OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE

4725

MATHEMATICS

Further Pure Mathematics 1

QUESTION PAPER

FRIDAY 1 June 2012: Morning
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4725
List of Formulae (MF1)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- **The Question Paper will be found in the centre of the Printed Answer Book.**
- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Answer ALL the questions.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

INFORMATION FOR CANDIDATES

- **This information is the same on the Printed Answer Book and the Question Paper.**
- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**
- **The total number of marks for this paper is 72.**

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

1 The complex numbers z and w are given by $z = 6 - \mathrm{i}$ and $w = 5 + 4\mathrm{i}$. Giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain them, find

(i) $z + 3w$, [2]

(ii) $\frac{z}{w}$. [3]

2 The matrices A and B are given by $\mathrm{A} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$ and $\mathrm{B} = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$. Find

(i) AB, [2]

(ii) $\mathrm{B}^{-1}\mathrm{A}^{-1}$. [3]

3 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is the complex number $4 - 3\mathrm{i}$. Find the values of a and b. [4]

4 Find $\sum_{r=1}^{n} (3r^2 - 3r + 2)$, expressing your answer in a fully factorised form. [7]

5 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} 4\times 3^r = 6(3^n - 1)$. [5]

6 The quadratic equation $2x^2 + x + 5 = 0$ has roots α and β.

(i) Use the substitution $x = \dfrac{1}{u+1}$ to obtain a quadratic equation in u with integer coefficients. [3]

(ii) Hence, or otherwise, find the value of $\left(\dfrac{1}{\alpha} - 1\right)\left(\dfrac{1}{\beta} - 1\right)$. [3]

7 The loci C_1 and C_2 are given by $|z - 3 - 4\mathrm{i}| = 4$ and $|z| = |z - 8\mathrm{i}|$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [6]

(ii) Hence find the complex numbers represented by the points of intersection of C_1 and C_2. [2]

(iii) Indicate, by shading, the region of the Argand diagram for which

$|z - 3 - 4\mathrm{i}| \leqslant 4$ and $|z| \geqslant |z - 8\mathrm{i}|$. [2]

8 (i) Show that $\dfrac{1}{r} - \dfrac{1}{r+2} \equiv \dfrac{2}{r(r+2)}$. [1]

(ii) Hence find an expression, in terms of n, for

$\displaystyle\sum_{r=1}^{n} \frac{2}{r(r+2)}$. [6]

(iii) Given that $\displaystyle\sum_{r=N+1}^{\infty} \frac{2}{r(r+2)} = \frac{11}{30}$, find the value of N. [4]

9 **(i)** **The matrix X is given by $\mathbf{X} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by X. [2]**

(ii) **The matrix Z is given by $\mathbf{Z} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}(2+\sqrt{3}) \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2}(1-2\sqrt{3}) \end{pmatrix}$.**

The transformation represented by Z is equivalent to the transformation represented by X, followed by another transformation represented by the matrix Y. Find Y. [5]

(iii) **Describe fully the geometrical transformation represented by Y. [2]**

10 The matrix D is given by $\mathbf{D} = \begin{pmatrix} a & 2 & -1 \\ 2 & a & 1 \\ 1 & 1 & a \end{pmatrix}$.

(i) Find the determinant of D in terms of a. [3]

(ii) Three simultaneous equations are shown below.

$$ax + 2y - z = 0$$
$$2x + ay + z = a$$
$$x + y + az = a$$

For each of the following values of a, determine whether or not there is a unique solution. If the solution is not unique, determine whether the equations are consistent or inconsistent.

(a) $a = 3$

(b) $a = 2$

(c) $a = 0$ [7]

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