OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4723

MATHEMATICS

Core Mathematics 3

QUESTION PAPER

THURSDAY 14 JUNE 2012: Morning
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the Centre. The Printed Answer Book may be enlarged by the Centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4723
List of Formulae (MF1)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (SLM/SW) 12MEP06474 65807

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- **The Question Paper will be found in the centre of the Printed Answer Book.**
- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Answer ALL the questions.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**
- **The total number of marks for this paper is 72.**

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

1 Solve the inequality $|2x - 5| > |x + 1|$. [5]

2 It is given that $p = e^{280}$ and $q = e^{300}$.

(i) Use logarithm properties to show that $\ln\left(\frac{ep^2}{q}\right) = 261$. [3]

(ii) Find the smallest integer n which satisfies the inequality $5^n > pq$. [3]

3 It is given that θ is the acute angle such that $\sec\theta \sin\theta = 36\cot\theta$.

(i) Show that $\tan\theta = 6$. [3]

(ii) Hence, using an appropriate formula in each case, find the exact value of

(a) $\tan(\theta - 45°)$, [2]

(b) $\tan 2\theta$. [2]

4 (a) Show that $\int_0^4 \frac{18}{\sqrt{6x+1}}\,dx = 24$. [4]

(b) Find $\int_0^1 (e^x + 2)^2\,dx$, giving your answer in terms of e. [4]

5 **(i)** **It is given that k is a positive constant. By sketching the graphs of**

$$y = 14 - x^2 \text{ and } y = k\ln x$$

on a single diagram, show that the equation

$$14 - x^2 = k\ln x$$

has exactly one real root. [3]

(ii) **The real root of the equation $14 - x^2 = 3\ln x$ is denoted by α.**

(a) **Find by calculation the pair of consecutive integers between which α lies. [3]**

(b) **Use the iterative formula $x_{n+1} = \sqrt{14 - 3\ln x_n}$, with a suitable starting value, to find α. Show the result of each iteration, and give α correct to 2 decimal places. [4]**

6 **The volume, V m^3, of liquid in a container is given by**

$$V = (3h^2 + 4)^{\frac{3}{2}} - 8,$$

where h m is the depth of the liquid.

(i) **Find the value of $\frac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 0.6$, giving your answer correct to 2 decimal places. [4]**

(ii) **Liquid is leaking from the container. It is observed that, when the depth of the liquid is 0.6 m, the depth is decreasing at a rate of 0.015 m per hour. Find the rate at which the volume of liquid in the container is decreasing at the instant when the depth is 0.6 m. [3]**

7 The function f is defined for all real values of x by $\mathrm{f}(x) = 2x + 5$. The function g is defined for all real values of x and is such that $\mathrm{g}^{-1}(x) = \sqrt[3]{x - a}$, where a is a constant. It is given that $\mathrm{fg}^{-1}(12) = 9$. Find the value of a and hence solve the equation $\mathrm{gf}(x) = 68$. [7]

8 (i) Express $3\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence

(a) solve the equation $3\sin\theta + 4\cos\theta + 1 = 0$, giving all solutions for which $-180° < \theta < 180°$, [4]

(b) find the values of the positive constants k and c such that

$$-37 \leqslant k(3\sin\theta + 4\cos\theta) + c \leqslant 43$$

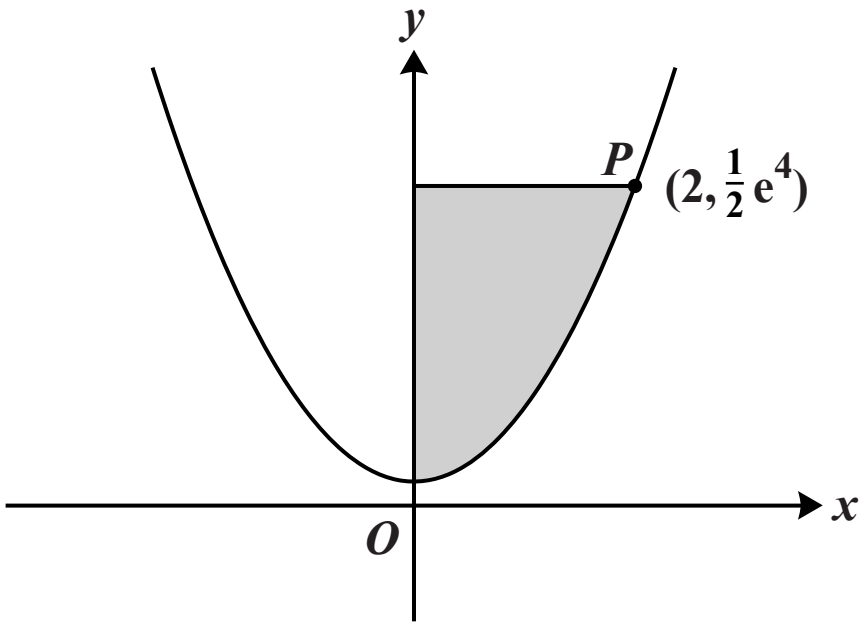
for all values of θ. [4]

9 (i) Show that the derivative with respect to y of

$$y \ln(2y) - y$$

is $\ln(2y)$. [3]

(ii) Look at the following diagram.



The diagram shows the curve with equation $y = \frac{1}{2}e^{x^2}$. The point $P(2, \frac{1}{2}e^4)$ lies on the curve. The shaded region is bounded by the curve and the lines $x = 0$ and $y = \frac{1}{2}e^4$. Find the exact volume of the solid produced when the shaded region is rotated completely about the y-axis. [6]

(iii) Hence find the volume of the solid produced when the region bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$ is rotated completely about the y-axis. [2]