OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4737/01

MATHEMATICS

Decision Mathematics 2

QUESTION PAPER

FRIDAY 17 MAY 2013: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4737/01
List of Formulae (MF1)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (NH/CGW) 2452951280 80226/1

INSTRUCTIONS TO CANDIDATES

- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.**
- **If you use the Printed Answer Book, write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Answer ALL the questions.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

INFORMATION FOR CANDIDATES

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**
- **The total number of marks for this paper is 72.**

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

1 **Six students, Alice (A), Beth (B), Correy (C), Drew (D), Edmund (E) and Fred (F) are moving into a shared house together at university. They have viewed the house and now they are choosing their rooms.**

Alice wants a room at the front of the house, so that she can keep a check on her car, but she does not want a ground floor room; Beth wants a room at the back of the house; Correy wants a ground floor room; Drew wants a first floor room, and he does not want a small room because he needs enough space for his drum kit; Edmund wants a room that is next to the bathroom; Fred wants a large room on the first floor, because he thinks that will be the best room in the house.

The table below shows the features of each of the six rooms available.

Room	Floor	Front or back	Size of room	Next to
1	Ground	Front	Small	Lounge
2	Ground	Back	Large	Kitchen
3	First	Back	Medium	Stairs
4	First	Front	Large	Stairs
5	First	Back	Medium	Bathroom
6	First	Front	Small	Bathroom

(i) **Draw a bipartite graph to show the possible pairings between the students (A, B, C, D, E and F) and the rooms (1, 2, 3, 4, 5 and 6). [2]**

Initially Alice chooses room 4, Beth chooses room 3 and Correy chooses room 2.

(ii) **Show how the remaining rooms can be allocated so that two of Drew, Edmund and Fred are happy with the choices. State which student is not happy with this arrangement. Draw a second bipartite graph to show the resulting incomplete matching between five of the students and five of the rooms. [2]**

(iii) **Construct an alternating path, starting from the student without a room and ending at the room that was not used, and hence find a complete matching between the students and the rooms. Write down a list showing which student should be given which room. [2]**

2 **Alice (*A*), Beth (*B*), Correy (*C*), Drew (*D*), Edmund (*E*) and Fred (*F*) live in a shared student house. They have decided that they will each do the cooking one day of the week, and on Saturdays they will have a takeaway meal. They each give a score from 1 to 10 to the different days of the week. The higher the score the more the student would like to cook on that day. The results are shown below.**

Day

Student		Sun	Mon	Tue	Wed	Thur	Fri
	Alice (*A*)	10	6	10	8	5	5
	Beth (*B*)	6	2	10	1	4	6
	Correy (*C*)	10	9	8	4	2	10
	Drew (*D*)	9	4	9	5	5	8
	Edmund (*E*)	9	8	9	8	7	8
	Fred (*F*)	3	4	10	3	1	4

(i) **Explain how to modify the table so that the Hungarian algorithm can be used to find the matching for which the total score is maximised. [1]**

(ii) **Show that modifying the table, and then reducing rows first and columns second gives the reduced cost matrix below. [3]**

		Day					
		Sun	**Mon**	**Tue**	**Wed**	**Thur**	**Fri**
Student	**Alice (A)**	**0**	**3**	**0**	**1**	**3**	**5**
	Beth (B)	**4**	**7**	**0**	**8**	**4**	**4**
	Correy (C)	**0**	**0**	**2**	**5**	**6**	**0**
	Drew (D)	**0**	**4**	**0**	**3**	**2**	**1**
	Edmund (E)	**0**	**0**	**0**	**0**	**0**	**1**
	Fred (F)	**7**	**5**	**0**	**6**	**7**	**6**

(iii) **Complete the application of the Hungarian algorithm, stating how each table was formed.**
Use your final matrix to decide on which day Fred cooks. [5]

Suppose that Alice cooks on Sunday.

(iv) **Write a list showing on which day each student cooks according to the matching in which the total score is maximised. [2]**

3 Molly is trying to book a holiday cottage for a short holiday. The activities involved, their durations (in minutes) and immediate predecessors are listed in the table below.

Activity	**Duration (mins)**	**Immediate predecessors**
A **Choose a weekend for the holiday**	**10**	**–**
B **Decide on a region to visit for the holiday**	**5**	**–**
C **Look at maps and find suitable locations**	**20**	***B***
D **Go online and find out what cottages are available**	**15**	***A, C***
E **Find out what there is to do near each location**	**25**	***C***
F **Decide how much she wants to spend on the cottage**	**5**	***D***
G **Look up train services to the nearest stations**	**15**	***A, C***
H **Choose a cottage and book it**	**30**	***E, F, G***

(i) **Draw an activity network, using activity on arc, to represent the project. [2]**

(ii) **Carry out a forward pass and a backward pass through the activity network, showing the early event time and the late event time at each vertex of your network. [3]**

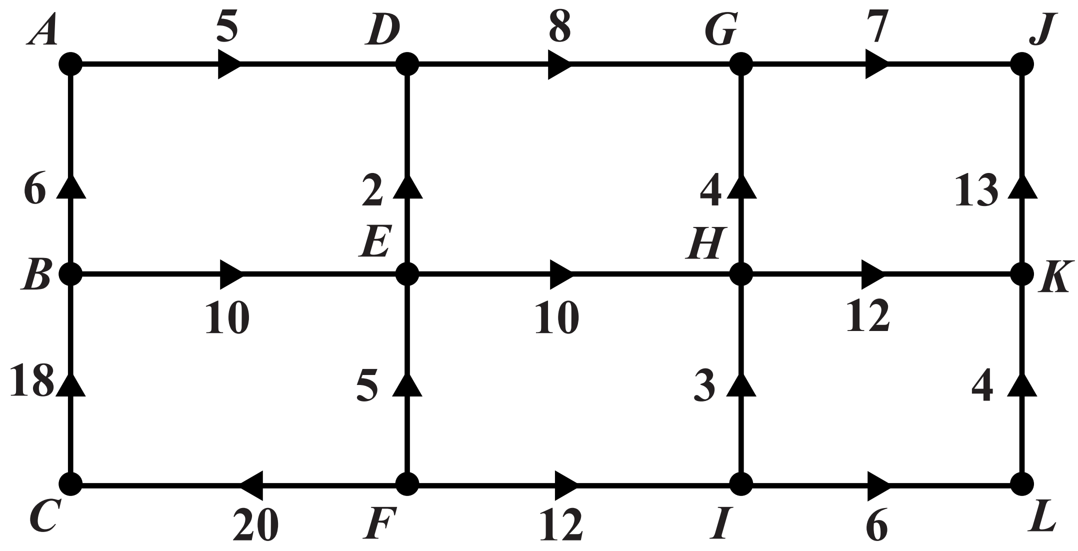
(iii) **State the minimum project completion time, assuming that Molly can share the tasks out with her friends, and list the critical activities. [2]**

Each activity requires one person and activities cannot be shared between people. Molly must do activities *A*, *B*, *F* and *H* herself.

(iv) **Draw up a schedule to show how Molly and two friends can complete the project in the minimum project completion time. [2]**

(v) **Draw up a schedule to show how Molly and just one friend can complete the project in just 5 minutes more than the minimum project completion time from part (iii). [2]**

4 The network below represents a system of pipes through which oil can flow. The weights on the arcs show the capacities for the pipes, in litres per second.



(i) Explain how you know that the source is at *F* and identify the sink. [2]

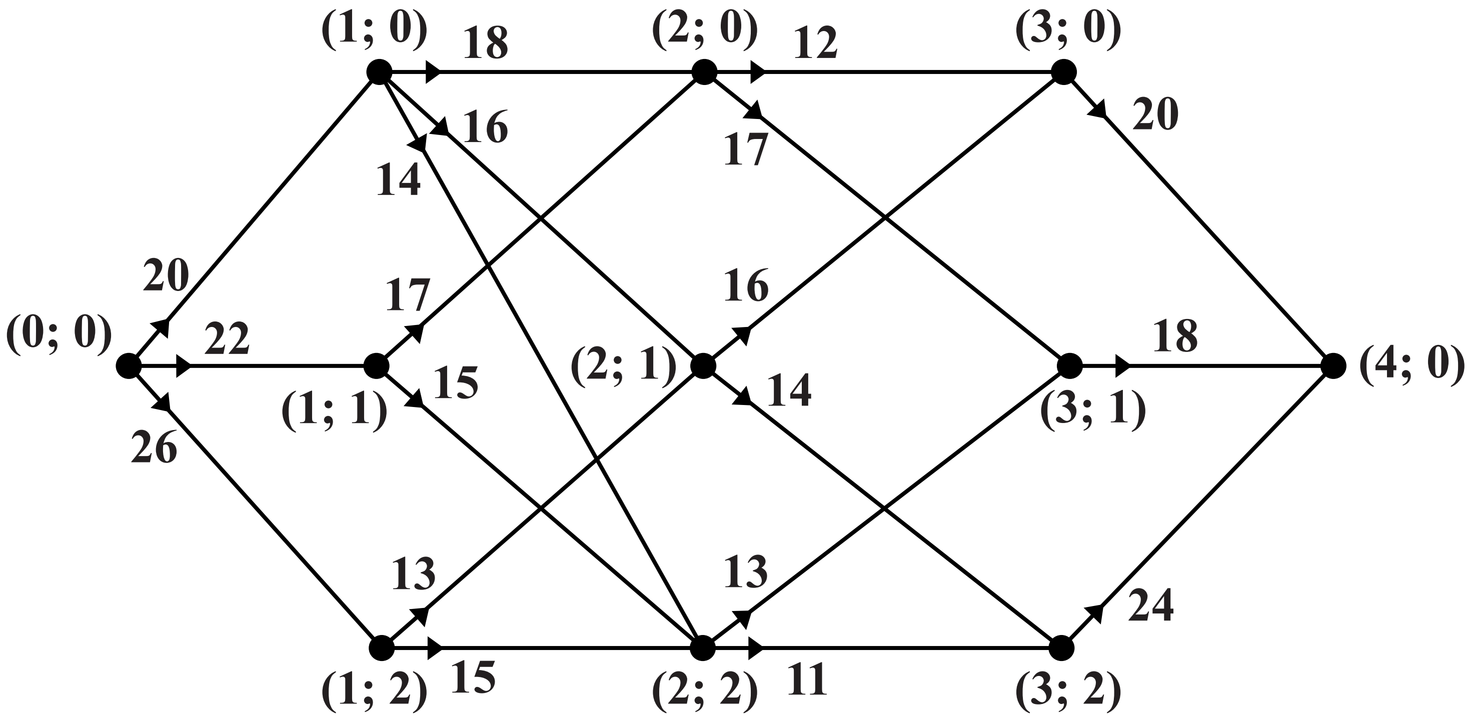
(ii) Calculate the capacity of the cut that separates $\{A, B, C, D, E, F\}$ from $\{G, H, I, J, K, L\}$. [1]

(iii) Explain why in any feasible flow pipe *BA* cannot be at its full capacity.
Explain why the flow in pipe *FC* cannot exceed 15 litres per second. [3]

(iv) Show a flow through the system in which the flows in pipes *FC*, *IL* and *KJ* are as large as possible and nothing flows in pipes *ED*, *FE* and *IH*. [2]

(v) Using your flows from part (iv), label the arrows on the diagram in the answer book to show the excess capacities and the potential backflows. Write down a flow augmenting path but DO NOT update the excess capacities and potential backflows on your diagram. Hence state the maximum flow through the system, in litres per second, and write down a cut that shows that the flow is maximal. [5]

5 A delivery company needs to transport heavy loads from its base to a depot. Each of the roads which it can use has a maximum weight limit. The directed network below represents the roads which can be used to get from the base to the depot. Road junctions are labelled with (stage; state) labels. The weights on the arcs represent weight limits in tonnes.



(i) Explain what a maximin route is. [1]

(ii) Set up a dynamic programming tabulation, working backwards from stage 4, to find the maximum load which one truck can carry (in tonnes, including the weight of the truck) in one journey from the base to the depot. Find all the routes for which this is the maximin. [12]

(iii) The road connecting (2; 0) to (3; 1) is to be strengthened so that it can carry 20 tonnes. Find the maximum load which one truck will be able to carry (in tonnes, including the weight of the truck), explaining how you know that no greater load can be carried. (Do NOT use a dynamic programming tabulation for this part.) [2]

6 **A team from the Royal Hotel have challenged a team from the Carlton Hotel to a darts competition. In this competition there are seven rounds, and the teams must choose a player for each round, although players may play more than one round.**

Each round is made up of five 'legs' and the team that WINS THE MOST LEGS (out of 35) wins the competition. On the basis of their performances so far, the number of legs that each member of the Royal team can expect to win (out of five) against each member of the Carlton team is shown below.

		Carlton		
	Legs won	**Jeff**	**Kathy**	**Leo**
	Greg	**2**	**3**	**4**
Royal	**Hakkim**	**4**	**3**	**1**
	Iona	**1**	**0**	**3**

The teams want to choose which player to put in for each round to maximise the number of legs they expect to win.

(i) **If the Royal team chooses Greg and the Carlton team chooses Jeff, how many legs will the Carlton team expect to win? [1]**

To convert the game into a zero-sum game, each value in the table is doubled and then 5 is subtracted.

(ii) **Construct the resulting table for the zero-sum game. [1]**

(iii) **Find the play-safe strategies for the zero-sum game, showing your working. Explain how you know that the game is not stable. State which player is best for the Carlton team if they know that the Royal team will play safe. [5]**

(iv) Use a dominance argument to explain why the Royal team should not choose Iona. [1]

The Royal team chooses a player for the next round by using random numbers to choose between Greg and Hakkim, where the probability of choosing Greg is p and the probability of choosing Hakkim is $1-p$.

(v) Show that the expected number of LEGS that the Royal team win when the Carlton team chooses Jeff is given by $4-2p$ and find the corresponding expressions for when Kathy is chosen and when Leo is chosen. [3]

(vi) Use a graphical method to find the optimal value of p for the Royal team, and calculate how many legs the Royal team can expect when this value of p is used. [4]

Suppose, instead, the team that WINS THE MOST ROUNDS wins the competition. The winner of each round is the team that wins the most legs (out of five) in that round and there are still seven rounds in the competition.

(vii) Give an example to show that it is possible to win the most legs without winning the most rounds. [1]

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