GCE

Mathematics

Advanced GCE **A2 7890 – 2**

Advanced Subsidiary GCE **AS 3890 – 2**

Examiners' Reports

January 2011

3890-2/7890-2/R/11J

Oxford Cambridge and RSA Examinations

This report on the Examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the Examination.

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EXAMINERS' REPORTS

Chief Examiner's Report – Pure Mathematics

At this session, units 4721, 4722, 4723 and 4725 required candidates to write their solutions in a Printed Answer Book and the marking was carried out online. At the next session, unit 4724 will also require candidates to use Printed Answer Books. Care is taken when Printed Answer Books are designed. A judgement is made of the space needed, at least by the vast majority of candidates, for each solution and, as far as possible, the need for candidates to turn a page in the middle of a solution is avoided. At the same time, the allocation of an excessive amount of space for solutions, resulting in a considerable waste of paper, is deemed inappropriate.

Most candidates coped well with the discipline of using Printed Answer Books. In many cases, the space allocated for a solution will be more than a candidate needs but candidates should not be concerned by this. Inevitably, there will be instances when the allocated space is not sufficient; then an extra single sheet should be used. It was disappointing to note that a few centres issued 8-page booklets, or even 12-page booklets, to candidates needing extra space. In most such cases, no more than one page of this extra booklet was used by the candidate. Consequently, this was wasteful of paper, wasteful because all the unused pages had to be scanned and wasteful of examiners' time because all the extra unused pages had to be checked.

Centres and candidates are advised to note the following points with respect to answering in a printed answer book.

(a) Avoid placing negative signs, decimal points, straight lines of a diagram, etc. so that they coincide with the rulings of the page. They may not be as visible on the scanned version of the page.
(b) Keep strictly to the space allotted for each solution. Do not allow a solution to extend into the space allotted for the next question. If there is insufficient space for the solution, continue on an extra single sheet.
(c) To make a second attempt at a question, use the allotted space if there is still room for the second attempt. Otherwise use an extra single sheet.
(d) Do not use graph paper.
(e) Do not use a pen containing ink which tends to seep through paper. This can affect what appears on the reverse side of the page.

4721 Core Mathematics 1

General Comments

The large majority of candidates were well prepared for this paper and relatively few appeared to run out of time, although there were a few omissions of the later parts of Q9. Most candidates attempted nearly all of the rest of the paper, with some omissions in the later parts of Q8.

It was pleasing to see that comparatively few candidates used additional sheets, indicating again that sufficient room was available in the answer book for solutions. Candidates who needed to re-do their graph sketch in Q5 did so on ordinary rather than graph paper.

Many candidates used sketches to support coordinate geometry questions, which is an encouraging trend that would have benefitted others. There were some difficulties caused by the arithmetical demands of the paper, notably with fractions and negative number arithmetic.

Comments on Individual Questions

1) (i) This easy starter proved very accessible to candidates, the overwhelming majority of whom secured both marks. Almost all attempted to use Pythagoras' theorem with only a few processing errors.

(ii) Again, the vast majority of candidates secured the method mark for finding the gradient of the line joining two points, although the negative coordinate did cause a small number of candidates to lose the accuracy mark.

(iii) Fully correct solutions were common, but some candidates had difficulty in presenting their determination of whether the lines were perpendicular in a clear manner. Many correctly established the gradient of the given line and compared this with their answer found in (ii), but a significant number only wrote "yes it is" without reference to the product of the gradients. Some candidates tried to find the equation of a line through *A* or *B* without referring to the given line at all.

2) This question proved demanding, with less than a third of candidates securing all three marks. Candidates who took the most direct route of expressing both sides as a product of three factors were the most likely to be successful. Those who expanded the expressions often made processing errors or failed to compare coefficients correctly, often omitting a term, or had difficulty forming and/or solving the resulting simultaneous equations. Substitution of values was less common and also prone to error. Some candidates spent a disproportionate amount of time on this question, producing large quantities of erroneous and/or irrelevant algebra; attempting to solve the "equation" or express p and q in terms of x was not uncommon.

3) (i) The overwhelming majority of candidates earned the mark.

(ii) Over 90% of candidates were able to gain this mark, although some candidates left their answer in the form $\frac{1}{8^2}$.

(iii) Most candidates started this question well, correctly using the addition rule to arrive at 2^8, or recognising 2^6 as equivalent to 8^2. Dealing with the fractional power proved much more demanding and many candidates who did arrive at $8^{\frac{2}{3}}$ then failed to combine this correctly with 8^2. As in previous sessions, candidates continue to have problems with these index rules; fewer than half of all candidates secured all three marks.

4) This question was well answered on the whole. The first three marks on this question were gained by most candidates; they followed the instructions on the paper and made the correct substitution and solved the resulting simple quadratic in u correctly, usually by factorisation, which was pleasing. Many candidates, however, then stopped and failed to reverse the substitution to find the values of x. Of those who attempted to do so, many were successful, but a significant number who took the square root neglected to find the negative solution and so only obtained two of the four final solutions. Those who expanded to form two more quadratics were generally successful, although some did not recognise 0 as a solution of $9x^2 - 12x = 0$. A significant minority made errors with the initial substitution and others tried to expand the whole expression to obtain a quartic, seldom producing the correct result.

5) (i) It was pleasing that this sketch of a negative cubic was better done than many sketches in previous sessions, although this remains a relatively weak area. Most candidates appeared to know the basic shape of $y = x^3$ and tried to reflect this in the y-axis, although zero gradient at the origin was extremely rare.

(ii) Graph transformations continue to be one of the most difficult areas of the specification for candidates. In part (ii), it was pleasing that most candidates gave their answer as an equation and not an expression, but less than half obtained the right answer. Common errors included $y = (-x - 3)^3$ and $y = -x^3 - 3$.

(iii) In part (iii), the use of correct mathematical vocabulary is improving with the vast majority of candidates using the word "stretch" (although this was often misspelt) but many still have problems describing the magnitude. "In the y-axis" was the most common error of description. A significant minority tried to describe the stretch parallel to the x-axis, usually incorrectly stating scale factor $\frac{1}{5}$.

6) (i) This question was started well, with even the candidates scoring lower totals both able to rewrite the given fractional terms involving negative powers of x and to differentiate individual terms correctly. The exception was term $\frac{1}{4x}$, which even many high-scoring candidates rewrote as $4x^{-1}$, so losing an accuracy mark and making the modal score for this question 3 marks out of 4.

(ii) Nearly all candidates recognised the notation and attempted to differentiate their answer from (i). Most candidates therefore obtained 1 mark out of 2 as their answer could not be fully correct because of the error in the previous part.

7) (i) Completing the square remains a difficult task for many candidates, although nearly all recognised what was expected of them and the vast majority obtained at least the first two marks for identifying both p and q. The usual processing errors occurred thereafter with many candidates forgetting to multiply by 4 when trying to combine the constants.

(ii) A significant proportion of candidates did see the connection between the first two parts of this question and tried to solve the quadratic by equating their expression from (i) to zero, with varying degrees of success. Common errors were not taking both the positive and negative square root and arithmetical errors processing the 12 and the 4. More commonly, however, candidates tried using the quadratic formula, many substituting correctly, but then being unable to simplify $\sqrt{192}$. Others used 3 instead of -3 for c or incorrectly arrived at a value of 196.

(iii) Candidates who approached this by putting the discriminant equal to zero were the most likely to succeed in this part, although the $-k$ term often caused errors in substitution, and many candidates who successfully obtained $144 + 16k = 0$ still went on to solve this incorrectly as $k = 9$. A large number of candidates tried to find a solution by attempting to complete the square again; only a small fraction of these successfully then put $x = \frac{3}{2}$ to obtain the correct solution.

8) (i) Most candidates scored highly on this familiar question on finding the equation of a tangent to a given curve, with nearly half scoring all 6 marks. The negative coefficient of x^2 caused some problems, with some candidates changing the signs of some or all the terms before starting. A disappointingly large number of candidates found the gradient correctly but went on to find the negative reciprocal and thus find the equation of the normal rather than the tangent. Sign errors were common, with even stronger candidates not succeeding in substituting $x = 5$ into the equation of the curve.

(ii) This question proved very demanding. Many candidates substituted $x = 0$ instead of $y = 0$ to find the point where the line met the y-axis and so lost accuracy marks. Most, however, did apply the correct rule to find the mid-point of their P and Q and so were able to gain a method mark.

(iii) A surprisingly large proportion of candidates tried to find the equation of the line of symmetry by completing the square; many of these had difficulty because of the $-x^2$. More successful were candidates who differentiated and equated the gradient function to 0; most of these scored both marks.

(iv) The candidates who saw the connection between parts (iv) and (iii) were the most likely to be successful in earning at least the method mark for this final part of the question. A very large number of candidates misinterpreted the question entirely and tried to find the values of x for which y was greater than 0; more than half of candidates scored zero for this part.

9) (i) Even the less successful candidates were able to successfully transform the given equation into the alternative form to find both the centre and the radius of the circle, full marks being very common here.

(ii) Many candidates found this taxing and made no attempt at all. Those who drew a sketch were most likely to succeed, realising that adding/subtracting the radius to the y-coordinate of the centre was an obvious solution. Those who substituted $x = 4$ into the equation of the circle were also likely to be successful; those who substituted k for y and then set the discriminant to zero were prone to algebraic and/or sign errors.

(iii) Many candidates did draw a sketch for this part, but comparatively few realised the need to complete a triangle and apply Pythagoras' theorem. Of those who did, mis-substitution was a common error. Only a quarter of candidates secured all three available marks.

(iv) This part question was quite well done with many fully correct solutions, but a surprising number of candidates who had scored fairly well on the rest of the paper failed to eliminate one of the variables. Of those who did, errors in expansion and arithmetic were the most likely causes of breaking down in the solution, with particular problems arising again from negative coefficients. Only a very small number of candidates attempted to use methods other than solution of simultaneous equations and these were seldom successful.

4722 Core Mathematics 2

General Comments

This paper was accessible to the majority of candidates, and gave them an opportunity to demonstrate their knowledge. Candidates performed well overall and the types of questions that, in the past, have tended to cause problems were attempted with confidence and usually answered accurately. Candidates should ensure that the working throughout the question is done to a sufficient degree of accuracy so as not to compromise the final answer. There were several questions on this paper where some candidates lost marks through lack of accuracy, either from rounding prematurely throughout the question or from not giving their final answer to the required degree of accuracy. Where candidates make multiple attempts at a question it is essential that they identify which is their final solution. Offering two or more solutions, sometimes even linked with 'or', is not in the candidate's best interests as it is the last attempt that will be credited. This is especially important if a second attempt is made on an additional sheet of paper.

This was the second time that C2 was answered using a booklet, and the majority of candidates coped well with this. There were fewer scripts where additional sheets had been used, and candidates confined their solutions to the relevant answer spaces. Candidates should appreciate that the size of the answer space for each question is a guide and they should not worry if they do not fill it, especially where a descriptive answer is required. There were some problems in reading the scripts where candidates had worked in pencil and then gone over it again, or rubbed out answers and written over the top. Candidates should also ensure that decimal points and negative signs are clearly visible in their working.

Comments on Individual Questions

1) (i) This proved to be a straightforward start to the paper and the majority of candidates gained full marks. The most common error was neglecting to square the 2 from the $2x$ term resulting in a third term of $42x^2$. Successful candidates made effective use of brackets to avoid this error.

(ii) The majority of candidates were able to correctly use their solution to part (i) to obtain the coefficient of x^2, though there were some candidates who either attempted only one term or who attempted one term and then added it to their coefficient of x^2 from part (i). A number of candidates attempted a full expansion and then picked out the relevant terms which was a slightly longer, but equally successful, method.

2) (i) Most candidates were able to write down the correct three terms, though a few treated it as an inductive sequence instead.

(ii) Almost all candidates could correctly identify the sequence as arithmetic.

(iii) This proved to be a challenging question, with only the most able candidates obtaining the correct answer. A number of candidates attempted only one of S_{100} and S_{200}. Other errors included attempting irrelevant sums, usually with 101 or 99 terms, and there were numerous inconsistencies between the value of n and the value of a used. Of those candidates who actually attempted a difference, the most common method was to evaluate $S_{200} - S_{101}$. The most efficient method was to find $50(u_{101} + u_{200})$, but only the most astute candidates employed this method.

3) (i) The trapezium rule was successfully applied by most candidates, and a pleasing number of fully correct solutions were seen with only a few losing the final mark due to a loss in accuracy. Some candidates omitted the outer brackets and others struggled to correctly place the y-values due to the initial value of 0 being forgotten. The most successful candidates wrote out a correct expression involving surds and then simply evaluated this in one step.

(ii) The majority of candidates could identify that it would be an underestimate, but a number then struggled to provide a convincing explanation. The reasons often lacked precision, referring only to a single trapezium or not making it clear where the gaps would be. Sketches tended to be of poor quality, with little effort made to place the top vertices of the trapezia on the curve and other attempts showing rectangles rather than trapezia.

4) (a) Candidates continue to be proficient in using logarithms to solve basic equations, and this question was very well done with many correct solutions seen. The occasional errors tended to occur when attempting to rearrange the equation, such as subtracting rather than adding 1 as a final step, rather than through a lack of topic knowledge.

(b) This part of the question was not done as well, though there were still a pleasing number of fully correct solutions seen. The majority of candidates seemed at least partially familiar with the relevant logarithm laws, but then struggled to apply them in the correct order. It was quite common to see the log $(x + 5)$ term incorrectly split into two terms, sometimes even when the other side of the equation had been correctly combined. A surprising number of candidates correctly obtained $\log(9x) = \log(x + 5)$, but then struggled to proceed.

5) (i) The majority of candidates were able to either state a correct equation and hence obtain the required value for r or substitute for r into the sum to infinity and obtain $4a$, though some candidates were not entirely convincing in their arguments. When asked to prove a given answer, candidates must ensure that they provide enough detail. Some candidates struggled to set up an initial equation, with $\frac{4a}{1-r}$ being a common error, and others substituted a numerical value for a, often chosen arbitrarily.

(ii) This part was very well done, with many fully correct solutions seen. Most candidates found an expression for the third term, equated it to 9 and then solved it, though some started with 9 and then used a common ratio of $^4/_3$. A surprising number of candidates correctly found a, but then carried out a further calculation to find u_1 with a few then concluding with a value other than 16.

(iii) The majority of candidates obtained full marks on this question, though a few spoiled a correct method with a rounding error. A few candidates used n - 1 rather than n in the sum formula.

6) (a) This question proved to be rather challenging, and fully correct solutions were in the minority. Many candidates simply integrated the numerator and the denominator and made no attempt to simplify the rational expression first. Of those who did attempt this step, a number only divided one of the two terms in the numerator by x. Others attempted to multiply throughout by x^{-1} but could not then accurately apply the relevant index law, with the first term often becoming x^{-3}. Having got this far, candidates generally then made a good attempt at the integration though some struggled to correctly simplify the coefficient in the second term. Most candidates gained a mark for including a constant of integration.

(b) (i) In contrast this question was rather better done, with the majority able to correctly integrate the given function, though simplifying the coefficient caused problems for some. Most candidates could then substitute the given limits correctly. Some candidates misunderstood the significance of being told that a was a constant greater than 2, and proceeded to state an equation or inequality, usually involving 0 or 2, which they then attempted to solve.

(b) (ii) Most candidates seemed familiar with what was required, and could write down either the required answer, or a correct answer following their expression in part (i). A surprising minority chose to do the integration again from the start. There was some confusion over where to place 2 and infinity in the integral, but most could then deduce the correct limit appropriately. Some candidates did not appreciate that a statement of the limit was required and used terminology that included 'approximately equal to' or 'is getting closer to'.

7) (i) Whilst most candidates carried out the correct solution method to find a value for x, fully correct solutions were rare. A common error was to only find the principal angle, which a number of candidates then spoilt by giving it to only 2 significant figures. Many candidates then did not even attempt a secondary angle, or added 180^o rather than 90^o. Some candidates confused their order of operations and consequently attempted $\tan^{-1}(^1/_6)$ or even $\tan^{-1}(1)$. The tan $2x$ identity was used successfully in a few cases.

(ii) Most candidates started by correctly replacing $\cos^2 x$ with $1 - \sin^2 x$, though some candidates substituted an incorrect rearrangement of the identity and others attempted to use $\sin x = 1 - \cos x$. Of those who did use the correct identity, most could then obtain a correct simplified equation but sign errors were common. The majority of candidates could then appreciate that the resulting equation was a quadratic in $\sin x$, and employ an appropriate method to solve it, though a few cancelled through by $\sin x$ thus losing two solutions. The two angles resulting from $3\sin x = 2$ were usually given correctly, as was $x = 0^o$, but a significant minority omitted $x = 180^o$. Whilst most candidates gained at least some credit on this question, fully correct solutions were not common.

8) (i) Most candidates were able to equate an attempt at the area of the triangle to 8 and attempt to solve it, either in degrees or in radians, but few appreciated that their angle did not satisfy the condition that it had to be obtuse. Of those who did attempt further work, adding $\frac{1}{2}\pi$ was a common error as was doubling the angle and subtracting from π. Only the most able candidates gained all three marks on this question.

(ii) Most candidates were able to make a good attempt at this part of the question, even if part (i) was incorrect. Attempts at finding the area of the sector were usually correct, though some omitted the $\frac{1}{2}$ and others used an angle in degrees. Some then neglected to subtract the area of the triangle and others chose to recalculate it rather than just using the given value of 8, but most candidates could gain at least two marks on this question.

(iii) Most candidates could correctly calculate the arc length following their angle, though a few used an angle in degrees in $r\theta$ instead. A number of candidates then offered the perimeter of the sector as their final answer, though it was unclear whether this was through a failure to read the question correctly or a lack of understanding of the correct terminology. Those candidates who did attempt to find the length of the chord AB usually applied the cosine rule correctly though some used alternative, equally valid, methods. In both parts (ii) and (iii) a significant minority of candidates used valid methods in degrees rather than working efficiently using the radian methods.

9) (i) Sufficiently detailed confirmation that f(3) = 0 was provided by almost all of the candidates though some attempted f(-3) and others failed to identify numerical errors in their working and still concluded with = 0. Whilst most candidates could correctly state a factor of f(x), some omitted to respond to this part of the question and others confused factor and root, with $x = 3$ being a common error.

(ii) This question was surprisingly well done, with many candidates not at all fazed by the awkward coefficient of x^3. Many could confidently use inspection, with others using coefficient matching or long division. The quadratic factor was usually correct though some ignored the request to write f(x) as a product of a linear and quadratic factor. Some candidates found the three correct roots, presumably from a calculator method, and then attempted to work backwards to find the required product but this was very rarely successful.

(iii) Most candidates could obtain the correct two roots, using a variety of methods. Some were confident when factorising a quadratic with a negative lead coefficient, whereas others multiplied through by -1 or used the quadratic formula. There were occasional sign errors when attempting to find the roots from a correct factorisation.

(iv) Only a minority of candidates gained full marks on this question, but the errors seemed to be more due to an inability to evaluate numerical expressions rather than to a lack of understanding of the processes required. The vast majority of candidates could correctly integrate the given function and then attempt use of limits, though a few failed to split it into the two relevant areas. Numerical errors when evaluating were then common, with -3^4 becoming $(-3)^4$ being a typical error. Candidates who worked in fractions and who were less reliant on their calculators tended to be more successful. Candidates who obtained a negative value for the region below the x-axis usually dealt with it appropriately.

4723 Core Mathematics 3

General Comments

All possible marks were recorded by candidates for this paper. It is pleasing to acknowledge the mathematical ability of those candidates obtaining full marks and, whilst there were a few candidates recording very low marks, there were fewer than has been the case in some recent examination sessions. Candidates seem to have had sufficient time to complete the paper and, when attempts at later questions were incomplete, this was due to uncertainty as to how to proceed rather than to lack of time.

A paper at A2 level will inevitably include some requests that are slightly different from questions the candidates will have met before. Success in tackling such questions involves a measure of judgement, mathematical awareness and care in reading the question; decisions need to be made about how to proceed and candidates are not always good at giving appropriate thought to such matters. With this paper, questions 3 and 5 were, in many instances, answered well and it seemed that candidates had given some thought to an appropriate strategy for solution. By contrast, solutions to questions 2, 8(b)(ii) and 9(i) would have been improved in many cases by some preliminary thought and planning.

This examination session was the first at which Core Mathematics 3 required candidates to use a Printed Answer Book for their solutions. This seemed to work well and most candidates found that the spaces allotted for their solutions were adequate.

Comments on Individual Questions

1) For many candidates, this was a straightforward opening question; two linear equations were formed and solved without difficulty. Others adopted the method of squaring both sides of the equation but some did then encounter difficulties in solving the quadratic equation accurately. Some candidates did no more than solve the equation $3x + 4a = 5a$. For a significant number of candidates, it was clear that the presence of the positive constant a caused problems. Some presented solutions giving a in terms of x and others assigned a particular value to a before attempting any solution. Only limited credit was available to such solutions. A number of candidates seemed to proceed as if, by sustained algebraic manipulation of various equations, they would determine a value for a. A few other candidates started with a double inequality $-5a < 3x + 4a < 5a$ and proceeded to solve this. Some candidates, having adopted a successful method of solution, concluded by rejecting one of the answers, arguing incorrectly that the presence of the modulus signs means that x must be positive.

2) Nearly half of the candidates recorded full marks on this question but a variety of errors was evident on the solutions from other candidates. Some ignored the need for a reflection and others incorrectly reflected the curve in the y-axis. Many attempted a translation parallel to the x-axis but some attempts involved a translation in the positive direction and, in other cases, the curve, whilst correctly passing through the point $(-10, 0)$, still passed through the origin. Many candidates would have benefited by breaking down the problem into separate stages and considering the different transformations involved.

3) In past examination sessions for this unit, questions on the topic of connected rates of change have not generally been answered well. It is therefore pleasing to note that, on this occasion, a majority of the candidates recorded full marks. The notation used was not always precise but candidates recognised the need to multiply the derivative $8\pi r$, with substitution of the appropriate value of r, by 12. The most common wrong answer was 3770, the result of substituting $r = 150$ into $8\pi r$ and ignoring the value 12 altogether. Others proceeded without any differentiation.

4) This question on a familiar topic enabled many candidates to record full marks. Part (i) caused no difficulties and the only error to occur with any frequency involved a value of α equal to $73.7°$, the result of equating $\tan\alpha$ to $\frac{24}{7}$ instead of $\frac{7}{24}$. The majority of candidates adopted the appropriate methods in part (ii). However, many candidates did not find the second angle and their incorrect method usually involved the subtraction of $12.4°$ from $180°$. A few candidates offered more than two answers between $0°$ and $360°$; no more than three of the available four marks were available for such conclusions.

5) A few candidates seemed confused between area and volume or thought that they could find the value of *a* by considering the volume. However, most candidates did know the appropriate steps to take, and many duly completed the solution successfully. Most of the errors which occurred resulted from uncertainty about powers or from errors with the integration of the form $(ax+b)^n$. Some started by integrating $6(3x-2)^{\frac{1}{2}}$ or simplified the integrand for the volume evaluation to $36(3x-2)^{\frac{3}{2}}$. The integral of $6(3x-2)^{-\frac{1}{2}}$ was often not correct and, in the latter part of the solution, errors in finding $\int 36(3x-2)^{-1}\,dx$ included $36\ln(3x-2)$, $108\ln(3x-2)$ and even expressions involving $(3x-2)^0$. Candidates making early errors with the integration or not solving the equation $4\sqrt{3a-2}-4=16$ appropriately found an incorrect value of *a*. There was little indication that candidates, having found a most unwieldy value of *a*, thought that they might have made a mistake and that it would be sensible to check their work.

6) (i) The vast majority of candidates attempted to use the quotient rule to find the first derivative but many were guilty of a lack of attention to detail. Brackets were not always provided and there were sign slips as well as elementary errors in differentiation. Of course, many candidates did produce a correct expression for the derivative but a significant number was unable to confirm the given equation. There was uncertainty about how to proceed and many resorted to showing by substitution that 2.4 more or less satisfied the given equation. With the equation given in the question, candidates had to provide convincing detail, showing clearly how equating the numerator of the derivative to zero does indeed lead to the given equation; most did so but there were cases where more care was needed.

(ii) Candidates were expected to have sufficient awareness of the question to start their iteration process with the value 2.4. Most did so but there were also instances where the initial value was taken as 1 or 2. The iteration process was usually carried out well, with sufficient detail being given to satisfy examiners. The final mark for the *y*-coordinate was often not earned. Some candidates either forgot to find it or did not realise that they had to do so. Others embarked on a second iteration process using an initial value of –1.6; this is a basic misunderstanding and does not lead to the *y*-coordinate of *P* but to the *x*-coordinate of another of the original curve's stationary points.

7) (i) This part was answered well; the correct equation $\ln(x^2+8)=8$ was formed and the exact root found without difficulty. Many candidates did go on to state an approximate value of 54.5 but, provided the correct answer had already been given, this extra step was ignored.

(ii) This part was not answered so well and many candidates opted for g as the function with an inverse and proceeded to give $\sqrt{x-8}$ as that inverse. About half of the candidates did correctly choose f and almost all realised that the inverse was e^x. However, very few completed the definition by indicating, in some acceptable form, that the domain of f^{-1} was all real numbers.

(iii) Most candidates used the correct expression for $\mathrm{gf}(x)$, i.e. $(\ln x)^2 + 8$. Some attempted no differentiation, merely substituting e^3 into this expression. Others thought that the expression could be rewritten as $2\ln x + 8$ and so gained no credit for any subsequent differentiation. It was not often that candidates used the chain rule correctly to obtain the derivative; a few opted to rewrite the expression as $(\ln x)(\ln x) + 8$ and used the product rule.

(iv) This part was answered well and most candidates applied Simpson's rule efficiently to obtain the correct value of 20.3. A few used values of $\ln 24$, $\ln 12$ and $\ln 8$ corrected to 3 significant figures in their calculation and this led to an incorrect answer. There is no reason why the calculation should not be carried out using $\ln 24$, $\ln 12$ and $\ln 8$ but, if approximate values are used, they should be to a greater degree of accuracy than is required of the final answer.

8) (a) Most candidates had a general idea of the nature of the graph in part (i). A few drew only two branches and others drew the graph of $y = \sec x$; one mark was available for such attempts. To gain all three marks, candidates were required to show some indication of scale on each axis. Most did so on the x-axis but the absence of 1 and -1 on the y-axis meant that some candidates did not earn the third mark.

Part (ii) proved to be a challenging request and correct answers were not seen very often. Many candidates made no attempt at this part and others proceeded no further than stating $\sin\alpha = \sin\beta$. Some mentioned symmetry about $x = \frac{3}{2}\pi$ but could not always develop this to a correct conclusion. Consideration of the graph and awareness of the symmetries involved should lead to the observation that $\beta = 2\pi + (\pi - \alpha)$ or that $\beta = \frac{3}{2}\pi + (\frac{3}{2}\pi - \alpha)$.

(b) The vast majority of candidates earned the mark for stating the double-angle identity; the surprise was that a significant number of candidates could not do this or stated an incorrect formula. There were some concise and accurate solutions to part (ii) from candidates who used the identity to find the exact value of $\tan 2\phi$; this was followed by use of the identity again, this time giving $\tan 4\phi$ in terms of $\tan 2\phi$ and obtaining $\tan 4\phi = \frac{240}{161}$. Many candidates succeeded in finding the value of $\tan 2\phi$ but then were unsure how to proceed to find $\tan 4\phi$; some just assumed that $\tan 4\phi = 2\tan 2\phi$. Many other candidates made little or no progress, gaining no more than one mark for stating $\tan\phi = \frac{1}{4}$. Their attempts, if pursued at all, tended to consist of increasingly complicated trigonometric expressions; in some cases, candidates seemed to forget what they were trying to do and ended up trying to solve an equation.

9) (i)(a) There were a few candidates who thought that the notation $f'(x)$ indicated an inverse function but the vast majority differentiated correctly. To earn the third mark, candidates were required to make some pertinent, suitably general, comment about exponential functions; merely commenting that e^x is always positive was sufficient. However, in an A2 unit, it was disappointing to note the approach taken by many candidates. Finding the particular values of $f'(x)$ for two numerical values of x proves nothing. Similarly, attempting to solve $f'(x) = 0$ and finding this impossible does not confirm the result.

(i)(b) Most candidates duly earned the first two marks for correct differentiation but, for many, there was either nothing more attempted or such attempts as were made involved nothing of merit. Some candidates seemed to misinterpret the question and proceeded to attempt a solution of the equation $f''(x) = f(x)$. For candidates making some relevant progress, there were difficulties in trying to solve an inequality such as $e^{2x} - 3e^{-2x} > 0$ caused usually by an inappropriate introduction of logarithms.

(ii) This was a suitably challenging final part to the paper and a pleasing number of candidates possessed the mathematical knowledge and technical expertise to answer it successfully. Others made some progress, often finding $\frac{1}{4}\ln k$, the value of x at the minimum point, but then mistakenly assuming that the range was given by $y \geq \frac{1}{4}\ln k$. However, many candidates had little idea how to proceed; for some, mention of the term 'range' triggered an attempt to 'complete the square'.

4724 Core Mathematics 4

General Comments

As usual, there was a wide range of responses. Many were excellent but there is still a not insignificant number of candidates who do not have a good grasp of the necessary techniques.
There seemed to be no problem with the length of paper.

Comments on Individual Questions

1) (i) Generally well answered but some candidates omitted the '2' in the third term $\frac{\frac{1}{2}.-\frac{1}{2}}{2}(-x)^2$; others incorporated an extra negative at the beginning of the term.

(ii) Here there was the expected muddle in signs; frequently 'x' in part (i) was replaced by $2y+4y^2$ or $-2y+4y^2$.

2) (i) The format of the partial fractions having been given, there was less likelihood of the corresponding identity being wrong. Almost all used the normal identity methods but other suitable ones, sensibly carried out, were also fully accepted.

(ii) The integration was better performed than usual; in the past, $\int \frac{A}{x-2}\,dx$ has quite often been given as $\frac{1}{A}\ln(x-2)$ but this error was much less common on this occasion. The most common mistake was to give both integrations as logarithms. As an **exact** answer was requested, a decimal value did not suffice – but any that was given was ignored provided the relevant logarithmic work was present. It is expected, however, that $-1+\frac{3}{2}$ would be simplified to $\frac{1}{2}$; any simplification of $-2\ln 3+2\ln 2$ was ignored as no request had been given in the question.

3) (i) In any question where the answer has been given, the solutions will be carefully examined and candidates must 'satisfy the examiner'. Each separate step should be carefully written down and negative signs should not be 'blocked out'. At the end, it was no use just writing $\frac{\sin x}{\cos^2 x}=\sec x.\tan x$; if that result had not been asked for, it would have been satisfactory just to write it down – but evidence was needed here.

(ii) Although a few candidates started by squaring numerator and denominator (presumably to 'make it look better'), most realised that the critical area was in the denominator and that a double angle formula should be used. Most did this satisfactorily until the stage where they wanted to write $\int \frac{\tan x}{\sqrt{2}\cos x}\,dx$ as $k\int \sec x.\tan x\,dx$ when k frequently appeared as $\sqrt{2}$ instead of $\frac{1}{\sqrt{2}}$.

4) (i) There was no real problem here for the vast majority.

(ii) The surprising difficulty here was the simplification of $\frac{4}{2t}$ from part (i) before the value of t was substituted. This, coupled with the fact that many confused 'normal' with 'tangent', meant that marks were often frittered away.

(iii) The elimination of t from the parametric equations was nearly always attempted. Many using $t = \frac{y}{4}$ finished with $x = 2 + \frac{y^2}{16}$; the majority using $x = 2 + t^2$ finished with $y = 4\sqrt{x-2}$, so producing only the upper portion of the curve.

5) (i) Here, as in 3(i), the answer was given and candidates were expected to show clear reasoning for all statements made. Some failed to mention why the limits had changed; others (but many fewer than in previous substitution questions) just changed 'dx' to 'du'.
The main problem was an algebraic one – how to show that $\frac{4-u^2}{2+u}.2u$ reduced to $4u - 2u^2$.The simple way, of course, was to change the fraction to $\frac{(2-u)(2+u)}{2+u}$, reduce to $2-u$ and multiply by $2u$ – but many did not see that. The majority decided to multiply out the numerator and so obtained $\frac{8u-2u^3}{2+u}$. If this became $4u-2u^2$ <u>without further work</u>, no marks were awarded for the simplification. (Could the result have been obtained by thinking of the fraction as $\frac{8u}{2} - \frac{2u^3}{u}$?) If, however, the actual long division was seen or $8u - 2u^3$ was re-written as $(2+u)(4u-2u^2)$, then these were accepted. A few left the value of I as $6 - \frac{14}{3}$.

(ii) Those who simplified (a) correctly usually managed to cope satisfactorily with (b), though there was some multiplying out of $(5-x)(2-\sqrt{x-1})$ in the numerator by those who failed to note the cancellation of $(5-x)$.

6) (i) Most used the correct direction vectors, losing marks only by arithmetical errors.

(ii) The method of approach was understood but the presentation of the work was scrappy. References to which equations were being used were often missing and Examiners were forced to delve into a mass of figures to see if the proposition had been proved successfully.

(iii) This prompted a very satisfactory response with more logic in evidence.

7) Candidates generally knew how to approach this question but there were many errors in mid-stream. These occurred for three reasons: (a) carelessness in integrating $\sin x$ and $\cos x$, (b) the idea that the integration of $(2x+5)\cos x$ at the second stage could be performed directly and not via 'integration by parts', and (c) a general lack of care with signs and brackets. A typical case of the latter frequently arose in the first stage: the final term was $-\int (2x+5).-\cos x \, dx$ but this was rarely simplified to $+\int (2x+5)\cos x \, dx$ so giving considerable scope for sign errors later. No doubt this question will be frequently used as an exercise and the variety of errors made by candidates will be evident.

8) (i) Implicit differentiation questions are now answered much more successfully; there was some carelessness of signs with the $-5xy$ term but the majority found $\frac{dy}{dx}$ correctly and equated it to $\frac{3}{8}$. Very few substituted $x=\frac{3}{8}$ as soon as the differentiation process had been completed, though this might have saved them from making a very common, but unexpected, mistake. Having obtained the equation $\frac{5y-4x}{2y-5x}=\frac{3}{8}$, it was very surprising how many said that $5y-4x=3$ and $2y-5x=8$.

(ii) The vast majority obtained $y^2=9$ and so y was ± 3 and x ± 6 (occasionally $\pm 1\frac{1}{2}$); what was surprising was how many (and this was a large proportion including many excellent candidates) gave the answer as (± 3, ± 6).

9) Quite a high proportion either separated the variables or inverted the equation correctly and proceeded to the correct answer in part (i). A few used definite integration but care was needed in setting corresponding limits.

Little careful thought was given in part (ii) to what the question was asking. Most assumed that $x = 0$ would be required but the flow stopping implied that $\frac{dx}{dt}=0$ and hence $x = 8$.

4725 Further Pure Mathematics 1

General Comments

Most candidates attempted all questions and there was no evidence of candidates being short of time. Candidates seemed well prepared for this paper, and correct solutions were seen to all questions. The space provided in the printed answer booklet was usually sufficient and only a few candidates needed additional answer paper.

Comments on Individual Questions

1) (i) This part was answered correctly by the majority of candidates, with a few arithmetic errors being seen.

(ii) A significant minority of candidates omitted the matrix brackets in their answer while some obtained a 2 x 2 matrix and others thought that the product was impossible to obtain.

(iii) Most candidates obtained a 2 x 2 matrix, with only a few arithmetic errors seen. As in part (ii), some candidates stated that the product could not be obtained.

2) (i) This was generally answered correctly.

(ii) Most candidates used both conjugates correctly and a few made arithmetic errors, the most common being obtaining 35 as the denominator. A small number of candidates simply divided real part and imaginary parts.

3) This question proved quite demanding. Most were able to establish the proof for $n = 1$ (or 2) from the given expression, but then did not use the recurrence relation to establish the proof for $n = k + 1$ correctly. The induction conclusion was often rather vague.

4) Most candidates answered this question by comparing coefficients, while a small number correctly used two values for n to obtain the correct answers. Some candidates misquoted the standard result for $\sum r^3$ and some gave an incorrect expression for $\sum r$.

5) A significant number of candidates did not show clearly that in the inverse of a product the matrices have to be reversed. Expressions such as $\mathbf{ABAB}^{-1} = \mathbf{A}^2$
showed that many candidates did not understand the non-commutativity of matrix multiplication.

6) (i) (a) This was generally well done, the most common errors being a vertical line through (8, 0), a horizontal line, or a circle.

(i) (b) This was generally well done, the most common errors being a complete line or a half-line starting at (0, 2).

(ii) This was generally well done, but some candidates failed to shade above a horizontal line through (0, -2) for the second region.

7) (i) This was not answered well by a significant proportion of candidates. They did not recognise that the image of (1, 0) is (1, 0) and those that did often gave the image of (0, 1) as (4, 1) rather than finding the image as (3, 1).

(ii) This was answered well by most candidates.

(iii)(a) This was answered well by the majority of candidates with clear indications of relevant coordinates. Some omitted the unit square from their diagram.

(iii)(b) Most found the determinant correctly and associated it with the scale factor for area, but some candidates were rather vague in their description.

8) (i) Most candidates used the symmetric functions correctly and showed sufficient working to justify obtaining the given answer.

(ii) Most dealt accurately with $\alpha^2 + \beta^2$, the most common error being using incorrect vales for $\alpha + \beta$ or $\alpha\beta$.

9) (i) Most candidates knew how to find the determinant of a 3 x 3 matrix, sign errors being the most common mistake. A small number found the inverse matrix, which meant that extra work had to be done.

(ii) Most candidates solved det **M** = 0 correctly.

(iii) A significant proportion of candidates did not appreciate that, when det **M** = 0, the equations have to be solved to determine whether there are no solutions or an infinite number of solutions and so did not earn much credit on this part of the question.

10) (i) Most candidates established the given result correctly.

(ii) Candidates who arranged terms in columns usually found the terms that cancelled, while those who displayed terms in a row usually had more difficulty.

(iii) Most candidates found the sum to infinity, subtracted the answer to part (ii) and showed sufficient working to obtain the given answer.

4726 Further Pure Mathematics 2

General Comments

The January session has a smaller entry than the June session, but there were quite a number of candidates who had clearly covered the whole of the specification thoroughly and tackled all the questions with considerable success. There were also those who appeared not to be familiar with even the most straightforward parts of questions, and could only be disheartened by their efforts. In between these two extremes, most candidates were able to make reasonable attempts at all the questions. The majority of candidates had time to complete the paper, but the small number who ran out of time in the last question had usually used some lengthier methods earlier. Some instances of these are noted in the comments below, mainly in the first four questions. Candidates should be familiar with the List of Formulae, and results included there should not be derived except where the question clearly requires them to be proved. In a significant number of scripts, presentation was poor or very poor. It does not help candidates or the examiners if figures are illegible or division lines in fractions are missing, for example. All integrals should include "d*x*" or equivalent but this was frequently omitted. Diagrams should be drawn using a ruler, where appropriate.

Comments on Individual Questions

1) Nearly all candidates knew what to do with this standard type of integral, and about 75% scored 4 or 5 marks. The expressions for the trigonometrical functions in terms of *t* are given in the List of Formulae (and so did not have to be derived, as a few did), and most were able to carry out the necessary simplification correctly. However, a very large number of answers were left in terms of *t,* instead of being changed back to *x*. A penalty was not imposed on this occasion for the omission of an arbitrary constant, but it is something which should be included as a matter of course in an indefinite integral.

2) (i) Most gained the first two marks easily by quoting the derivative of $\tanh^{-1} x$ and then differentiating it. A small number wasted time by using the logarithmic form of $\tanh^{-1} x$. Some used the quotient rule instead of the simpler chain rule for finding the second derivative. The third derivative was often found correctly using the product or quotient rule, although some carelessness with signs and basic algebra was seen in tidying the result to the given form.

(ii) The technique for finding the Maclaurin series was well known and full marks were often obtained. This series is in the List of Formulae, but those who simply quoted it scored no marks: "Hence find" requires use of the first part and appropriate working. A few wasted time starting from $\mathrm{f}(x) = a + bx + cx^2 + dx^3$.

3) (i) The asymptote was often given incorrectly: the existence of an asymptote parallel to the *x*-axis did not seem to occur to some candidates and it was quite common to see attempts at doing something with $x^2 + a^2 = 0$. The range of values was done much better and at least the critical values were usually found correctly. This was nearly always done by the $b^2 - 4ac$ technique, although precision with the details was sometimes lacking. The alternative use of differentiation is more awkward and also requires consideration of the shape of the curve to find whether the range is between or outside the values at the stationary points. The precisely correct inequality signs were required for the final mark, and some did not earn this.

(ii) Part (a) was quite often answered correctly but candidates were rather less successful with parts (b) and (c). For the maximum and minimum values all that had to be done was to take the + and – square roots of the positive critical value from part (i). There was some lack of precision, as the maximum and minimum were sometimes given in the form of a range or even as ±, although both of these were condoned. Some gave a minimum of 0 or squared the critical value instead, while others spent a considerable time starting from scratch ("Write down" means what it says). This is a part of the specification where candidates need to think and not to rely on their graphical calculators to show them the form of the curves.

4) (i) This hyperbolic identity was often proved correctly. The usual plan was to find $8\sinh^4 x$ in terms of exponentials and then to express the right hand side in similar form. It was surprising how few used the binomial expansion of $(e^x - e^{-x})^4$, preferring instead to square twice: admittedly this was often done correctly, but it does take more time and the binomial result simplifies it more easily. The few who worked entirely from right to left had more difficulty, as the factorisation was not so obvious. Answers which did not use the exponential definitions, as instructed, did not receive any credit.

(ii) The equation to be solved clearly had some connection with the identity in part (i) and this led most candidates to derive a quartic in sinh*x*. Mistakes were sometimes made here, but it usually worked well. There was some carelessness in solving the quartic, and both real values of sinh*x* were not always given. For the final stage it was expected that the logarithmic form of $\sinh^{-1} x$ would be quoted from the List of Formulae, but time was sometimes spent in deriving it. Similar working led some to a quartic in cosh*x* instead. Others expressed cosh4*x* in terms of cosh2*x*, leading to an easy quadratic in cosh2*x*. Those who expressed everything in terms of exponentials did not fare so well, as the resulting equation was quite tricky to factorise. In all methods great care was needed with the alternative + and – signs when they occurred. There was a variety of correct forms of the answers, depending on the method of solution, including some not shown in the mark scheme (obtained by those who derived the logarithmic forms).

5) (i) Although candidates should have been very familiar with the Newton-Raphson method, even the first part of this question was not done well by many. It appeared that some were confused by the appearance of F(*x*). For the first part the standard first stage of Newton-Raphson had only to be tidied into one fraction, but some tried to do something with F(*x*). Other simply rearranged equation (A) to give *x* = F(*x*), or worked back from that to (A): this was not what the question asked for, and no credit was given.

(ii) The differentiation of F(*x*) was usually done accurately, but this was as far as many candidates went. Some were able to see the relevance of attempting to factorise the numerator, but it was only the best answers which showed a factor of $(x^2 - 5x + 3)$ and how this implied that $F'(\alpha) = 0$. Some attempted verification by using one of the numerical roots, but this did not earn the final mark.

(iii) This should have been a straightforward application of the Newton-Raphson process, but confusion was evident here also, with some using F(*x*) instead of f(*x*): this earned no marks. Others used the correct iteration, but chose a starting point well away from 2 and found a different root, for which two of the three marks were available. Yet others used an iteration of their own for solving the equation: this gained no credit. In a few cases examiners were unable to determine what iteration was being used, as no details were given.

6) The modal mark for this question was 5, out of 10: these were the marks for parts (ii) and (iii) together with the upper bound in part (iv).

(i) Those who could convert the equation $y = x^x$ to logarithmic form correctly and then use well-known differentiation techniques often gained full marks. Others wrote x^x as $e^{x \ln x}$ and differentiated similarly. Incorrect attempts at differentiation such as $x x^{x-1}$ were sometimes seen. But many quickly realised that it would be more profitable to continue with the other parts of the question.

(ii) This was an easy calculation which was usually correct, including the necessary evidence as the answer was given.

(iii) Most answers obtained the correct answer here also. Although it was not necessary to draw a sketch, some did so in order to be sure that their boundaries and heights were right.

(iv) This part was not done at all well by the majority of candidates. Applying the techniques of lower and upper bounds to a function which was not entirely increasing or decreasing was perhaps unfamiliar, but many did not realise the implication of there being a stationary point within the interval specified. Diagrams were often drawn poorly, without a ruler and with little regard being given to the form shown on the question paper. In order for any mark to be awarded for the lower bound it was necessary to show a rectangle whose height was the value of y at the stationary point where $x = e^{-1}$, which had to be marked. Even some of the better candidates failed to mark this value on their diagrams: perhaps their diagrams were drawn better, but the omission still cost them 2 marks. The rectangle(s) for the upper bound did not present a problem, and most scored the mark for it.

7) (i) Those who noted that $\cos 3\theta = \cos(-3\theta)$, or that the cosine is an even function, or equivalent, usually gained the method mark. But in order to show the symmetry of the curve it is necessary to deal with the whole equation, and not just the trigonometrical part of it; so not all gained the second mark. However, it was common to see consideration of one or two values of θ, or for a sketch of the curve to be drawn: neither of these methods gained any credit.

(ii) Tangents suggested differentiation to quite a number of candidates, but this is not how to find the tangents at the pole. A majority of answers started correctly by taking $r = 0$ and solving the resulting equation in θ. But of the three possible values, only one or two were usually found, with π being omitted most frequently.

(iii) In contrast, this part was often done very well. Nearly all had the right expression for the area, the integration was usually carried out correctly, and the substitution of limits done accurately. Perhaps the most common mistake was a sign error in converting $\cos^2 3\theta$ to an expression in $\cos 6\theta$. A few lost the factor of $\frac{1}{2}$ in the course of their working. Changing the limits to the range of 0 to $\frac{1}{3}\pi$, in order to simplify the substitution of limits, was not seen very often: this should have occurred in more answers, following the symmetry in part (i).

8) (i) About two-thirds of candidates made a serious attempt at this part. The most common method was the rather lengthy Method 3 of the mark scheme, using the logarithmic form for $\cosh^{-1} x$ and the exponential definition of sinh$x$. Many then found themselves entangled in expressions containing $\sqrt{3}$ and did not always simplify legitimately, preferring to work backwards from the given value. Both of the other methods were seen, with the first one being less common.

(ii) The recurrence relation was often done correctly, quite commendably so as the end of the paper was drawing near. Nearly all split $\cosh^n x$ up appropriately and proceeded with confidence, though not always with neatness, and only a few failed to use $\sinh^2 x = \cosh^2 x - 1$ or made sign errors. Occasionally $\cosh^n x$ was split as $\cosh^2 x \,.\, \cosh^{n-2} x$, but it was only one or two exceptionally able candidates who could make it work.

(iii) The numerical part was also correct in very many cases. Just a few failed to realise that they had to start with $n = 1$ rather than $n = 0$. The initial value for recurrence relations usually has to be done as a special case, and this one was no exception, but the majority showed their working for I_1 correctly. A small number attempted to use the recurrence relation just found with $n = 1$, despite this being outside the range of values of n stated. This appeared to give the correct value of $\sqrt{3}$, but as the method includes a term $0 \,.\, I_{-1}$, which is invalid, such answers were penalised.

4727 Further Pure Mathematics 3

General Comments

Although the entry was small, as usual in the January session, there were quite a number of candidates who were well prepared for this paper. Questions 4 and 6 (ii) made the paper a little more demanding than usual, but those who were unable to make much progress did not waste time on them. There did not appear to be any problems with the time allocated for the paper, although some spent more time than necessary on Question 7. Presentation was generally at least fair, but a certain amount of carelessness in using mathematical notation properly was noted: some details are in the comments below.

Comments on Individual Questions

1) The majority of candidates gained 2, 3 or 6 marks for this question, with the modal mark being 2. This was because the integration of $x\mathrm{e}^{x^2}$ caused much more trouble than had been anticipated. It may not have been a function which was very familiar, but most failed to see that it was an exact integral and launched into unprofitable attempts at integration by parts or tried analogies with the integral of e^{mx}. None of these came to anything, but some gained another mark by making an attempt at part (ii), using their incorrect general solution. Those who did know how to integrate the function did the whole question in as little as seven short lines, usually scoring full marks. With regard to presentation, omission of "dx" in the integrals was condoned, but it is not good practice to miss it out.

2) (i) Virtually all candidates realised that the instruction to "obtain" the equation meant that verification was not acceptable. The standard method of using the vector product was almost always used correctly, with sufficient working being shown to obtain the given equation. The only criticism is that the statement $\begin{pmatrix}10\\-5\\5\end{pmatrix}=\begin{pmatrix}2\\-1\\1\end{pmatrix}$ occurred too often: although it was not penalised, it shows poor attention to detail to omit the multiplying factor.

(ii) There are several straightforward applications of vectors to planes, and it is not always easy to choose the most appropriate method in the stress of the examination. But it was surprising how few used the simple technique of finding the difference between 21 and 3, and then dividing by the modulus of the normal vector. Longer methods were more common: most of them worked, apart from those which found the distance between (1, 3, 4) and an arbitrary point on the plane q, without any reference to a scalar product or the normal.

3) (i) The derivation of trigonometrical identities is well practised, and many answered this one correctly. Those who omitted the i in the denominator of the expression for $\sin\theta$ lost the first mark, but all others were available.

(ii) None had any difficulty with the integral, and full marks were very common. Occasionally the factor of $\frac{1}{8}$ was lost by carelessness.

4) (i) The verification of $1+\omega+\omega^2=0$ is a standard result which can be obtained by a variety of methods. All those shown in the mark scheme were seen, with the last one being most common. The quickest is to use the sum of the roots of the cubic equation, and this was often the choice of the best candidates.

(ii) Many candidates seemed unfamiliar with this section of the specification. The first mark, for stating that multiplication by ω represents a rotation of $\frac{2}{3}\pi$, was as much as many gained, and even this was more often stated to be clockwise. Few were able to relate the expressions $z_1 - z_3$ and $z_3 - z_2$ to the diagram correctly and to deduce the given result. The best candidates answered this part well, giving proper attention to the directions of the vectors.

(iii) Some were able to gain the marks here by rearrangement of the result in part (ii). The key element was the use of the result of part (i), even if some answers were obtained by a roundabout route.

5) The solution of this differential equation was much more familiar territory, and high marks were usually awarded. There were rather more numerical and algebraic errors than expected, but few marks were lost as many of the marks were for method or for follow-through from previous answers. On this occasion lack of "y =" in the solutions was not penalised.

(i) The solutions of the auxiliary equation and the complementary function were almost always correct, as was the form of the particular integral. There was some carelessness in solving the equations for the constants, but the mark for the general solution was usually gained.

(ii) Again, most candidates knew what to do, but it was their algebraic accuracy which sometimes let them down.

(iii) The final part really only made sense if the particular solution comprised an exponential term with a negative index and a linear term, but the follow-through mark was awarded for a solution valid from the candidate's particular solution.

6) (i) Many correct answers were seen to this part. Consideration of powers of a and r gave the required orders quite easily, although some showed no working at all.

(ii) This question was done well only by the most able candidates. The specification expects familiarity with the structure of the groups G and H, and it was not too difficult to construct the two tables, showing the number of elements with each of the possible orders. Some answers curiously contained numbers which did not add up to 4 and 6 respectively. The purpose of the tables was, firstly, to draw attention to the fact that G and H are the only non-cyclic groups of order up to 6, and for the first mark it was necessary to note this. Then the tables were used to compare the number of elements of order 2 with that in the table for group Q. It was good to see very concise answers to this part from some candidates; it was probably a result, and perhaps a method, with which few would have been familiar.

7) (i) Most candidates answered this part easily. It was, of course, necessary to note that the two normal vectors were multiples of each other. Statements such as $\begin{pmatrix} -3 \\ 15 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix}$ were seen less frequently than in Question 2. A few used lengthier methods, sometimes doing work which was then used in part (iii).

(ii) Nearly all gained the mark here, although some thought that this line of intersection was perpendicular to l and m.

(iii) A variety of approaches was seen to this part, some more elegant than others, but it was regrettably common for algebraic errors to be made. The neatest solutions came from those who considered the equations as a whole and obtained a contradiction almost immediately. Others set about solving the equations or finding points on two of the parallel lines which had, for example, $z = 0$. Some credit was given to those whose algebra went wrong if their general method was correct. A few stated only that the determinant of the coefficients was zero: this did not earn any marks as it should have been obvious from the fact that the lines of intersection were parallel, and work was then needed to determine whether the planes had a common line or formed a prism.

8) Responses to this question as a whole were good, although it was only the best candidates who gained at least 10 of the 12 marks.

(i) Examiners were pleased to find that the verification of associativity was almost always done correctly.

(ii) This part was done well, although some did not realise that $a + b = b + 1$ implied that b could take any value.

(iii) Most solved the equations arising from the given relationships correctly, to find p and q.

(iv) This part was not always done completely. Those who knew what to do obtained a pair of equations in two unknowns, but sometimes missed the fact that a had two possible values. The general form $(-1, b)$ was found quite often, but many omitted to state that the identity $(1, 0)$ was also self-inverse.

(v) The most common response to the final part was to say that "some elements have no inverse". As the set was clearly closed and had the associative property and an identity, this was fairly obvious, and unless a specific example of an element with no inverse was given, the mark was not awarded.

Chief Examiner's Report - Mechanics

The standard of work was good, and few candidates struggled throughout the papers they sat.

When candidates drew careful diagrams of the situations set out in the questions, they greatly benefitted from the initial thinking this requires. Conversely, confused or inconsistent solutions were often associated with an inadequate picture of the problem described in a question. This was noted in each level of the suite of mechanics syllabuses.

There were specific parts of questions which seemed unfamiliar to many, as described below. When setting papers, the language of mechanics used is that found in the syllabus. In this way it is hoped that variations in terminology found in different texts or used by different teachers will not affect candidates' performance.

4728 Mechanics 1

General Comments

Candidates were well prepared for the more routine parts of the paper, and scored well on the majority of questions. In some scripts there was a tendency for work to be done in a very fragmentary way, with an answer emerging without any coherent strategy applied. This extended to numbers appearing which had no apparent relationship to data in the question paper.

Some candidates who used specific letters to represent unknown vector quantities used the same letter to represent their magnitude. In general no penalty was imposed in cases where the vector value was negative.

Few correct solutions were seen to Q7(ii). In January 2010 candidates were asked to find the components of a contact force, and sensible attempts were often made to do so. It seemed much harder to work from the relevant components towards a unified contact force, as requested at this session.

Comments on Individual Questions

1) (i) Candidates usually obtained a correct answer, though (-)1.1 was often seen when candidates overlooked the change in direction of *P*.

(ii) Rather than use the result from (i), candidates usually treated this as a traditional conservation of momentum problem. When their equation yielded v = -0.125, it was common to see the answer expressed as v = 0.125. Most marks were lost as a result of sign errors.

2) Though slightly unusual, this question was well answered, with nearly all candidates gaining full marks. The working showed that (ii) was often done before (i), and many candidates kept their answers to (i) and (ii) independent by using trigonometry in (i) and Pythagoras' Theorem in (ii).

3) At least half of the entry for the paper treated the upwards and downwards motion separately.

(i) The most popular approach was finding the greatest distance the particle attained above the point of projection, then finding correctly the speed at which the particle struck the ground, working with u = 0.

(ii) Again, often the times for upward and downward motions were found, and then added. Candidates using their answer from (i) in $v = u + at$ usually obtained the correct answer, though confusion over signs for 8.6, 5 and 9.8 was seen. Candidates approaching the solution via $2.5 = -5t + 9.8t^2/2$ usually obtained the correct answer.

(iii)(a) Perhaps reflecting the perception of upward and downward motion being distinct, the majority of candidates drew a V shaped graph. A few candidates drew a graph with a negative intercept on the vertical axis, in conflict with the question giving a positive value for the velocity on projection.

(iii)(b) Though nearly all candidates drew a graph finishing below its starting point, these graphs often consisted of two straight line segments joined at a sharp apex.

4 (i) Most candidates attempted this question sensibly, considering the motion of *B* alone, and using Newton's Second Law (N2L). It was quite common however for *T* (tension) to appear in the equation where $T\cos 10$ was appropriate.

(ii) The majority of marks for accurate work were made available to candidates who had found an inaccurate value of *T*. A significant number of scripts contained the error that the reaction on *P* equalled its weight. The most common fault in finding the frictional force was to forget to resolve *T*. Inaccurate work by candidates who used μR as the notation for a frictional force could lead to further problems when the value of μR was negative.

5 The explicit structure given to part (i) of the question was expected to help candidates. Unfortunately many candidates sought to use the answer to (i)(a) in the answer (i)(b). Scripts frequently showed the given value in (i)(c) used to find the value of θ in (i)(b), and a circular argument ensued.

(i)(a) Candidates answered this first part of the question well.

(i)(b) The only acceptable way to tackle this part of the question was to relate the acceleration of *P* and the angle θ with the acceleration due to gravity. Candidates who completed this part of the question successfully often failed to see the significance of $\theta = 30$ in part (i)(c).

(i)(c) Though correct solutions to this part of the question were seen, based on the ratio of distances in (i)(a) and the value of θ from (i)(b), they were rare.

(ii) Candidates who made little progress on (i) were able to score full marks here using the value of *T* given in (i)(c), and many did so.

6 Candidates showed a good knowledge of the relationship between mechanics and calculus. The main problem faced by candidates was dealing logically with an arbitrary constant arising after the integration of velocity. There was a minority who wrongly used constant acceleration formulae in parts of the question, having correctly tackled variable acceleration work earlier.

(i) Most candidates gained full marks here, though in some scripts the displacement was given a negative value.

(ii) Nearly all candidates found the correct time to use in this part of the question, and fully correct answers were often seen, although the answer was quite frequently left as -2.67 or -18.

(iii) It was common for answers to this part of the question to gain 4/5 marks. The final mark could only be gained by clear thought about the distances and times involved.

7 (i) The most interesting feature of solutions was the appearance of a force in the direction of motion. Thus friction (often described as μR or confused with ma) was shown acting up the plane in many scripts. However a majority of candidates obtained the correct value for the magnitude of the frictional force. A minority of scripts contained a wrong value for the normal component of force exerted on the particle by the plane.

(ii)(a) When considering the "contact force exerted on the particle by the plane", some candidates considered only the normal component of reaction, while others included the component of weight parallel to the plane. Selecting the two appropriate values (for the normal component of reaction and the frictional force), and combining them as two component forces at right angles was rare.

(ii)(b) A significant minority of candidates grasped the notion that, because the particle was remaining at rest, it must experience a force equal and opposite to its weight.

4729 Mechanics 2

General Comments

Many candidates demonstrated a good knowledge of the topics in the specification, and accordingly gained high marks on the paper. Only a small minority of candidates were unprepared for the demands of the paper. Candidates need to be mindful that poor or non-existent diagrams frequently lead to misunderstanding, particularly with Q2(ii), Q5(ii) and Q7**.**

Comments on Individual Questions

1) (i) This question was usually well answered by the majority of candidates. A minority of candidates failed to find the required distance having found the position of the centre of mass.

(ii) A few candidates were confused about which axis the frame was rotating about, but most were able to use an appropriate method to find the required speed.

2) (i) Very few candidates encountered significant difficulty with this question. Examiners were pleased that the majority of candidates showed sufficient detail in their solutions to get the given answer.

(ii) This part proved more challenging with frequent errors being made either in the signs in the Newton's 2nd Law equation or the number of terms. A clear force diagram might have avoided many of the errors seen.

3) (i) This question was well answered by the majority, with only a minority of candidates only giving one of the tensions when the request was for both. Some candidates misread the relationship between the tensions as $T_B = 2T_A$. There were a few cases where candidates omitted the weight and formed a second equation by 'equating' the horizontal components to form simultaneous equations.

(ii) This proved to be a good question for the well-prepared candidate. There were examples of some candidates incorrectly using T = $2\pi\sqrt{(l / g)}$ or $T = ml\omega^2$.

4) (i) The request in this question was standard, but examiners saw surprisingly many errors. The most common errors were including the 70 N (and sometimes also 25*g*) or failing to use the component of tension.

(ii) This was well answered by the majority.

(iii) There were two possible approaches to this question, both of which were equally successful for candidates. In the energy approach, a common error was to include the same energy twice, omit one of the energy terms, or use incorrect signs. In the Newton's 2nd Law approach, the usual error was to omit one of the forces.

5) (i) There were many good solutions to this question. However, candidates should be reminded that where an answer is given in the question, examiners will assess their work to ensure that the answer is obtained logically and accurately.

(ii)(a) This was the least well-answered question on the paper. Candidates would be better served by the inclusion of a full force diagram as well as indicating about which points they are taking moments. Invariably, those who attempted to take moments about the centre of the common face omitted the moment of the friction at the contact point. Various other points were used for taking moments but most omitted the moment of at least one of the forces.

(ii)(b) Of those who found a mass in (a), most could find a value for μ, but few candidates realised that the range of values comes from use of friction being less than or equal to the normal reaction.

6) (i) This question was often fully correct. Various methods were used either using constant acceleration directly or by finding the time first. Some candidates quoted the formula for the greatest height, but this method is without value if the formula is quoted incorrectly.

(ii) This question was answered particularly well by the majority of candidates. The common errors included using an initial velocity of 0, or not using the component of the 14ms^{-1}.

(iii) Examiners saw some very good solutions to this question. However some candidates used a particularly long method to show that the vertical speed was 7 ms^{-1} on return. A significant number of candidates ignored the fact that the projectile's horizontal velocity had been changed due to the impulse found earlier and seemed unsurprised to get a speed of 14 ms^{-1} and an angle of 30º.

(iv) Candidates were often more successful in obtaining the time than the distance. Many thinking they could simply apply the range formula using a component of 14 ms^{-1}, or use their total time with a component of 14 ms^{-1}. Not all realised that the times of ascent & descent were the same and proceeded to calculate the times separately.

7) (i) Only a minority of candidates were successful in solving this problem. The momentum and restitution equations were usually correct. Many were unable to cope with tying together the condition that the speed of sphere *B* was greater than that of sphere *C*. Many had *B* and *C* with the same direction and so imagined that sphere *B* would somehow pass through sphere *C*. The presence of what were regarded as three unknowns meant that weaker candidates then tried to find another equation in order to 'solve' for a, b and e instead of using the inequality $a > b$.

(ii) Candidates seemed more confident in their approach to this part, perhaps because they now had a value for e and so were on 'familiar ground'. Some only considered one of the two possible directions of motion of *C* after collision. For those who found two values of c, although one case was often considered correctly, the second value of m was frequently incorrect, as the appropriate adjustment was not made to the momentum equation. Even when this was done correctly a surprising number gave the solution of the equation $0.5m = 0.7$ as $m = 0.35$.

4730 Mechanics 3

General Comments

Many candidates gave a good account of themselves in this paper, and there were few who had been entered for an examination for which they were ill-prepared.

Comments on Individual Questions

1) (i) This proved a very accessible question for almost all candidates.

(ii) A small minority either omitted the mass of the ball, or assumed 1 kg, and found the speed to be half the correct value.

2) There were many excellent solutions, with some candidates providing a general solution for when the string made angle θ with the downward vertical, and then substituting $\pi/2$ and π to find the required solutions.

3) (i) This was done well.

(ii) Some candidates lost marks on part (ii) by using the vertical force acting on *QR* at *Q* in their solution without explaining why it was 36 N.

(iii) A significant number of candidates unnecessarily worked out the length *QR* and the angle it made with the vertical (or horizontal). Such solutions were more likely to lead to errors.

4) (i) All but a small minority of candidates were able to tackle part (i) competently, with few sign errors in the conservation of momentum equation or in the use of Newton's Experimental Law.

(ii) Most candidates realised that they needed to find the angle that the direction of motion of *A* made either with the line of centres or else with a line perpendicular to this. After that, many candidates did not correctly give the angle turned through by the direction of motion of *A*, with the wrong answer of 134° being seen as frequently as the correct answer of 46°.

5) (i) Few candidates had any difficulty, with most finding the extension rather than verifying it.

(ii) Some candidates omitted the weight in part (ii), and some also omitted the part of the tension in the string due to the extension in the equilibrium position.

(iii) Most candidates quoted equations for SHM in part (iii), though some needlessly tried to solve the equation established in part (ii). Candidates using $v^2 = \omega^2(a^2 - x^2)$ were required to establish the direction of the velocity.

6 (i) Most candidates realised that they needed to work out the extension of the elastic rope when *P* is in its equilibrium position and then V^2, where V is the velocity of *Q* when it reaches *P*. A small number of candidates then wrongly used conservation of energy to try to find the speed of the combined particles after the impact.

(ii) A large number of candidates had varying amounts of difficulty with part (ii), though there were also many excellent solutions. Most candidates realised that they needed to consider the kinetic, potential and elastic energy at the point *Q* became attached to *P* and the point where the particles were instantaneously at rest. The most common errors were in not including the elastic energy at the beginning, and not taking account of $\sin\theta$ when working out the change in potential energy.

7 (i) There was little difficulty in establishing the given result.

(ii) Most candidates realised the integral came out as a logarithmic function. Some omitted the minus sign, some obtained '400' when they should have had '200' and a proportion failed to find the constant required. There were further problems writing the solution with v^2 as the subject. Many candidates who got to this point then failed to show convincingly that $v^2 <$ 3920 for all values of x. The required answer was that $e^{\frac{-x}{200}}$ is always positive, not that the function tends to 0 as x tends to infinity.

(iii) Part (iii) was attempted by only a minority of candidates, most of whom realised that it was necessary to find the distance travelled from the start to the time when the acceleration was 5.8 ms^{-2}, and that finding v or v^2 first was a sensible (though not absolutely essential) route. After that the candidates who found the work done from the potential energy lost and the kinetic energy gained were rather more successful than those who used Work Done = Force x Distance , since many of the latter forgot that the force varied, so that calculus was essential.

Chief Examiner's Report – Statistics

The increase in the number of units marked online has worked well in general. Centres should, however, bear in mind the following points.

Candidates who fail to write their answers in the specified parts of the Answer Book make life difficult for the Examiner who, when marking, say, question 4 part (i) can see only the region of the Answer Book corresponding to that question part. A few candidates wrote their answers consecutively with no regard for the Answer Book layout, which made their scripts very hard to mark.

The scanning process can pick up deleted work such as erased pencil lines as if it had not been deleted. Candidates are advised, therefore, not to rub out wrong work but to cross it out and start again.

The word "random" has several different specific meanings within the A-level specifications and candidates should use it with considerable caution. In particular it is not a synonym for "independent".

1) A **random sample** is a sample selected according to one of two (different) criteria: *either* every possible sample of size n is equally likely to be chosen, *or* each member of the population is equally likely to be chosen and the selections are made independently.

2) **Random numbers** are a sequence of numbers or digits which obey "binomial" properties: each is equally likely to be any of the possibilities, and each is independent of all the others. "Select numbers randomly" is not a synonym for "select by using random numbers", and an exam question that requires the latter answer will not usually allow credit for the former.

3) The statement "events occur **randomly**" is very weak; it amounts to no more than "events are not exactly predictable" and certainly does not include any implication of "independence". A **random variable** is a variable whose outcomes are not exactly predictable.

One simple consequence of all this is that "events occur randomly" is not a valid condition for a Poisson distribution to be a good model.

4732 Probability & Statistics 1

General Comments

Because this paper is now marked online, candidates are required to answer in the answer book with spaces allocated for each part-question. It was pleasing to note that very few candidates answered questions in the wrong space. Some candidates ran out of space and continued on an extra sheet, but without any indication that the examiner needed to look at the extra sheet. Centres should note that if candidates run out of space for a particular answer, they should ask for extra sheets and they should indicate in the normal space for the particular question that there is work on additional sheets. These sheets should then be attached at the back of the answer book.

Many candidates showed a reasonable understanding of a good proportion of the mathematics in this paper. There were some very good scripts, although very few candidates gained full marks. There were several questions that required an interpretation to be given in words, and these were not answered as well as in some previous years.

The only question that made a significant call upon candidates' knowledge of Pure Mathematics were question 4(ii) where a quadratic equation needed to be formed and solved, and question 7(ii), where some elementary algebraic manipulation was required. Notation and manipulation were often poor. Questions 2(iv) and 6(ii)(b) required some clear, logical thinking and many candidates found these difficult.

Few candidates appeared to run out of time.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae and tables

The formula booklet, MF1, was useful in questions 3(i), 3(iv), 5(for binomial tables) and 8. It was good to note that very few candidates appeared to be unaware of the existence of MF1. Some candidates tried to use the given formulae, but clearly did not understand how to do so properly (e.g. Σd^2 was sometimes misinterpreted as $(\Sigma d)^2$ in question 8). In questions 3(i) and 3(iv) a few candidates quoted their own (usually incorrect) formulae for r and b, rather than using the one in MF1. Some thought that, eg, $S_{xy} = \Sigma xy$. Some candidates used the less convenient version, $b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ from MF1, but most of these completely misunderstood this formula, interpreting it as, for example, $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{(\Sigma x-\bar{x})^2}$. Some candidates' use of the binomial tables showed that they understood the entries to be individual, rather than cumulative, probabilities. Others did not know how to use the tables to handle $P(X = 2)$.

Responses to question 5 gave evidence that many students (understandably!) prefer to use the binomial formula rather than the tables. In part (ii)(a), the answer can be written down immediately from the table, but a few candidates went into detailed and lengthy working, sometimes making errors.

It is worth noting yet again, that candidates would benefit from direct teaching on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1 (except in the case of b, the regression coefficient). They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

Comments on Individual Questions

1) (i) Most candidates answered this correctly although a few took the total frequency to be 225 (the top of the graph paper) instead of 200.

(ii) Most candidates answered correctly, but a few gave bogus reasons for their choice, for example: "Paper 2 because the curve is steeper" or "Paper 2 because everyone scored higher marks." A disappointingly large number of candidates failed to notice that the answers to part (i) gave them sufficient material to answer this part.

(iii) Most candidates calculated the inter-quartile ranges correctly, although a few took the total frequency to be 225 instead of 200. Having found these, some candidates made a comment that did not answer the question, such as "The inter-quartile range for Paper 2 is less than that for Paper 1". A few failed to calculate the inter-quartile ranges and just commented on the level of variation that they thought they could see in the two papers' results.

(iv) This part was generally well answered. A few candidates found the minimum mark for grade A in Paper 2, but did not go on to find the number of candidates gaining at least this mark. Others found the number of candidates who failed to gain grade A on Paper 2, but did not go on to subtract from 200. A few had the minimum mark for grade A on paper 1 at 10 marks higher, rather than lower, than that for Paper 2.

(v) The overwhelming majority of candidates answered this correctly. Some wasted much time and effort by calculating one or both quantity using Σx, Σx^2 etc, but many of these made errors. A few added 1 to the standard deviation.

2) (i) This was usually answered correctly.

(ii) Most candidates did not appreciate that all they had to do was to find $P(X = 3, 4 \text{ or } 5)$. All kinds of long methods were used, including $P(X = 1, 2, 3, 4, \text{ or } 5) – P(X = 1 \text{ or } 2)$. Candidates who used this method often included a bogus $P(X = 0) = 0.2\times0.8^{-1}$. Candidates who tried to use a more subtle method (subtracting two probabilities) usually made errors such as $P(X = 3) – P(X = 5)$ or $1 – P(X = 5) – P(X = 2)$ or $(1 – 0.8^5) – (1 – 0.8^3)$ or, more commonly, $(1 – 0.8^5) – 0.8^2$.

(iii) Candidates, as usual, found this simple question difficult. Some used the "long" method, finding $1 - P(X = 1, 2, 3, \text{ or } 4)$ (possibly including the bogus $P(X = 0)$). These were often successful. Others knew that there is a short cut, but made errors such as $1 – 0.8^4$ or just 0.8^5. Some candidates tried to use their answer to part (ii), which is not relevant here.

(iv) Confusion was common here. Many included the pair 0, 3 (which is impossible) as well as 1, 2. Some added $P(X = 1) + P(X = 2)$ instead of multiplying. Some considered only 1, 2 and not 2, 1. A few candidates tried to use a binomial distribution with $n = 3$.

3) (i) This was answered correctly by the vast majority of candidates, showing careful substitution without premature rounding. A few made arithmetical error or copied the figures incorrectly from the question paper. A few used incorrect versions of the formulae, as detailed above.

(ii) The most common error was not including any mention of the context, eg "There is strong correlation". Some candidates who did include the context just explained that the correlation between spend on advertising and profit is positive, which is an inadequate answer. To gain the mark candidates had to mention that the relationship is strong, and to include the context. Acceptable answers are given on the mark scheme.

(iii) Many candidates merely repeated the words of the question with answers such as "spending money on advertising may not result in greater profits." To gain both marks candidates needed to state that extrapolation is unreliable (or words to that effect) and that correlation does not imply causation (or words to that effect). Acceptable answers are given on the mark scheme.

(iv) This question was well answered. A few candidates fell into the traps mentioned above.

(v) The majority of candidates missed the fact that the data in the question is given in thousands and so substituted 7400 instead of 7.4. These gained no marks. A few substituted 7.4 but did not convert their answer to thousands.

4) (i) Many candidates gave Jenny a second attempt even after she had succeeded on her first attempt, assigning a probability of either 0.6 or 0.7 to this. This leads to the correct answer and was condoned. Some candidates multiplied 0.6 by 0.4×0.7 instead of adding. A few candidates used the elegant method of 1 – 0.4×0.3.

(ii) Many candidates omitted brackets in their first line: $p + p \times 1 - p$. Many candidates started correctly, but were unable to solve the resulting quadratic equation. Some did not rearrange it into the form " . . . = 0". Others did this, but made errors in the formula. Some seemed just baffled by the fact that the coefficients are not integers. A few solved the equation correctly but gave two answers: $p = 0.3$ or 1.7. A few started incorrectly with, for example, $p^2 = 0.51$ or $p + p^2 = 0.51$ or $p + p(p - 1) = 0.51$. Some candidates used the elegant method: $(1 - p^2) = 0.49$ etc. A few used trial and improvement, often successfully, although this method is not recommended. Generally speaking, trial and improvement methods lead almost inevitably to no marks being scored at all.

5) (i) Many candidates gave the conditions in general rather than in context. Some of these appeared to have learnt the conditions for a binomial distribution off by heart, directly from a text book. These gained no marks. Lack of thought was evidenced by the frequent references to "repeated trials" and "two possible outcomes" both of which are features implicit in the context and are therefore not conditions at all.

(ii)(a) The answer can be written down immediately from the table, but a few candidates went into detailed and lengthy working, sometimes making errors.

(ii)(b) Most candidates answered this correctly, using either the table or the formula. Some just read the value for 2 from the table.

(iii) A large number of candidates used $p = 0.3$, with $n = 5$ or 35 or 15. Some others correctly realised that the answer to part (ii)(b) is the required value of p here, but many used it in a binomial calculation with $n = 7$ or 15 or 35. A few just multiplied their answer to part (ii)(b) by 3 or $\frac{3}{7}$.

6) (i) Many correct answers were seen, but 7!, 4!, and $^7P_4$ or $\frac{7!}{3!}$ were also frequent incorrect attempts.

(ii)(a) The most common error was adding $^5C_3 + {}^{10}C_4$. Some candidates found $^5P_3 \times {}^{10}P_4$. Others correctly found $^5C_3 \times {}^{10}C_4$ but then divided by a "total" (eg $^{15}C_7$) to obtain a probability.

(ii)(b) Few candidates were successful here. Common errors involved products such as $^5C_3 \times {}^{10}C_4$ or (slightly better but still incorrect) $^4C_3 \times {}^{10}C_4$. A few candidates tried using $^{13}C_6$. Others used permutations instead of combinations. Some used fractions rather than combinations, but these usually thought that P(The three white cards contain A) = $\frac{1}{5}$. Many candidates thought that the number of combinations of the three white cards that include 'A' is $^5C_3 \times \frac{1}{5}$. A good number of candidates found $\frac{1}{5}\times\frac{1}{4}\times\frac{1}{3} + \frac{1}{10}\times\frac{1}{9}\times\frac{1}{8}\times\frac{1}{7}$. Some used a binomial calculation such as $^5C_3 \times (\frac{1}{5})^3 \times (\frac{4}{5})^2$.

7) In both parts of this question, many candidates used incorrect algebraic notation such as $2 \times 1 - a$ or $(2 \times 1 - a)$ or $1 - a \times 4$ or $2 - 2a^2$.

(i) Most candidates were successful in this part, although a few felt they had to give a a value. A few made the usual error of dividing by 2. Some candidates found E(X) = $2 - 2a$ correctly, but went on to form an equation ($2 - 2a = 1$ or $2 - 2a = 0$) and solve it.

(ii) Most candidates found E(X^2) correctly (although a few divided by 2 or found Σxp^2). Most attempted to subtract $(E(X))^2$. Basic algebraic manipulation was often weak, with omitted terms and/or incorrect signs, eg $(2 - 2a)^2 = 4 - 4a^2$. But most candidates nevertheless managed to massage their expressions so that they ended with the correct answer (which is given in the question).

8) (i) Most candidates answered this correctly, although some had to check their answer by evaluating r_s. A few seemed to understand the point and wrote 1, 2, 3, 4, 5 alongside 5, 4, 3, 2, 1, but did not actually answer the question. Others drew a scatter diagram showing perfect negative linear correlation, but did not answer the question.

(ii)(a) This was well answered, either starting with 0.9 and finishing with 0.2 or vice versa. A few candidates showed $r = 1 - \frac{6\times 2}{5(5^2-1)}$ but then just wrote "= 0.9" with no intermediate steps. These only scored one mark. Some candidates showed how $\Sigma d^2 = 2$ can be found from two orders, but did not connect 0.2 with 0.9 at all. A few candidates used an incorrect formula such as $\frac{6\times\Sigma d^2}{5(5^2-1)}$ or $\frac{1-6\times\Sigma d^2}{5(5^2-1)}$.

(ii)(b) This was also generally well answered. A few candidates misread the question and compared the third race with the second instead of the first.

4733 Probability & Statistics 2

General Comments

Many excellent scripts were seen and it is pleasing to note good work especially on some topics that have often been weak, such as hypothesis tests using the binomial distribution.

Attention is drawn to the comments on the use of the word "random" in the Chief Examiner's Report. In subsequent examinations it is likely that candidates who use the word "random" when the correct term is "independent" will not gain credit.

Examiners noted an increase in use of calculators that give probabilities without working. It is a high-risk strategy to give a calculator answer without supporting evidence, and this was particularly true in questions which involve the use of the normal distribution. There is generally a method mark for showing standardisation using $\frac{x-\mu}{\sigma}$, and a wrong answer obtained without showing this step may well lose several marks.

Candidates are reminded that conclusions to hypothesis tests must include mention of the context (such as "the mean number of job applications") and also to avoid over-assertive statements such as "the mean has increased". "There is significant evidence that …" is needed. Those candidates who lost marks for this reason seemed to be casual rather than ignorant. "There is significant evidence that H_0 is correct" is wrong.

Comments on Individual Questions

1) Almost everyone found this a simple start.

2) Essentially a very standard question, and many completed it well. Few failed to obtain *z*-values by using the tables back-to-front. The most common mistakes were with the sign of *z*. A few, having obtained $\sqrt{n} = 10$, wrote $n = \sqrt{10}$, and others gave answers such as "100.0 (1 dp)", failing to appreciate that *n* has to be an integer.
As before, Examiners remain surprised at how many candidates make heavy weather of solving a pair of simultaneous equations of the form $a-\mu = b\sigma$, $c-\mu = d\sigma$. Addition or subtraction is much easier than substitution.

3) Many scored full marks here. Some failed to justify the approximation fully or appropriately. Rules such as "*n* large, *p* small" are acceptable but if numerical inequalities are given they must be the ones in the Specification, namely "$n > 50$, $np < 5$". Weaker candidates used a normal approximation or the exact binomial and scored few marks.

4) (i) This was generally well done, with few omitting the $\sqrt{50}$. To obtain full marks candidates needed to make an explicit comparison, either between –2.608 and –2.576 or between 0.0047 and 0.005, and also to state in their conclusion something like "significant evidence that the mean is not 230".

(ii) This question not only tested knowledge of what the Central Limit Theorem (CLT) says but also the difference between "necessary" and "sufficient". It is necessary to use the CLT because the parent distribution is unknown, and *not* because the sample size is large which merely shows that the CLT *can* be used. In any case the CLT is very poorly understood. A very common misconception was illustrated by: "As it is a continuous distribution, it is normal already."
Other misconceptions were illustrated by "Yes as we are using the sample mean" *or* "estimating the sample variance".

5) Hypothesis tests involving discrete distributions have generally been one of the weaker areas in this Specification. On this occasion fewer candidates than usual committed the serious errors of calculating P(≤ 19) or P(= 19) instead of the correct P(≥ 19).

6) (i) This was not very well answered. A typical good answer was: "if one customer arrives, it does not change the probability that another one does, which is not true as customers tend to arrive in groups" or "if the restaurant is full no more can arrive". "Customers enter on their own" is not sufficient as the "singly" condition is only part of "independence". A common misunderstanding was to consider what happened on different days of the week, which is wrong here.

(ii) Almost always correct.

(iii) Very often correct, apart from the use of the wrong, or no, continuity correction.

7) (i) Full marks were quite common, but many candidates lost marks either by failing to relate the two graphs so that they had approximately the same areas beneath them (which means that the curves must cross), or by continuing the curves beyond the limits of [1, 3], heedless of the full definitions of the PDFs. Erased lines on graphs, picked up by the scanning, caused some problems for Examiners here.

(ii) Very well done apart from algebraic mistakes.

(iii) Very well done apart from errors with the integration. In particular, some thought that the integral of x^{-1} involved x^0, and others who wrote the integrand as $3/2x$ thought that the integral was 3 ln ($2x$) or similar.

(iv) This was by far the least well answered question on the paper. The correct answer is simply that T is equally likely to take any value between 1 and 3. The "answer" given on the paper illustrated a commonly-held, if vague, misconception. Many candidates do not see that x just represents values of T; they seem to think that whether or not T (which they imply is an *event*) "occurs" depends on the value of x, which appears to be some completely unrelated variable. It is hoped that the explicit appearance of this question will help to eradicate this serious misunderstanding.

8) (i) Many confused the size of the population (3600) and the sample (40). The correct distribution is B(40, 0.225) and not B(3600, 0.225). As elsewhere, some candidates failed to justify the approximation: the correct conditions are either "n large, p close to 0.5" or $np > 5$ and $nq > 5$" It is not $npq > 5$, and the relevant values (in particular, $nq = 31$) should be shown if the inequality is used. Those who started with B(40, 0.225) generally got most of the remaining marks, apart from the usual problem with the continuity correction.

(ii) It is disappointing that there are still candidates who attempt to use "hats" to pick random samples, when almost every Report to Centres for many years has spelt out that candidates are expected to demonstrate that they know about random numbers. There is a distinction between "select using random numbers" (which scores a mark) and "select numbers randomly" (which doesn't). It is also pointed out that taking a three-decimal-place random number such as 0.123 from a calculator and multiplying it by 3600 is a *biased* method as it can produce only 1000 different answers. For further comments, please see the Chief Examiner's Report.

9) (i) Calculation of the critical region was poorly done. For a start, some do not know whether the expression refers to the acceptance or the rejection region. Some, regardless of the fact that they stated the alternative hypothesis as $p > 0.7$, found a left-hand tail such as $R \leq 6$. Weaker candidates attempted to use a normal approximation. Others found the correct entry in the tables but wrote down the wrong region, typically > 11 instead of > 12. In order to obtain full marks it was necessary not only to state the critical region "≥ 13", or equivalent, but also to give a relevant supporting probability.

(ii) This was often poorly done. It was not necessary to do any calculations other than to compare the sample value 12 with the critical region, but many calculated a probability and often failed to find the correct one, which was P(≥ 12) = 0.1608. The conclusion was often not correct; it should be "there is insufficient evidence that the proportion who show a substantial improvement is greater than 0.7" and not "insufficient evidence of a substantial improvement".

(iii) Most knew to use B(14, 0.8), but answers were often not consistent with the critical region in part (i) and there was clearly much confusion. Some candidates do altogether the wrong thing by finding a "new critical region" corresponding to the new distribution, as if they were answering part (i) again.

4734 Probability & Statistics 3

General Comments

The overall standard was again high with only a small number who were unable to attain a reasonable score. Some questions (Q6) required understanding and some tricky algebra both of which were often seen.

Procedures for carrying out hypotheses tests are now well-known, and a majority of candidates give conditions of validity and test conclusion in context.

Printed on the Question Paper in INSTRUCTIONS TO CANDIDATES it is stated that non-exact numerical answers be given correct to 3 significant figures. This applies to requested answers and not intermediate values used to find a later answer.
Some candidates lost marks from the latter.

Comments on Individual Questions

1) (i) This proved to be an easy start, as was expected.

(ii) Only a few candidates used a t-distribution and some calculated a value of σ.

2) This was another high-scoring question. Most use the distribution of $G - M$ and some $M - G$ but there were often associated sign errors here.

3) (i) There were occasional difficulties with the integrals but the given answer often helped. It was sometimes forgotten that e is a constant.

(ii) This involved setting up an equation for Q_3 and the completion involved logs, of which most candidates were aware.

4) The first part involved test for a difference in proportions and here it was a minority of candidates who did not find a pooled estimate for p. The completion of the procedure involved finding the smallest critical value for the test and not just the nearest value found in the tables. This proved to be a discriminator.

5) (i) Most candidates could calculate the confidence interval for p and were usually accurate.

(ii) Some calculated it afresh, despite the *deduce* and the fact that only 1 mark could be earned.

(iii) Many candidates were aware that p is a constant and so probability does not come into the question. We hoped to see *about* 90% *of all such confidence intervals for p will contain p*, or equivalent.

(iv) Most candidates knew what was required.

6) (i) Many needed to show that functions F and G were identical, and they could start with $G(y) = P(Y \le y)$. The main difficulty was with $1 - F(1/y)$, but many managed this successfully.

(ii) Some found differentiating F difficult and a few used $F(x)$ rather than $f(x)$ in finding $E(X+1)$. Part(i) showed that $E(1/X)=E(X)$ which was often realised.

7) (i) It was pleasing that so many candidates knew when to apply Yates' correction.

(ii) The test was mostly applied accurately but Yates' correction was not always correctly applied. *Comment on the result* was often ignored. It was hoped that candidates would note that there was very strong evidence (< ½ % significance level) or that vaccine B appeared to be more successful.

8) (i) Some misunderstood the question, giving conditions more appropriate to Part (ii). A pooled estimate of variance is found when (independent) samples are drawn from populations with a common variance.

(ii) Validity conditions were not always given in context, which then led to the loss of a mark. Not all candidates used a pooled estimate of variance but it was usually found accurately by those who did. Some used a z-distribution rather than t and lost a considerable number of marks.

The confidence interval required the same variance as that for the test but this was not always seen. A different t value was also required. Candidates often seemed unaware that the conditions for validity of the confidence interval were the sam as those for the test.

4736 Decision Mathematics 1

General Comments

A wide range of marks were seen with some thoughtful and carefully worded responses to the more challenging questions from the best candidates.

Candidates who had learnt the basic algorithms and definitions coped well with the first parts of the questions but could not always apply their knowledge to the given situations.

Candidates need to write their answers in the correct spaces in the answer book. If candidates need to rework a part, or cannot fit their answer into the space available, they should use an extra sheet, labelled with the question number and part. In this case it is very helpful if they indicate in the answer booklet that the answer continues on an extra sheet.
Some candidates' writing is almost impossible to read and sometimes candidates appear to misread their own working.

Comments on Individual Questions

1) (i) Generally well answered, except that some candidates forgot to write down the route. Some candidates had given extra temporary labels which incurred a small loss of marks.

(ii) Most candidates identified the odd nodes and paired them. Some only wrote down the minimum pairing and did not show the working for the others, and quite a few gave the total (65+10) instead of stating that 10 were repeated.

(iii) Done well by the candidates who had read the question and understood what was happening, but most candidates just treated this as either asking for the route from part (ii) or asking them to combine the answers from (i) and (ii).

2) (i) Most candidates could find the minimum spanning tree, although most listed the vertices in the order chosen rather than the arcs, as asked for in the question.
Some candidates had clearly used Kruskal's algorithm, and a few used nearest neighbour to construct a path rather than a tree.

(ii) For the lower bound several candidates knew that they needed to add the two shortest arcs from *F* to the weight of the minimum spanning tree for the other five vertices, although a few used 29 and 31 instead of 29 and 30. Some candidates did not appreciate that an extra vertex had been added, and some started again from scratch by deleting *A*, rather than using their previous working.

The upper bound was answered well by the majority of candidates, but some only got as far as *ABDEC* and then either went back to *A*, leaving *F* out, or went to *A* but did not close the cycle by finishing at *A*.

3) (i) Some good answers, but also a number of incorrect responses describing specific cases or assuming that the graph had to be simply connected.

(ii) Several candidates gave minimal answers that only just achieved the mark. Ideally they should have realised that the graph could not be simple because the vertex of order 4 would either connect twice to another vertex or would connect back to itself.

(iii) Most candidates were able to answer this part, although some did not label their vertices and a few did not deal with loops correctly.

(iv) Not well answered in general, with answers often being either vague or sometimes just wrong. Some candidates assumed that the parts were cumulative and ended up with just one specific case.

4) (i) Many very good descriptions, but a few very bad ones and some who did not know the difference between bubble sort and shuttle sort.

(ii) Most candidates managed the first three passes correctly, but several did not carry out a fourth pass. Candidates should be careful to label the results at the end of each pass, rather than expecting examiners to find them in amongst the working.

(iii) Many correct responses, but some candidates put the 2 in the second plank instead of going back and fitting it in the first plank.

(iv) Many correct responses to first-fit decreasing, but the written explanations were often vague and suggested that there was less waste (when the total waste was the same in both cases).

(v) Often answered well, although several candidates thought that there were 6 cuts not 4.

5) (i) Most candidates realised that x, y and z corresponded to 'new', 'occasional' and 'regular' respectively. However few were able to identify that the variables represented the number of parcels of each type checked per hour, even though this was virtually given in the question.

(ii) Most candidates gave the non-negativity constraints, but several were not able to put the other three constraints together, often just giving a single constraint corresponding to the total checking time being no more than an hour. The question had said that the checks were carried out by different people; this meant that each type of check had up to 60 minutes available.

(iii) Several candidates gave up on question 5 at this point. Those who continued usually realised that the value of z could be set as 0 and the effect this had on the objective, but for some reason they tended to treat the situation as if z had a value of 1 in the constraints.

(iv) Few candidates attempted this part, and those who did often had only one non-trivial constraint resulting in graphs that were too simple to be of any use. Candidates should note that the axes need to be scaled and labelled and that using a truncated axis will result in incorrect graphs when it comes to finding the vertices of the feasible region.

In this part of the question the checking was an hour taken from a continuous process, so fractional answers were feasible.

(v) Few candidates attempted this part, those who did often gave the solution to the continuous problem from part (iv). Because there was now just a single hour available this became an integer programming problem. Checking the *P* value at integer points near the boundaries of the feasible region shows that the optimum for this problem is not an integer valued point adjacent to the optimum vertex.

(vi) Very few responses, and those candidates who gave answers usually queried the timings or points values given in the question. Some candidates said that the last parcel would not be able to complete all the checks, but this assumed that the checks were carried out in a fixed order. The issue here was one of resourcing, there may not be enough parcels of the required types available.

6) (i) When attempted this was often done well, although some candidates gave up after sorting out the objective. The question directed candidates to substituting for *a*, *b* and *c* so those who took the answers and worked back to the start were not given the marks.

(ii) The use of the Simplex algorithm was generally done well. Some candidates had the signs wrong in the objective row ($P - 2x + 4y - 5z = 0$) and a few got confused by the position of the 0 in the second constraint. Some candidates omitted the *P* column, but fewer than in previous sessions. The choice of the pivot entry was sometimes haphazard, with negative (and even zero) pivots sometimes being claimed. A few candidates did not indicate the equations being used to form the new rows, these are easiest when expressed in the form *current row* $\pm$ *multiple of (new) pivot row*, although some amount of contraction may be used (eg row 1 + 2 row 3).

4737 Decision Mathematics 2

General Comments

The candidates for this paper were, in general, well prepared and were able to show what they knew. However, as in previous reports, candidates should be reminded to read the questions carefully as several dropped marks for not answering exactly what had been asked.

This is the last paper for this module that will be marked from the actual scripts, in future they will be marked from scanned scripts. Centres should alert their candidates that work drawn on diagrams or tables in pencil or coloured pens will all show up the same as work in black ink.

Comments on Individual Questions

1) (i) Nearly all the candidates were able to draw the bipartite graph correctly.

(ii) Many candidates had problems finding the shortest possible alternating path, $N - A - K - C - O - D$, although some gave the resulting matching even without writing down the path. Candidates were required to write down the matching and write down the path, so diagrams were not accepted as being full solutions.

(iii) Most candidates were able to find a complete matching, although some of them still included the owl.

2) Most candidates followed the instructions to assign a cost of £25 to the missing entries before commencing row and column reduction, and nearly all the candidates did reduce rows first.

A small number of candidates only reduced rows and then set out on an elaborate set of augmentations, when had they reduced columns as well they would only have needed one augmentation.

Some candidates were not able to carry out the augmenting operations correctly, the most common error in this case being to reduce uncrossed values by, say, 4 but only increase the values crossed through twice by 1. A few only augmented by 1 at a time.

Most candidates who successfully achieved a reduced cost matrix were able to give the complete matching, some read the rows and columns the wrong way round (essentially interchanging 'from' and 'to') and some had Amir giving a present to himself.

3) (i) A lot of candidates gave diagrams in which at least one precedence was violated, often having both H and I following from F and G. The diagram needed five dummy activities to preserve all the precedences. Some candidates had large numbers of unnecessary extra dummy activities.

Most candidates drew directed arcs, without these it was very difficult to try to follow through the forward and backward passes.

A tiny number of candidates used activity on node, this has not been in the specification for some time now.

(ii) The passes were usually correct, apart from the odd numerical slip or candidates forgetting to deal with dummy activities correctly.

Most candidates listed the critical activities, although not always correctly, but some did not state the minimum project completion time.

(iii) Most of the resource histograms were correct, or very nearly correct. Only a few had 'holes' or activities hanging out over empty space. Many candidates chose to label the activities, which was not required but could have been helpful to them in answering the next part of the question.

(iv) Some candidates chose to show this on a diagram, which was not required but was acceptable, if correct. Some amended their diagram from part (iii), which sometimes made it very difficult to work out which parts of the diagram were the answers to which parts of the question. Most candidates gave a brief written description of delaying *G* by 2 hours, some did not realise, however, that then *I* had to be moved by an hour because it had to follow *G*.

4) (i) Most candidates gave *B* as the source and *E* as the sink.

(ii) Apart from arithmetic errors, most candidates were able to calculate the capacity of this simple cut.

(iii) The most common correct answer was that the source and sink were both the same side of the cut. Some candidates thought that this could not be a cut because 'all the cut arcs flow vertically' or similar reasoning that suggested that the candidates had only ever experienced situations where the diagram was arranged so that the source was at the left hand side and the sink at the right hand side.

(iv)(a) The upper and lower capacities of arc *DG* are both 3, so the flow must be 3.

(b) The minimum flow from vertex *D* equals the maximum possible flow into *D*, so arc *AD* must be at its upper capacity.

Some candidates did not answer the second request about the consequence for the flow in the arc *AB*.

(c) Explaining why the flow in arc *BC* must be at least 7 required tracing round from *I* to *F* to *C* as well as considering the arc *CE*.

(v) Only a few candidates tried to use a labelling procedure type of diagram. Most candidates tried to show the flows, and several were successful.

(vi)(a) Many candidates were able to find a flow of 19 litres per second.

(b) Many candidates listed the saturated arcs correctly, a few missed out one arc and some duplicated an arc. The required cut was $\{B, C\}$, $\{A, D, E, F, G, H, I\}$

(vii) The candidates who had found a valid flow of 19 litres per second and a valid cut through saturated arcs were usually able to explain how these showed that this was the maximum flow.

5) (i) Most candidates were able to explain what a 'zero-sum' game is. Only a few were able to say that the consequence of this was that there was nothing to be gained by trying to collaborate.

(ii) Some candidates calculated the row minima and column maxima, and sometimes even indicated the row maximin and col minimax, but did not state which strategies were the play-safe choices. Several candidates were able to state that this game was unstable, with a correct reason, but only a few were able to say what 'stable' and 'unstable' mean for the way in which the players play a game in general.

If a game is stable then, in the long run, players cannot expect to do any better than by agreeing to always choose their play-safe strategies. If the game is unstable then the long run optimal strategy will be a mixed strategy, using a randomising method to choose between the different strategies with probabilities that can be determined.

(iii) Many candidates gave the appropriate pair of comparisons, and often they also explained how this showed that circle dominated square on the reduced game. However, some candidates then went on to say that the second player should not choose circle and several candidates either said that 'square should not be played very often' or did not give an interpretation of dominance for the way in which the game is played.

(iv) Most candidates were able to construct the appropriate expressions, although some tried to find expressions for all four possible play choices, some thought that they should be giving expressions for the first player, and some now eliminated circle despite having said that the second player should not play square. The majority of the candidates who had the correct expressions then found the optimal value of p as 0.6, but very few thought to check the extreme values ($p = 0, 1$) as well, the best answers came from the candidates who had given sketch graphs of the expected winnings against p.

(v) The majority of the candidates who attempted this part were able to explain how the given expression had been achieved.

(vi) Almost all candidates who attempted this part were able to substitute correctly into the three expressions. Some then thought that these values were M rather than m, and several chose to calculate M using the greatest value of m instead of the least.

6) (i) Several candidates just added the times in the top row of the table, others added the travel times between birds but did not include travel to and/or from the entrance/exit.

(ii) Many candidates who attempted this part realised that the solution involved visiting the kite twice, or that it missed out the nightjar (which sometimes morphed into a nightingale at this point).

(iii) There were some good answers to this part, explaining how the value 18 had been obtained from the suboptimal minimum for (3, 4(13)) and the value 6 came from the travel time between the kite and the moorhen in the original table.

(iv) Inevitably some candidates were put off attempting this question by the size of the table to be filled in. Those who attempted it were usually able to complete the action column, and several successfully transferred the suboptimal minimum values from stage 3. There were several slips in transferring the travel times from the original table, often candidates slipped a row and used the times from the entrance instead of times from the kite.

Some candidates completed the table correctly but then forgot to write down the journey time or list the birds in the order they should be visited to achieve this time.

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