - (17.2/12)^2)$aef
	EITHER: $t = \frac{\bar{x}}{\sqrt{s^2/12}}$ (+ or -)	M1		With 12 or 9.309/11
	=1.558	A1		Must be positive. Accept 1.56
	1.363 seen	B1		Allow CV of 1.372 or 1.356
	1.558 > 1.363, so reject H_0 and accept that there evidence that the readings from the aneroid device overestimate blood pressure on average	B1√		Explicit comparison of CV(not - with +) and conclusion in context.
	OR: For critical region or critical value of $\bar{x}$			
	$1.363\sqrt{(s^2/12)}$	M1B1		B1 for correct ***t***
	Giving 1.25(3)	A1		
	Compare 1.43(3) with 1.25(3)			
	Conclusion in context	B1√	**8**	

4	(i) Proper (columns P, F); Trial (rows P, F): P: **31**, **11**, 42 F: **5**, 13, **18** Totals: 36, **24**, 60	B1 B1	**2**	Two correct Others correct
	(ii) (H_0: Trial results and Proper results are independent.) E-values: 25.2 16.8 10.8 7.2	M1 A1		One correct. Ft marginals in (i) All correct
	$\chi^2 = 5.3^2(25.2^{-1}+10.8^{-1}+16.8^{-1}+7.2^{-1})$ = 9.289	M1 A1 A1		Allow two errors With Yates' correction art 9.29
	Compare correctly with 7.8794 There is evidence that results are not independent.	M1 A1 √	**7**	Or 7.88 Ft χ^2_{calc}.
5	(i) $e^{-\mu} = 0.45$ $\mu_G = 0.799 \approx 0.80$ AG	M1 A1	**2**	0.799 or 0.798 or better seen
	(ii) $\mu_U \approx 1.8$ Total, $T \sim Po(2.6)$ P(>3) = 0.264	B1 M1 A1	**3**	May be implied by answer 0.264 From table or otherwise
	(iii) $e^{-2.6}2.6^6/6!$ $e^{-5.2}5.2^4/4!$ Multiply two probabilities Answers rounding to 0.0053 or 0.0054	B1 B1 M1 A1	4	Or 0.318 from table

6	(i) $\hat{p}$ = 62/200=0.31	B1		aef
	Use $\hat{p}_\alpha \pm z\sqrt{\frac{\hat{p}_\alpha(1-\hat{p}_\alpha)}{200}}$	M1		With 200 or 199
	z=1.96	B1		Seen
	Correct variance estimate	A1√		ft $\hat{p}$
	(0.2459,0.3741)	A1	**5**	art (0.246,0.374)
	(ii)EITHER: Sample proportion has an approximate normal distribution OR: Variance is an estimate	B1	**1**	Not $\hat{p}$ is an estimate, unless variance mentioned
	(iii) H_0: $p_\alpha = p_\beta$, H_1: $p_\alpha \neq p_\beta$ $\hat{p}$ =(62+35)/(200+150)	B1		aef
	EITHER: $z = (\pm)\frac{62/200 - 35/150}{\sqrt{\hat{p}\hat{q}(200^{-1}+150^{-1})}}$	M1		s^2 with, $\hat{p}$, 200, 150 (or 199,149)
		B1√		Evidence of correct variance estimate. Ft $\hat{p}$
	=1.586	A1		Rounding to 1.58 or 1.59
	(-1.96 <) 1.586 < 1.96	M1		Correct comparison with ± 1.96
	Do not reject H_0- there is insufficient evidence of a difference in proportions.	A1		SR: If variance $p_1q_1/n_1+p_2q_2/n_2$ used then: B0M1B0A1(for z=1.61 or 1.62)M1A1 Max 4/6.
	OR: $p_{s\alpha} - p_{s\beta} = zs$	M1		
	s = √(0.277×0.723(200^{-1}+150^{-1}))	B1√		Ft $\hat{p}$
	CV of $p_{s\alpha} - p_{s\beta}$= 0.0948 or 0.095	A1		
	Compare $p_{s\alpha} - p_{s\beta}$ = 0.0767 with their 0.0948	M1		
	Do not reject H_0 and accept that there is insufficient evidence of a difference in proportions	A1	**6**	Conditional on z=1.96

7	(i) $G(y) = P(Y \le y)$	M1		May be implied by following line
	$= P(X^2 \ge 1/y)$ [or $P(X > 1/\sqrt{y})$]	A1		Accept strict inequalities
	$= 1 - F(1/\sqrt{y})$	A1		
	$= \begin{cases} 0 & y \le 0, \\ y^2 & 0 \le y \le 1, \\ (1 & y > 1.) \end{cases}$	A1	4	Or $F(x)=P(X \le x) = P(Y \ge 1/x^2)$ M1 $=1 - P(Y < 1/x^2)$ A1 $=1-G(y)$;etc A1 A1
	(ii) Differentiate their $G(y)$	M1		
	to obtain $g(y)= 2y$ for $0 < y \le 1$ AG	A1	2	Only from G correctly obtained
	(iii) $\int_0^1 2y(\sqrt[3]{y})dy$	M1		Unsimplified, but with limits
	$=[6y^{7/3}/7]$	B1		OR: Find f(x), $\int_1^\infty x^{-2/3} f(x)dx$ M1
	$= \frac{6}{7}$	A1	3	$=[4x^{-14/3}/(14/3)]$; $\frac{6}{7}$ B1A1 OR: Find H(z), $Z= Y^{1/3}$
8	(i) $P(20 \le y < 25) = \Phi(0) - \Phi(-5/\sqrt{(20)})$	M1		
	Multiply by 50	A1		
	to give 18.41 AG	A1		
	18.41 for $25 \le y < 30$ and 6.59 for $y < 20$, $y \ge 30$	A1	4	
	(ii) H_0: N(25,20) fits data	B1		OR $Y \sim N(25,20)$
	$\chi^2 = 3.59^2/6.59 + 8.59^2/18.41 + 6.41^2/18.41 + 1.41^2/6.59$	M1√		ft values from (i)
	$=8.497$	A1		art 8.5
	$8.497 > 7.815$	M1		
	Accept that N(25,20) is not a good fit	A1	5	
	(iii) Use $24.91 \pm z\sqrt{(20/50)}$	M1		With $\sqrt{(20/50)}$
	$z = 2.326$	B1		
	(23.44,26.38)	A1	3	art (23.4,26.4) Must be interval
	(iv) No- Sample size large enough to apply CLT	B1		Refer to large sample size
	Sample mean will be (approximately) normally distributed whatever the distribution of Y	B1	2	Refer to normality of sample mean

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Statistics 4

1 (i) Use $P(A' \cap B') = 1 - P(A \cup B)$ Use $P(A \cap B) = P(A) + P(B) - P(A \cup B)$ $= c - 0.1$	M1 M1 A1 **3**	Or $c = 1 - P(A \cup B)$
(ii) $P(B \mid A) = (c - 0.1)/ 0.3$ Use $0 \leq p \leq 1$ to obtain $0.1 \leq c \leq 0.4$ AG	B1√ M1 A1 **3**	Shown clearly
2 H_0: $m_n = m_s$, H_1: $m_n \neq m_s$ Use Wilcoxon rank sum test 59 64 68 77 80 85 88 90 98 N N N S N S N S S $R_m = 4 + 6 + 8 + 9 = 27$ $40 - 27 = 13$ $W = 13$ Compare correctly with correct CV, !2 Do not reject H_0. There is no evidence of a difference in the median pulse rates of the two populations.	B1 M1 A1 B1 B1 M1 A1 **7**	Medians; both hypotheses 'Population medians' if words Rank and identify M0 if normal approx. used Quote critical region or state that 13 > 12. M0 if W=27 Conclusion in context.
3 (i) Use marginal distributions to obtain E(X) = -0.4, E(Y)= 1.5 E(XY) = -0.24 + 0.04 – 0.52 + 0.12 Cov(X,Y) = -0.6 + 0.6 = 0 AG	M1 A1A1 M1 A1 **5**	
(ii)$P(X = -1 \mid Y = 2) = 0.26/0.5 = 0.52$ $P(X = 0 \mid Y = 2) = 0.18/0.5 = 0.36$ $P(X = 1 \mid Y = 2) = 0.12$	M1 A1 **2**	Correct method for any one All correct SR: B1 if no method indicated

4 (i) H_0: m = 2.70, H_1: m > 2.7 Subtract 2.70 from each value and count the number of positive signs Obtain 13 Use $B(20, {}^1/_2)$ to obtain $P(X \geq 13) = 0.1316$ (0.132) Compare correctly with 0.05 Do not reject H_0. Conclude that there is insufficient evidence to claim that median level of impurity is greater than 2.70	B1 M1 A1 M1 A1 M1 A1 **7**	In terms of medians Allow just 'medians' here For finding tail probability Or CR: $X \geq 15$ M1A1 Or: N(10, 5), p=0.132
(ii) Wilcoxon signed rank test Advantage: More powerful (uses more formation) Disadvantage: This test requires a symmetric population distribution, not required for sign test	B1 B1 B1 **3**	 Smaller P(Type II) Not 'more time taken'
5 (i) $\int_0^\infty \frac{1}{(\alpha-1)!} x^{\alpha-1} e^{-x} dx = 1$, result follows	B1 **1**	
(ii) $M_X(t) = \int_0^\infty \frac{1}{(\alpha-1)!} x^{\alpha-1} e^{-x} e^{xt} dx$ $= \int_0^\infty \frac{1}{(\alpha-1)!} x^{\alpha-1} e^{-x(1-t)} dx$ $x = u/(1-t)$, $dx = du/(1-t)$ and limits unchanged $= \int_0^\infty \frac{1}{(\alpha-1)!} \frac{u^{\alpha-1}}{(1-t)^{\alpha-1}} \frac{e^{-u}}{1-t} du$ $= \frac{1}{(\alpha-1)!(1-t)^\alpha} \int_0^\infty u^{\alpha-1} e^{-u} du$ $= (1-t)^{-\alpha}$ AG	M1 M1 A1 A1 A1 **5**	 Attempt to differentiate With evidence
(iii) EITHER: $M'(t) = \alpha(1-t)^{-\alpha-1}$ $M''(t) = \alpha(\alpha+1)(1-t)^{-\alpha-2}$ Substitute t=0 $E(X) = \alpha$ $Var(X) = \alpha(\alpha+1) - \alpha^2$ $= \alpha$ OR: $(1-t)^{-\alpha} = 1 + \alpha t + ½\alpha(\alpha+1)t^2 + \ldots$ $E(X) = \alpha$ $Var(X) = E(X^2) - [E(X)]^2$ $= \alpha(\alpha+1) - \alpha^2$; α	B1 B1 M1 A1 M1 A1 M1A1 B1 M1 A1A1 **6**	 AEF M0 if t involved

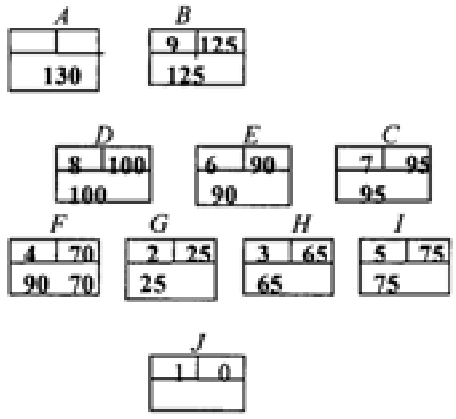
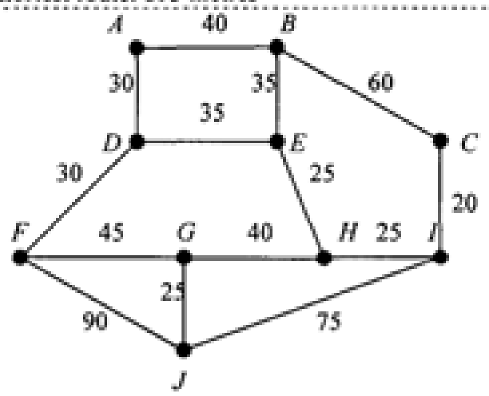
6 (i) $q+pt$	B1 **1**	Accept qt^0+pt^1
(ii) $(q+pt)^n$ (= $G_S(t)$) Binomial	B1 B1 **2**	
(iii) $E(S)=G'(1) = np(q+p)$ $= np$ $Var(S) = G''(1)+G'(1) - [G'(1)]^2$ $= n(n-1)p^2(p+q) + np - n^2p^2$ $= npq$	M1A1 A1 M1 A1 A1 **6**	AEF, properly obtained
(iv) $(½ + ½t)^{10}e^{-(1-t)}$ Find coefficient of t^2 $(1/2^{10})(1 + 10t + ½\times10\times9t^2)$ $e^{-1}(1 + t + ½t^2)$ Required coefficient $= e^{-1}2^{-10}(1/2 + 10 + 45)$ = 0.0199	M1 M1 A1 A1 M1 A1 **6**	Seen May be implied OR: P(Y=0)P(Z=2)+….M1, Z is Po(1) M1 Ans:A1A1A1;A1 Not from $e^{-(1-t)}=1-(1-t)+(1-t)^2/2$ No more than one term missing--
7 (i) $E(T_1) = 2E(\bar{X}) = 2\times\frac{1}{2}\theta = \theta$ (So T_1 is an unbiased estimator of θ)	M1A1 2	SR: B1 if $\bar{X} = \int_0^{\theta}\frac{x}{\theta}\,d\theta$
(ii) $E(U) = \int_0^{\theta}\frac{nu^n}{\theta^n}du$; $\left[\frac{nu^{n+1}}{\theta^n(n+1)}\right]$; $\frac{n\theta}{n+1}$ $E(U^2) = \int_0^{\theta}\frac{nu^{n+1}}{\theta^n}du$; $\frac{n}{n+2}\theta^2$ $Var(U) = E(U^2) - [E(U)]^2$ $= \frac{n\theta^2}{(n+1)^2(n+2)}$ AG	M1A1A1 M1A1 A1 6	
(iii) $Var(T_2) = \theta^2/[n(n+2)]$ $Var(T_1) = 4Var(X)/n$; $\theta^2/3n$ $Var(T_2)/Var(T_1)$ $3/(n+2)$ < 1 for $n > 1$ So T_2 is more efficient than T_1	B1 M1A1 M1 M1A1 A1 **7**	For comparison of var. T_1, T_2 Idea used.

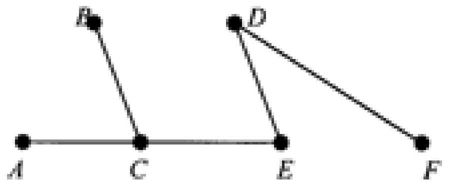
Mark Scheme 4736
June 2007

SOLUTIONS 4736 D1 June 2007 FINAL

1	(i)	Example: $N-P-Q-T-S-R-N$ or: $P-Q-S-P$	B1	1	Any valid cycle (closed and does not repeat vertices, need not be a Hamiltonian cycle)
	(ii)	It passes through Y twice	B1	1	Or, it includes a cycle (accept 'loop')
	(iii)	5	B1	1	
	(iv)	A: neither B: semi-Eulerian	B1 B1	2	If graphs are not specified, assume A is first
	(v)	A: 2 B: 1	B1 B1	2	If graphs are not specified, assume A is first A: 1, B: 0 $\Rightarrow$ B1 only
	(vi)	There are 4 odd nodes (N, P, S and Z) To connect these we must add 2 arcs	M1 A1	2 [9]	Seen or implied For 2
2	(i)	$d+f+g=120$	B1	1	For this equality. Condone an inequality
	(ii)	"(Area of) grass is not more than 4 times (area of) decking"	B1	1	Identifying the constraint in words (not just 'grass is less than or equal to 4 times decking' though)
	(iii)	$d \le f$	B1	1	Do not accept $d < f$
	(iv)	$g \ge 40$ min $d = 10$ min $f = 20$	B1 B1 B1	3	Do not accept $g > 40$ $d \ge 10$ $f \ge 20$
	(v)	$5g + 10d + 20f$ or $g + 2d + 4f$	B1	1	Or any positive multiple of this
	(vi)	Minimise $g + 2d + 4f$ Subject to $d + f + g = 120$ $g - 4d + s = 0$ $d - f + t = 0$ $g \ge 40$, and $d \ge 10, f \ge 20, s \ge 0, t \ge 0$	M1 B1 A1	3 [10]	For a reasonable attempt at setting up the minimisation problem using their expressions For dealing with this slack variable correctly (variables on LHS and constant on RHS) For a completely correct formulation (accept d and $f \ge 0$, or their min values for d, f)
3	(i)	8 6 9 7 5 — Comps — Swaps After 1st pass: 6 8 9 7 5 — 1 — 1 After 2nd pass: 6 8 9 7 5 — 1 — 0 After 3rd pass: 6 7 8 9 5 — 3 — 2 After 4th pass: 5 6 7 8 9 — 4 — 4 Comparisons must be 1, 2, 3 or 4 with total ≤ 10 Swaps must be 0, 1, 2, 3 or 4 and no more than corresponding number of comparisons	 M1 M1 M1 A1 B1 B1	6	Bubble sort or decreasing order loses first 4 marks 1st pass correct 2nd pass correct, follow through from 1st pass 3rd pass correct, follow through from 2nd pass 4th pass correct Counting comparisons for at least three passes Counting swaps for at least three passes
	(ii)	Step 1 A = 8 6 9 7 5 Step 2 A = 6 9 7 5 X = 8 Step 3 A = 9 7 5 B = 6 Step 4 A = 7 5 C = 9 Step 4 A = 5 B = 6 7 Step 4 A is empty B = 6 7 5 Step 6 N = 3 Step 7 A = 6 7 5 8 9 Step 8 Display 6 7 5 8 9	 M1 M1 M1 M1 A1	5 [11]	 For identifying that 6 $\rightarrow$ B or the sublist {6} For identifying that 9 $\rightarrow$ C or the sublist {9} For identifying that 7 $\rightarrow$ B For identifying that 5 $\rightarrow$ B For the final A list or the display correct

4	(i)	<table><tr><td>*P*</td><td>*x*</td><td>*y*</td><td>*s*</td><td>*t*</td><td>*u*</td><td></td></tr><tr><td>1</td><td>-3</td><td>5</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>5</td><td>1</td><td>0</td><td>0</td><td>12</td></tr><tr><td>0</td><td>1</td><td>-5</td><td>0</td><td>1</td><td>0</td><td>10</td></tr><tr><td>0</td><td>3</td><td>10</td><td>0</td><td>0</td><td>1</td><td>45</td></tr></table>	B1 B1 B1 3	For correct use of three slack variable columns For $\pm$ (-3 5) in objective row For 1 5 12, 1 -5 10 and 3 10 45 in constraint rows
	(ii)	Pivot on second 1 in *x* column *x* column has a negative entry in objective row $12 \div 1 = 12$, $10 \div 1 = 10$, $45 \div 3 = 15$ Least non-negative ratio is 10 so pivot on the second 1	B1 B1 2	For correct pivot choice (cao) For 'negative in top row for *x*', or equivalent, and a correct explanation of choice of row 'least ratio $10 \div 1$' (ft their pivot column)
	(iii)	<table><tr><td>*P*</td><td>*x*</td><td>*y*</td><td>*z*</td><td>*s*</td><td>*t*</td><td></td></tr><tr><td>1</td><td>0</td><td>-10</td><td>0</td><td>3</td><td>0</td><td>30</td></tr><tr><td>0</td><td>0</td><td>10</td><td>1</td><td>-1</td><td>0</td><td>2</td></tr><tr><td>0</td><td>1</td><td>-5</td><td>0</td><td>1</td><td>0</td><td>10</td></tr><tr><td>0</td><td>0</td><td>25</td><td>0</td><td>-3</td><td>1</td><td>15</td></tr></table> $x = 10, y = 0$ $P = 30$	M1 M1 M1 A1 B1 B1 6	ft their tableau if possible for method marks For correct method evident for objective row For a correct method evident for pivot row For a correct method evident for other rows For correct tableau CAO For correct values from their tableau For correct value from their tableau
	(iv)	$11 + 5(0.2) = 12$ or $s = 0$ $11 - 5(0.2) = 10$ or $t = 0$ $3(11) + 10(0.2) = 35$ or $u = 10$ so all the constraints are satisfied $P = 3(11) - 5(0.2) = 32$ which is bigger than 30 from (iii)	B1 B1 2 13	For showing (not just stating) that constraints are satisfied For calculating 32, or equivalent (eg 3*x* has increased by 3 but -5*y* has only decreased by 1)

5	(i)				ANSWERED ON INSERT
			M1		For correct initial temporary labels at *F*, *G*, *I*
			M1		For correctly updating *F* and label at *H*
			A1		For all temporary labels correct (including *A*) (allow extra 100 at *C*, 105 at *D*, 75 at *H* only)
			B1		For order of becoming permanent correct
			B1		For all permanent labels correct (*A* need not have a permanent label)
		Shortest path from *J* to *B*: *J G H E B*	B1		For correct route (condone omission of *J* or *B*)
		Length of path: 125 metres	B1	7	For 125
	(ii)	Odd nodes: *B C E J*	B1		For identifying or using *B C E J* or implied
		BC = 60 *BE* = 35 *BJ* = 125 *EJ* = <u>90</u> *CJ* = <u>95</u> *CE* = <u>70</u> 150 130 195	M1		For any three of these weights correct, or implied or ft from their (i)
		Repeat *BE* and *CJ* (or *BE*, *JI*, *IC*)	A1		For <u>identifying</u> the pairing *BE*, *CJ* to repeat or 130 (not ft)
		130 + 765	M1		For 765 + their 130 (a valid pairs total)
		Shortest route: 895 metres	A1	5	For 895 (cao)
	(iii)		B1		For graph structure correct
			M1		For a reasonable attempt at arc weights (at least 9 correct, including the three given)
			A1	4	For all arc weights correct
		Travelling salesperson problem	B1	16	For identifying TSP by name

<table>
<tr><td>6</td><td>(i)</td><td>
<table>
<tr><td></td><td>1</td><td>5</td><td>2</td><td>4</td><td>3</td><td>6</td></tr>
<tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td></tr>
<tr><td>A</td><td>-</td><td>6</td><td>3</td><td>-</td><td>-</td><td>-</td></tr>
<tr><td>B</td><td>6</td><td>-</td><td>5</td><td>6</td><td>-</td><td>14</td></tr>
<tr><td>C</td><td>3</td><td>5</td><td>-</td><td>8</td><td>4</td><td>10</td></tr>
<tr><td>D</td><td>-</td><td>6</td><td>8</td><td>-</td><td>3</td><td>8</td></tr>
<tr><td>E</td><td>-</td><td>-</td><td>4</td><td>3</td><td>-</td><td>-</td></tr>
<tr><td>F</td><td>-</td><td>14</td><td>10</td><td>8</td><td>-</td><td>-</td></tr>
</table>
</td><td></td><td>ANSWERED ON INSERT</td></tr>
<tr><td></td><td></td><td></td><td>M1</td><td>For choosing row C in column A</td></tr>
<tr><td></td><td></td><td></td><td>M1 dep</td><td>For choosing more than one entry from column C</td></tr>
<tr><td></td><td></td><td></td><td>A1</td><td>For correct entries chosen</td></tr>
<tr><td></td><td></td><td>Order: A C E D B F</td><td>B1</td><td>For correct order, listed or marked on arrows or table, or arcs listed AC CE ED CB DF</td></tr>
<tr><td></td><td></td><td>Minimum spanning tree:
</td><td>B1</td><td>For tree (correct or follow through from table, provided solution forms a spanning tree)</td></tr>
<tr><td></td><td></td><td>Total weight: 23 miles</td><td>B1 6</td><td>For 23 (or follow through from table or diagram, provided solution forms a spanning tree)</td></tr>
<tr><td></td><td>(ii)</td><td>MST for reduced network = 18
Two shortest arcs from B = 5 + 6 = 11
Lower bound = 29 miles</td><td>M1
M1
A1 3</td><td>For their 18 seen or implied
For 11 seen or implied
For 29 (cao)</td></tr>
<tr><td></td><td>(iii)</td><td>F – D – E – C – A – B – F

8 + 3 + 4 + 3 + 6 + 14
= 38 miles</td><td>M1
A1
M1 4
A1 [13]</td><td>For F–D–E–C–A–B
For correct tour
For a substantially correct attempt at sum
For 38 (cao)</td></tr>
</table>

Mark Scheme 4737
June 2007

SOLUTIONS 4737 D2 June 2007 FINAL

1 (i)

	house 1	house 2	house 3	house 4
A	500	400	700	600
B	300	200	400	350
C	500	300	750	680
D	0	0	0	0

B1 — For copying the table, with row and column headings (accept consistent scalings)

B1 **2** — For dummy row (Daniel) with all equal values

(ii) Reduce rows

100	0	300	200
100	0	200	150
200	0	450	380
0	0	0	0

M1 — For a substantially correct attempt at reducing rows and columns

A1 — For correct reduced cost matrix (ft scalings)
Do not treat as MR

Columns are already reduced **2**

Cross out using two lines

100	[redacted]	300	200
100	[redacted]	200	150
200	[redacted]	450	380
[redacted]			

M1 — For covering zeros using minimum number of lines, clearly seen or implied from augmenting

Augment by 100

M1 dep — For a single augmentation by 100 (ft their matrix) (accept either way of augmenting by 100)

0	0	200	100
0	0	100	50
100	0	350	280
0	100	0	0

A1 ft — For a correct augmented matrix (ft their matrix) **3**

(ii) Cross out using three lines

[redacted]		200	100
[redacted]		100	50
[redacted]		350	280
[redacted]			

M1 — For covering zeros using minimum number of lines a second time, clearly seen or implied from augmenting

Augment by 50

M1 dep — For a single augmentation by 50 (ft their matrix) (accept either way of augmenting by 50)

0	0	150	50
0	0	50	0
100	0	300	230
50	150	0	0

A1 ft — For a correct augmented matrix (ft their matrix)

Complete matching

0	0	150	50
0	0	50	**0**
100	**0**	300	230
50	150	**0**	0

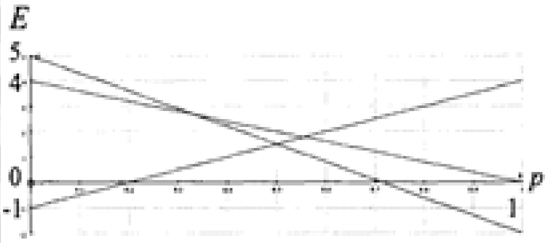
B1 **4** — For a complete matching achieved, must follow from an attempt at reducing or augmenting a matrix, not just implied from a list of the matching

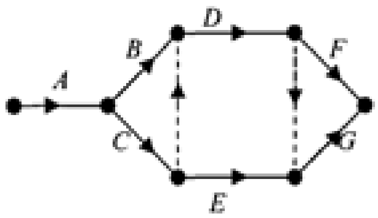
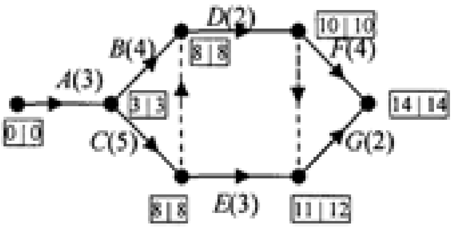
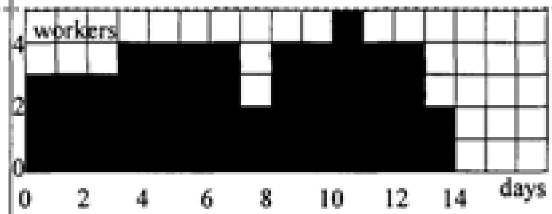
(iii)
Allclean should clean house 1
Brightenupp should clean house 4
Clean4U should clean house 2

B1 **2** — For A = 1, B = 4, C = 2 (may also list D = 3) cao

Cost = £1150

B1 **13** — For 1150 cao

2	(i)	$4p-(1\text{-}p)$ $=5p-1$	M1 A1		For $4p-1(1\text{-}p)$ or equivalent, seen or implied For $5p-1$ or $-1+5p$ cao
		$-2p+5(1\text{-}p)=5-7p$ $4(1\text{-}p)=4-4p$	B1 B1	4	For any form of this expression cao For any form of this expression cao
	(ii)		M1 A1 A1 A1	 ft ft ft 4	For correct structure to graph with a horizontal axis that extends from 0 to 1, but not more than this, and with consistent scales. For line $E=5p-1$ plotted from (0,-1) to (1, 4) For line $E=5-7p$ plotted from (0, 5) to (1,-2) For line $E=4-4p$ plotted from (0, 4) to (1, 0) In all three cases, correct or ft from (i)
		$p=0.5$	B1	1	For this or ft their graph
	(iii)	$5(0.5)-1$ $=1.5$ points per game Bea may not play her best strategy	M1 A1 B1	 3	For substituting their p into any of their equations (must be seen, cannot be implied from value) For 1.5 cao For this or equivalent Describing a mixed strategy that involves Z
	(iv)	1.5 If Amy plays using her optimal strategy, Bea should never play strategy Z Assuming that Bea knows that Amy will make a random choice between P and Q so that each has probablility 0.5, it does not matter how she chooses between strategies X and Y.	B1 M1 A1	ft 3 **15**	Accept -1.5, ft from (iii) For identifying that she should not play Z For a full description of how she should play (If the candidate assumes that Bea does not know then Bea should play P with probability $\frac{7}{12}$ and Q with probability $\frac{5}{12}$).

3 (i)		M1	A substantially correct network Condone arrows missing or wrong way round, no end and/or extra dummies Do NOT allow activity on node formulation
		A1	A correct network, with arrows on at least the dummy activities, with no extra dummies and a single end point.
	A dummy is needed after *C* because *D* follows both *B* and *C*.	B1	A valid explanation
	A dummy is needed after *D* because *F* and *G* both follow *D*.	B1 4	A valid explanation
(ii)		M1	A substantially correct forward pass
		A1	Early event times correct (ft their network if possible)
		M1	A substantially correct backwards pass
		A1	Late event times correct (ft their network if possible)
	Minimum completion time = 14 days	B1	For 14 cao
	Critical activities are *A*, *C*, *D*, *F*	B1	For these four activities and no others cao
		6	In both cases these need to be stated, not implied from the diagram
(iii)		M1	For a reasonable attempt at using the number of workers for the different activities Scales and labels required and some days with 4 workers.
		M1 dep	For a reasonable attempt with no overhanging blocks
		A1 3	For an entirely correct histogram
(iv)	*E* cannot happen until after *C* has finished so must overlap with *F*.	B1	Earliest finish for *E* > latest start for *F*
	Start *E* immediately after *C* but delay the start of *F* for 1 day (until after *E* has finished).	B1 2 15	For delaying the start of *F* (by 1 day)

4 (i) ANSWERED ON INSERT

Values only credited when seen in table

stage	state	action	working	minimax
1	0	0	4	4
	1	0	3	3
	2	0	2	2
2	0	0	max(6,4) = 6	3
		1	max(2,3) = 3	
		2	max(3,2) = 3	
	1	0	max(2,4) = 4	4
		1	max(4,3) = 4	
		2	max(5,2) = 5	
	2	0	max(2,4) = 4	3
		1	max(3,3) = 3	
		2	max(4,2) = 4	
3	0	0	max(5,3) = 5	
		1	max(5,4) = 5	
		2	max(2,3) = 3	3

Mark		Guidance
M1		For calculating the maxima as 4, 4, 5
A1	2	For calculating the minimax as 4
B1		For completing 4, 3, 2 in the brackets
M1		For calculating the maxima as 4, 3, 4 (method)
A1	3	For calculating the minimax as 3 cao
B1		For using their minimax values from stage 2
M1		For calculating the maxima for their values
A1		For calculating the maxima as 5, 5, 3 cao
A1	4	For calculating the minimax as 3 cao

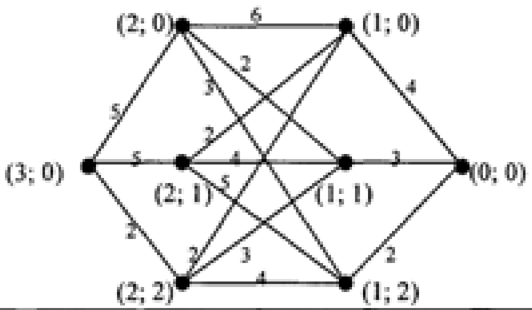
(ii)

3

(0; 0) - (1; 1) - (2; 2) - (3; 0) (or in reverse)

Mark		Guidance
M1		For the value from their tabulation
A1		For 3 (irrespective of their tabulation) cao
M1 dep		For reading route from their tabulation
A1	4	For this route (irrespective of their tabulation) cao

(iii)



Mark		Guidance
B1		For the graph structure correct
M1		For a substantially correct attempt at the weights (no more than two definite errors or omissions)
A1	3	For weights unambiguously correct
	16	

5	(i)				**ANSWERED ON INSERT**
		$S-E-I-T$	B1	1	For this route (not in reverse) cao
	(ii)	6 litres per second	B1		For 6
		From A to G	B1	2	For direction AG
	(iii)	$6+2+4+0+8$	M1		For a substantially correct attempt with $DF=0$
			M1		For dealing with EI (= 8 or = 2 + 6)
		= 20 litres per second	A1	3	For 20 cao
					Method marks may be implied from answer
	(iv)	eg flow 5 along $S-A-G-T$	M1		For describing a valid flow augmenting route
		and 2 along $S-C-F-H-G-T$	A1	2	For correctly flowing 7 from S to T
		Diagram correctly augmented	M1		For a reasonable attempt at augmenting a flow
			M1		For correctly augmenting a flow
			A1	3	For a correct augmentation by a total of 7
		Cut $\{S, A, B, C, D, E, F, G, H, I\}$, $\{T\}$	B1		For identifying cut or arcs GT and IT
		This cut has a value of 13 and the flow already found is 6 + 7 = 13 litres per second. Or This is the maximum flow since the arcs GT and IT are <u>both saturated</u>, so <u>no more can flow into T</u>.	B1	2 13	For explaining how this shows that the flow is a maximum, but NOT just stating max flow = min cut

Advanced GCE Mathematics (3892 – 2, 7890 - 2)
June 2007 Assessment Series

Unit Threshold Marks

Unit		**Maximum Mark**	**a**	**b**	**c**	**d**	**e**	**u**
4721	Raw	72	60	52	44	36	29	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	56	48	40	33	26	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	57	50	43	36	29	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	61	54	47	40	33	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	54	46	39	32	25	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	60	53	46	39	33	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	57	50	43	36	29	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	57	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	59	51	44	37	30	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	62	54	46	38	31	0
	UMS	100	80	70	60	50	40	0
4731	Raw	72	51	43	36	29	22	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	55	48	42	36	30	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0

4734	Raw	72	56	49	42	36	30	0
	UMS	100	80	70	60	50	40	0
4735	Raw	72	60	51	43	35	27	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	62	55	48	42	36	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890/3891/3892	300	240	210	180	150	120	0
7890/7891/7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	31.2	47.9	62.0	74.4	84.9	100	13873
3891	20.0	20.0	20.0	20.0	20.0	100	10
3892	58.5	75.6	87.9	94.7	97.5	100	1384
7890	45.3	66.9	82.2	92.4	97.7	100	9663
7891	0	0	0	100	100	100	1
7892	58.2	78.1	89.1	96.0	98.8	100	1487

For a description of how UMS marks are calculated see;
http://www.ocr.org.uk/exam_system/understand_ums.html

Statistics are correct at the time of publication

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