OCR
RECOGNISING ACHIEVEMENT

Friday 24 May 2013 – Morning

AS GCE MATHEMATICS

4736/01   Decision Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
*   Printed Answer Book 4736/01
*   List of Formulae (MF1)

Other materials required:
*   Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

*   The Question Paper will be found in the centre of the Printed Answer Book.
*   Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
*   **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
*   Use black ink. HB pencil may be used for graphs and diagrams only.
*   Answer **all** the questions.
*   Read each question carefully. Make sure you know what you have to do before starting your answer.
*   Do **not** write in the bar codes.
*   You are permitted to use a scientific or graphical calculator in this paper.
*   Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

*   The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
*   **You are reminded of the need for clear presentation in your answers.**
*   The total number of marks for this paper is **72**.
*   The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

*   Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

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1 The list below is to be sorted into increasing order using bubble sort, starting at the left-hand end of the list.

<div align="center">24   57   9   31   16   4</div>

    (i) Show which values are compared and which are swapped in the first pass. Write down the list that results at the end of the first pass. **[2]**

    (ii) Without showing the individual comparisons and swaps, write down the lists that result after the second pass and after the third pass. **[2]**

    (iii) In total there will be five passes made in carrying out bubble sort on the list. Write down how many swaps are made in each pass. **[2]**

2 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself. A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex. A *simply connected* graph is one that is both simple and connected.

    (i) (a) Draw a connected Eulerian graph that has exactly four vertices and five arcs but is not simple. **[1]**

        (b) Explain why it is not possible to have a simply connected Eulerian graph with exactly four vertices and five arcs. **[2]**

A simply connected Eulerian graph is drawn that has exactly eight vertices and ten arcs.

    (ii) (a) Explain how you know that the sum of the vertex orders must be 20. **[1]**

        (b) Write down the minimum and maximum possible vertex order and draw a diagram that includes both the minimum and the maximum cases. **[3]**

        (c) Draw a diagram to show a simply connected Eulerian graph with exactly eight vertices and ten arcs in which the number of vertices of order 4 is as large as possible. **[1]**

3 Holly has written an algorithm.

Step 1   Input two positive integers A and B
Step 2   Let $C = A - B$
Step 3   If $C < 0$, let $D = B$ then let $E = B + C$
Step 4   If $C = 0$, jump to Step 10
Step 5   If $C > 0$, let $D = A$ and let $E = B$
Step 6   Let $F = D - E$
Step 7   If $F < 0$, let $D = E$ then let $E = F + D$ and go back to Step 6
Step 8   If $F = 0$, let $F = D$ then jump to Step 11
Step 9   If $F > 0$, let $D = F$ then go back to Step 6
Step 10   Let $F = A$
Step 11   Let $G = A \div F$
Step 12   Let $M = G \times B$
Step 13   Print the values F and M

(i) Work through Holly's algorithm using the input values $A = 30$ and $B = 18$. Set out your working using the table in the answer book. Use one row for each step where any values change and only write down values when they change. Write down the values that are printed. **[6]**

(ii) Describe what happens when $A = 18$ and $B = 30$. You need only record enough rows of the table to be able to show what happens. **[2]**

(iii) Without doing further working, state the output values of F and M when $A = 12$ and $B = 8$. **[2]**

4 A simplified map of an area of moorland is shown below. The vertices represent farmhouses and the arcs represent the paths between the farmhouses. The weights on the arcs show distances in km.
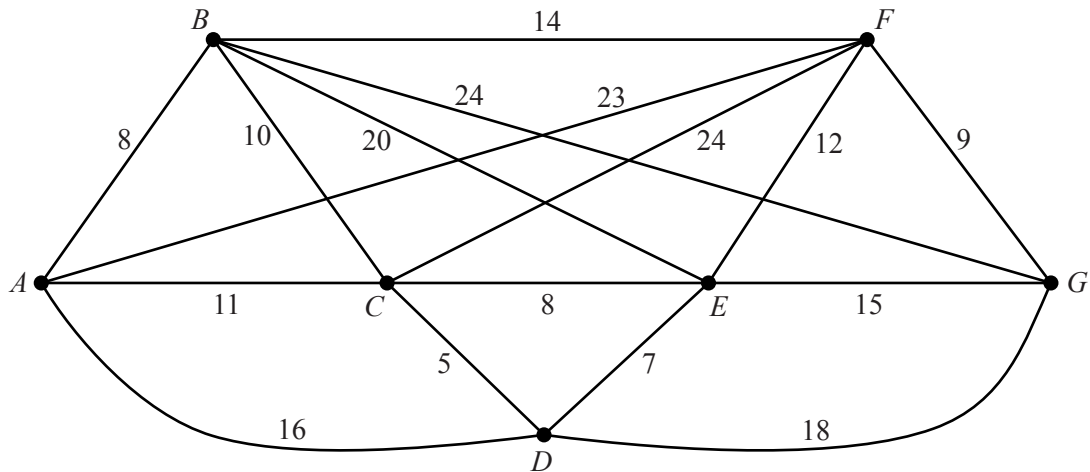
Ted wants to visit each farmhouse and then return to his starting point.

(i) In your answer book the arcs have been sorted into increasing order of weight. Use Kruskal's algorithm to find a minimum spanning tree for the network, and give its total weight. Hence find a route visiting each farmhouse, and returning to the starting point, which has length 82 km. **[4]**

(ii) Give the weight of the minimum spanning tree for the six vertices *A*, *B*, *C*, *E*, *F*, *G*. Hence find a route visiting each of the seven farmhouses once, and returning to the starting point, which has length 81 km. **[2]**

(iii) Show that the nearest neighbour method fails to produce a **cycle** through every vertex when started from *A* but that it succeeds when started from *B*. Adapt this cycle to find a complete cycle of total weight less than 70, and find the total length of the shorter cycle. **[6]**

5 This question uses the same network as question **4**. The total weight of the arcs in the network is 224.
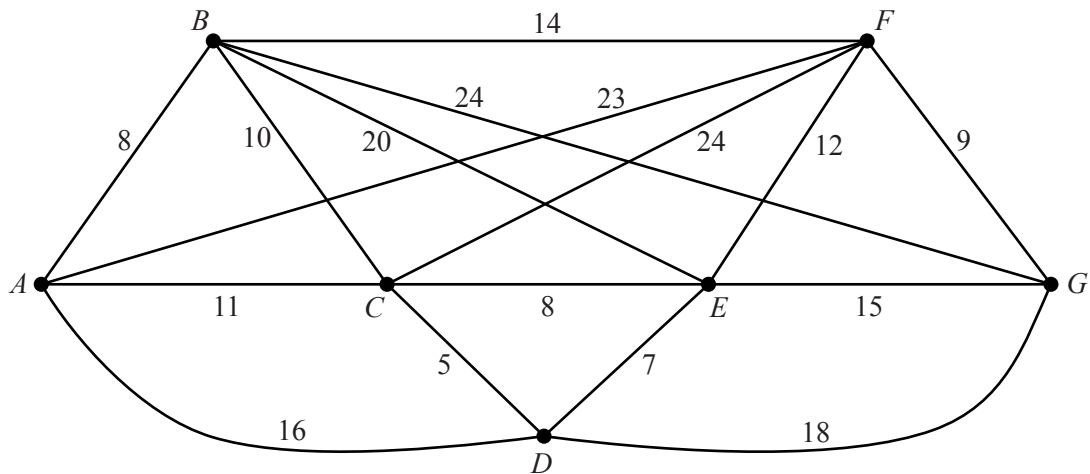
(i) Apply Dijkstra's algorithm to the network, starting at *A*, to find the shortest route from *A* to *G*. **[5]**

(ii) Dijkstra's algorithm has quadratic order (order n^2). It takes 2.25 seconds for a certain computer to apply Dijkstra's algorithm to a network with 7 vertices. Calculate approximately how many hours it will take to apply Dijkstra's algorithm to a network with 1400 vertices. **[2]**

(iii) How much shorter would the path *CE* need to be for it to become part of a shortest path from *A* to *G*? **[2]**

Following a landslip, the paths *AC* and *CE* become blocked and cannot be used. A warden needs to travel along all the remaining paths to check that there are no more landslips.

(iv) Find the shortest distance that the warden must travel, assuming that she starts and ends at vertex *C*. Show your working. **[6]**

6   Consider the following linear programming problem.

Maximise $P = 5x + 8y$,

subject to $3x - 2y \leqslant 12$,
$\qquad\qquad\ 3x + 4y \leqslant 30$,
$\qquad\qquad\ 3x - 8y \geqslant -24$,
$\qquad\qquad\ x \geqslant 0, y \geqslant 0$.

(i)   Represent the constraints graphically. Shade the regions where the inequalities are **not** satisfied and hence identify the feasible region. **[4]**

(ii) Calculate the coordinates of the vertices of the feasible region, apart from the origin, and calculate the value of P at each vertex. Hence find the optimal values of x and y, and state the maximum value of the objective. **[4]**

(iii) Suppose that, additionally, x and y must both be integers. Find the maximum feasible value of y for **every** feasible integer value of x. Calculate the value of P at each of these points and hence find the optimal values of x and y with this additional restriction. **[4]**

Tom wants to use the Simplex algorithm to solve the original (non-integer) problem. He reformulates it as follows.

Maximise $P = 5x + 8y$,

subject to $3x - 2y \leqslant 12$,
$\qquad\qquad\ 3x + 4y \leqslant 30$,
$\qquad\qquad -3x + 8y \leqslant 24$,
$\qquad\qquad\ x \geqslant 0, y \geqslant 0$.

(iv) Explain why Tom needed to change the third constraint. **[1]**

(v) Represent the problem by an initial Simplex tableau. **[2]**

(vi) Perform **one** iteration of the Simplex algorithm, choosing your pivot from the **y column**. Show how the pivot row was used to calculate the other rows. Write down the values of x, y and P that result from this iteration.

Perform a second iteration of the Simplex algorithm to achieve the optimum point. **[6]**

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