OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE

4726/01

MATHEMATICS

Further Pure Mathematics 2

QUESTION PAPER

THURSDAY 13 JUNE 2013: Morning
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or on any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4726/01
List of Formulae (MF1)
Insert for question 5 (inserted)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (AC/SW) 2376015131 79877/1

INSTRUCTIONS TO CANDIDATES

- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.**
- **IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **Answer ALL the questions.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

INFORMATION FOR CANDIDATES

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**
- **The total number of marks for this paper is 72.**
- **Any blank pages are indicated.**

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

1 By using the substitution $t = \tan\frac{1}{2}\theta$, find $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{1}{1+\cos\theta}\,\mathrm{d}\theta$. [5]

2 (i) Using the definitions for $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x}, show that $\cosh^2 x - \sinh^2 x \equiv 1$. [3]

(ii) Hence solve the equation $\sinh^2 x = 5\cosh x - 7$, giving your answers in logarithmic form. [5]

3 It is given that $\mathrm{f}(x) = \tanh^{-1}\left(\frac{1-x}{3+x}\right)$ for $x > -1$.

(i) Show that $\mathrm{f}''(x) = \frac{1}{2(x+1)^2}$. [6]

(ii) Hence find the Maclaurin series for $\mathrm{f}(x)$ up to and including the term in x^2. [4]

4 It is given that $I_n = \displaystyle\int_0^{\frac{1}{2}\pi} \cos^n x\,\mathrm{d}x$ for $n \geqslant 0$.

(i) Show that $I_n = \frac{n-1}{n} I_{n-2}$ for $n \geqslant 2$. [5]

(ii) Hence find I_{11} as an exact fraction. [3]

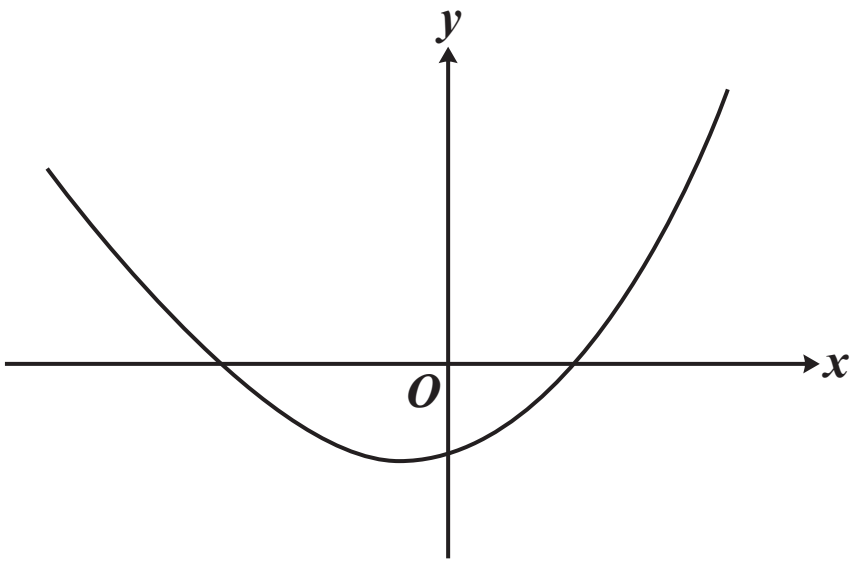
5 You are given that the equation $x^3 + 4x^2 + x - 1 = 0$ has a root, α, where $-1 < \alpha < 0$.

(i) Show that the Newton-Raphson iterative formula for this equation can be written in the form

$$x_{n+1} = \frac{2x_n^3 + 4x_n^2 + 1}{3x_n^2 + 8x_n + 1}.$$ [3]

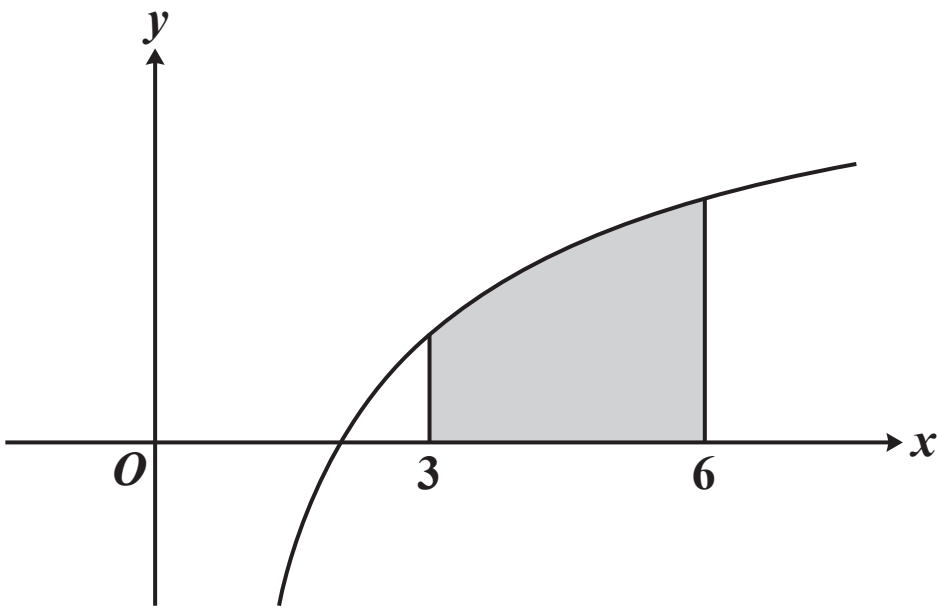
(ii) Using the initial value $x_1 = -0.7$, find x_2 and x_3 and find α correct to 5 decimal places. [3]

(iii) The diagram below shows a sketch of the curve $y = x^3 + 4x^2 + x - 1$ for $-1.5 \leqslant x \leqslant 1$.



Using the copy of the diagram on the insert provided or the copy in your answer book, explain why the initial value $x_1 = 0$ will fail to find α. [2]

6 Look at the following diagram.



The diagram above shows part of the curve $y = \ln\big(\ln(x)\big)$. The region between the curve and the x-axis for $3 \leqslant x \leqslant 6$ is shaded.

(i) By considering n rectangles of equal width, show that a lower bound, L, for the area of the shaded region is $\frac{3}{n}\sum_{r=0}^{n-1} \ln\left(\ln\left(3 + \frac{3r}{n}\right)\right)$. [3]

(ii) By considering another set of n rectangles of equal width, find a similar expression for an upper bound, U, for the area of the shaded region. [1]

(iii) Find the least value of n for which $U - L < 0.001$. [4]

7 The equation of a curve is $y = \dfrac{x^2 + 1}{(x+1)(x-7)}$.

(i) Write down the equations of the asymptotes. [3]

(ii) Find the coordinates of the stationary points on the curve. [5]

(iii) Find the coordinates of the point where the curve meets one of its asymptotes. [3]

(iv) Sketch the curve. [3]

8 The equation of a curve is $x^2 + y^2 - x = \sqrt{x^2 + y^2}$.

(i) Find the polar equation of this curve in the form $r = \mathrm{f}(\theta)$. [3]

(ii) Sketch the curve. [2]

(iii) The line $x + 2y = 2$ divides the region enclosed by the curve into two parts. Find the ratio of the two areas. [6]