RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4723

Core Mathematics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

4723

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

2R–9K03

1 Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = x^3 e^{2x}$, **[2]**

(ii) $y = \ln(3 + 2x^2)$, **[2]**

(iii) $y = \frac{x}{2x+1}$. **[2]**

2 The transformations R, S and T are defined as follows.

R : reflection in the x-axis

S : stretch in the x-direction with scale factor 3

T : translation in the positive x-direction by 4 units

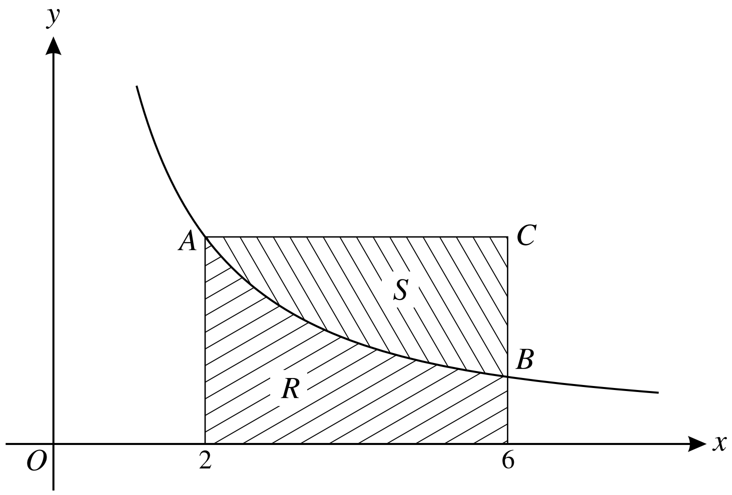
(i) The curve $y = \ln x$ is transformed by R followed by T. Find the equation of the resulting curve. **[2]**

(ii) Find, in terms of S and T, a sequence of transformations that transforms the curve $y = x^3$ to the curve $y = \left(\frac{1}{9}x - 4\right)^3$. You should make clear the order of the transformations. **[2]**

3 **(i)** Express the equation $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$ in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants. **[3]**

(ii) Hence solve, for $-180^\circ < \theta < 180^\circ$, the equation $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$. **[3]**

4



The diagram shows part of the curve $y = \frac{k}{x}$, where k is a positive constant. The points A and B on the curve have x-coordinates 2 and 6 respectively. Lines through A and B parallel to the axes as shown meet at the point C. The region R is bounded by the curve and the lines $x = 2$, $x = 6$ and $y = 0$. The region S is bounded by the curve and the lines AC and BC. It is given that the area of the region R is $\ln 81$.

(i) Show that $k = 4$. **[3]**

(ii) Find the exact volume of the solid produced when the region S is rotated completely about the x-axis. **[4]**

5 **(i)** Solve the inequality $|2x+1| \leqslant |x-3|$. **[5]**

(ii) Given that x satisfies the inequality $|2x+1| \leqslant |x-3|$, find the greatest possible value of $|x+2|$. **[2]**

6 **(i)** Show by calculation that the equation

$$\tan^2 x - x - 2 = 0,$$

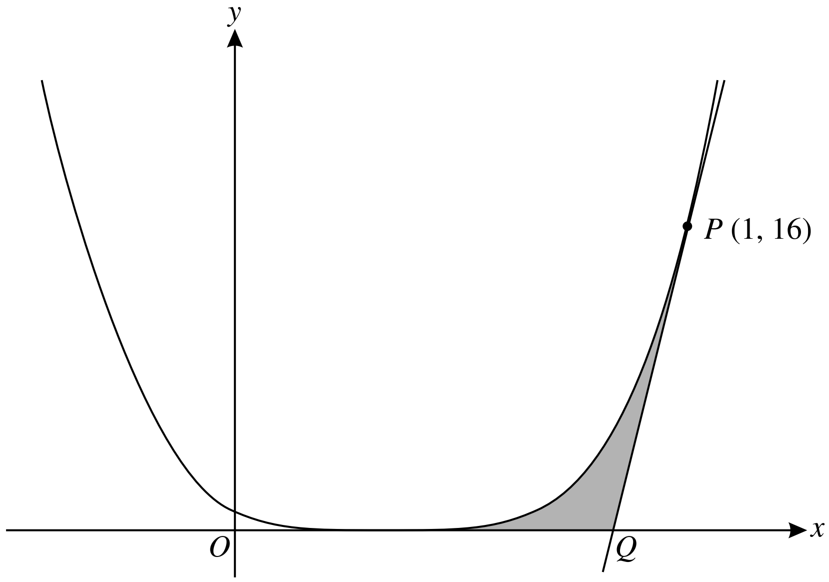
where x is measured in radians, has a root between 1.0 and 1.1. **[3]**

(ii) Use the iteration formula $x_{n+1} = \tan^{-1}\sqrt{2+x_n}$ with a suitable starting value to find this root correct to 5 decimal places. You should show the outcome of each step of the process. **[4]**

(iii) Deduce a root of the equation

$$\sec^2 2x - 2x - 3 = 0.$$ **[3]**

7



The diagram shows the curve with equation $y = (3x-1)^4$. The point P on the curve has coordinates $(1, 16)$ and the tangent to the curve at P meets the x-axis at the point Q. The shaded region is bounded by PQ, the x-axis and that part of the curve for which $\frac{1}{3} \leqslant x \leqslant 1$. Find the exact area of this shaded region. **[10]**

8 **(i)** Express $3\cos x + 3\sin x$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **[3]**

(ii) The expression $\mathrm{T}(x)$ is defined by $\mathrm{T}(x) = \dfrac{8}{3\cos x + 3\sin x}$.

(a) Determine a value of x for which $\mathrm{T}(x)$ is not defined. **[2]**

(b) Find the smallest positive value of x satisfying $\mathrm{T}(3x) = \frac{8}{9}\sqrt{6}$, giving your answer in an exact form. **[4]**

[Question 9 is printed overleaf.]

9 The functions f and g are defined for all real values of x by

$$\mathrm{f}(x) = 4x^2 - 12x \qquad \text{and} \qquad \mathrm{g}(x) = ax + b,$$

where a and b are non-zero constants.

(i) Find the range of f. **[3]**

(ii) Explain why the function f has no inverse. **[2]**

(iii) Given that $\mathrm{g}^{-1}(x) = \mathrm{g}(x)$ for all values of x, show that $a = -1$. **[4]**

(iv) Given further that $\mathrm{gf}(x) < 5$ for all values of x, find the set of possible values of b. **[4]**