OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4734/01**

MATHEMATICS

Probability & Statistics 3

TUESDAY 5 JUNE 2007 Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} a & 0 \leqslant x \leqslant 1, \\ \dfrac{a}{x^2} & x > 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find the value of the constant a. [4]

2 Two brands of car battery, 'Invincible' and 'Excelsior', have lifetimes which are normally distributed. Invincible batteries have a mean lifetime of 5 years with standard deviation 0.7 years. Excelsior batteries have a mean lifetime of 4.5 years with standard deviation 0.5 years. Random samples of 20 Invincible batteries and 25 Excelsior batteries are selected and the sample mean lifetimes are $\overline{X}_I$ years and $\overline{X}_E$ years respectively.

(i) State the distributions of $\overline{X}_I$ and $\overline{X}_E$. [2]

(ii) Calculate $\mathrm{P}(\overline{X}_I - \overline{X}_E \geqslant 1)$. [5]

3 A nurse was asked to measure the blood pressure of 12 patients using an aneroid device. The nurse's readings were immediately checked using an accurate electronic device. The differences, x, given by x = (aneroid reading − electronic reading), in appropriate units, are shown below.

−1.3 4.7 −0.9 3.8 −1.5 4.0 −1.9 4.4 −0.8 5.5 −2.9 4.1

$[\Sigma x = 17.2, \quad \Sigma x^2 = 136.36.]$

Stating any assumption you need to make, test, at the 10% significance level, whether readings with an aneroid device, on average, overestimate patients' blood pressure. [8]

4 The students in a large university department take a trial examination some time before the proper examination. A random sample of 60 students took both examinations during a particular course. 42 students passed the trial examination, 36 passed the proper examination and 13 failed both examinations.

(i) Copy and complete the following contingency table. [2]

		Proper		
		Pass	Fail	Total
Trial	Pass			42
	Fail		13	
	Total	36		60

(ii) Carry out a test of independence at the $\frac{1}{2}$% level of significance. [7]

5 A music store sells both upright and grand pianos. Grand pianos are sold at random times and at a constant average weekly rate λ. The probability that in one week no grand pianos are sold is 0.45.

(i) Show that $\lambda = 0.80$, correct to 2 decimal places. [2]

Upright pianos are sold, independently, at random times and at a constant average weekly rate μ. During a period of 100 weeks the store sold 180 upright pianos.

(ii) Calculate the probability that the total number of pianos sold in a randomly chosen week will exceed 3. [3]

(iii) Calculate the probability that over a period of 3 weeks the store sells a total of 6 pianos during the first week and a total of 4 pianos during the next fortnight. [4]

6 Random samples of 200 'Alpha' and 150 'Beta' vacuum cleaners were monitored for reliability. It was found that 62 Alpha and 35 Beta cleaners required repair during the guarantee period of one year. The proportions of all Alpha and Beta cleaners that require repair during the guarantee period are p_α and p_β respectively.

(i) Find a 95% confidence interval for p_α. [5]

(ii) Give a reason why, apart from rounding, the interval is approximate. [1]

(iii) Test, at the 5% significance level, whether p_α differs from p_β. [6]

7 The continuous random variable X has (cumulative) distribution function given by

$$F(x) = \begin{cases} 0 & x < 1, \\ 1 - \dfrac{1}{x^4} & x \geqslant 1. \end{cases}$$

(i) Find the (cumulative) distribution function, $G(y)$, of the random variable Y, where $Y = \dfrac{1}{X^2}$. [4]

(ii) Hence show that the probability density function of Y is given by

$$g(y) = \begin{cases} 2y & 0 < y \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$ [2]

(iii) Find $E(\sqrt[3]{Y})$. [3]

[Question 8 is printed overleaf.]

8 The continuous random variable Y has a distribution with mean μ and variance 20. A random sample of 50 observations of Y is selected and these observations are summarised in the following grouped frequency table.

Values	$y < 20$	$20 \leqslant y < 25$	$25 \leqslant y < 30$	$y \geqslant 30$
Frequency	3	27	12	8

(i) Assuming that $Y \sim N(25, 20)$, show that the expected frequency for the interval $20 \leqslant y < 25$ is 18.41, correct to 2 decimal places, and obtain the remaining expected frequencies. [4]

(ii) Test, at the 5% significance level, whether the distribution N(25, 20) fits the data. [5]

(iii) Given that the sample mean is 24.91, find a 98% confidence interval for μ. [3]

(iv) Does the outcome of the test in part **(ii)** affect the validity of the confidence interval found in part **(iii)**? Justify your answer. [2]