OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE

4723/01

MATHEMATICS

Core Mathematics 3

QUESTION PAPER

WEDNESDAY 23 JANUARY 2013: Morning
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED 18pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4723/01
List of Formulae (MF1)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (LK/SW) 1465581013 72436/1

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- **The Question Paper will be found in the centre of the Printed Answer Book.**
- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Answer ALL the questions.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**
- **The total number of marks for this paper is 72.**
- **The Printed Answer Book consists of 12 pages. The Question Paper consists of 8 pages. Any blank pages are indicated.**

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

1 For each of the following curves, find the gradient at the point with x-coordinate 2.

(i) $y = \dfrac{3x}{2x+1}$ [3]

(ii) $y = \sqrt{4x^2+9}$ [3]

2 The acute angle A is such that $\tan A = 2$.

(i) Find the exact value of $\operatorname{cosec} A$. [2]

(ii) The angle B is such that $\tan(A+B) = 3$. Using an appropriate identity, find the exact value of $\tan B$. [3]

3 (a) Given that $|t| = 3$, find the possible values of $|2t-1|$. [3]

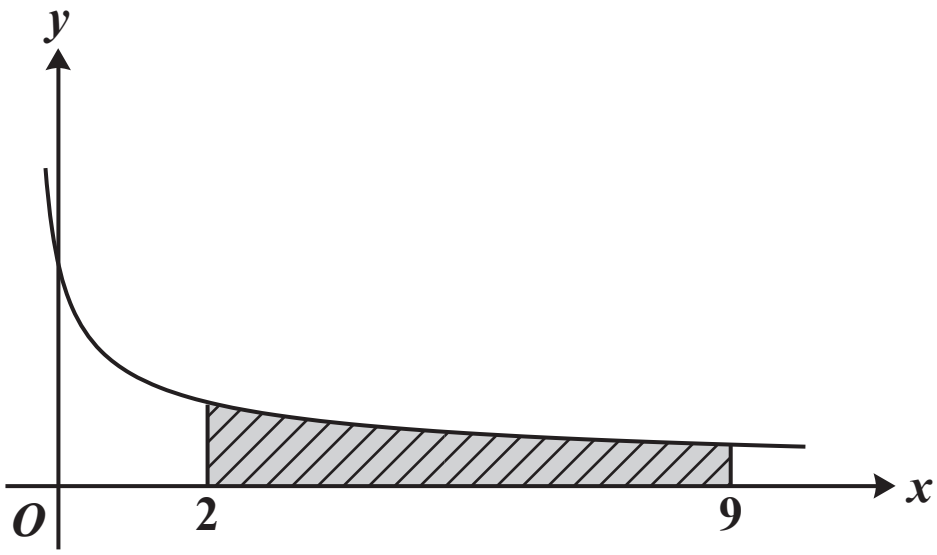
(b) Solve the inequality $|x-\sqrt{2}| > |x+3\sqrt{2}|$. [4]

4 The mass, m grams, of a substance is increasing exponentially so that the mass at time t hours is given by $m = 250e^{0.021t}$.

(i) Find the time taken for the mass to increase to twice its initial value, and deduce the time taken for the mass to increase to 8 times its initial value. [3]

(ii) Find the rate at which the mass is increasing at the instant when the mass is 400 grams. [3]

5 Look at the following diagram.



The diagram above shows the curve $y = \frac{6}{\sqrt{3x+1}}$.

The shaded region is bounded by the curve and the lines $x = 2$, $x = 9$ and $y = 0$.

(i) Show that the area of the shaded region is $4\sqrt{7}$ square units. [4]

(ii) The shaded region is rotated completely about the x-axis. Show that the volume of the solid produced can be written in the form $k\ln 2$, where the exact value of the constant k is to be determined. [5]

6 (i) By sketching the curves $y = \ln x$ and $y = 8 - 2x^2$ on a single diagram, show that the equation

$$\ln x = 8 - 2x^2$$

has exactly one real root. [3]

(ii) Explain how your diagram shows that the root is between 1 and 2. [1]

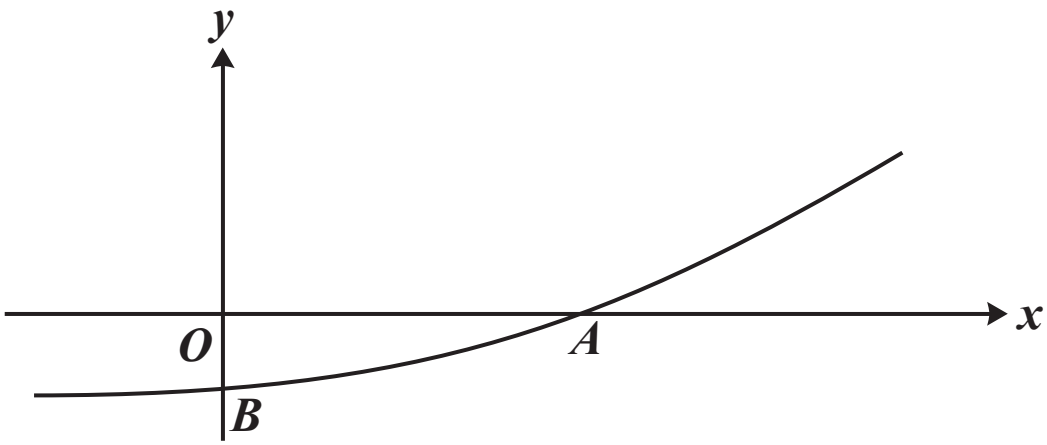
(iii) Use the iterative formula

$$x_{n+1} = \sqrt{4 - \tfrac{1}{2}\ln x_n},$$

with a suitable starting value, to find the root. Show all your working and give the root correct to 3 decimal places. [4]

(iv) The curves $y = \ln x$ and $y = 8 - 2x^2$ are each translated by 2 units in the positive x-direction and then stretched by scale factor 4 in the y-direction. Find the coordinates of the point where the new curves intersect, giving each coordinate correct to 2 decimal places. [3]

7 Look at the following diagram.



The diagram above shows the curve with equation

$x = (y + 4)\ln(2y + 3)$.

The curve crosses the x-axis at A and the y-axis at B.

(i) Find an expression for $\frac{dx}{dy}$ in terms of y. [3]

(ii) Find the gradient of the curve at each of the points A and B, giving each answer correct to 2 decimal places.[5]

8 The functions f and g are defined for all real values of x by

$f(x) = x^2 + 4ax + a^2$ **and** $g(x) = 4x - 2a$,

where a is a positive constant.

(i) Find the range of f in terms of a. [4]

(ii) Given that $fg(3) = 69$, find the value of a and hence find the value of x such that $g^{-1}(x) = x$. [6]

9 (i) Prove that

$$\cos^2(\theta + 45°) - \tfrac{1}{2}(\cos 2\theta - \sin 2\theta) \equiv \sin^2\theta.$$ **[4]**

(ii) Hence solve the equation

$$6\cos^2(\tfrac{1}{2}\theta + 45°) - 3(\cos\theta - \sin\theta) = 2$$

for $-90° < \theta < 90°$. **[3]**

(iii) It is given that there are two values of θ**, where** $-90° < \theta < 90°$**, satisfying the equation**

$$6\cos^2(\tfrac{1}{3}\theta + 45°) - 3(\cos\tfrac{2}{3}\theta - \sin\tfrac{2}{3}\theta) = k,$$

where k **is a constant. Find the set of possible values of** k**.** **[3]**