OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE

4724/01

MATHEMATICS

Core Mathematics 4

QUESTION PAPER

TUESDAY 18 JUNE 2013: Morning
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR supplied materials:

Printed Answer Book 4724/01
List of Formulae (MF1)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

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INSTRUCTIONS TO CANDIDATES

- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.**
- **IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Answer ALL the questions.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **Do NOT write in the bar codes.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

INFORMATION FOR CANDIDATES

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**
- **The total number of marks for this paper is 72.**

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 Express $\dfrac{(x-7)(x-2)}{(x+2)(x-1)^2}$ in partial fractions. [5]

2 Find $\int x^8 \ln(3x)\,\mathrm{d}x$. [5]

3 Determine whether the lines whose equations are

$$\mathbf{r} = (1+2\lambda)\mathbf{i} - \lambda\mathbf{j} + (3+5\lambda)\mathbf{k}$$
$$\text{and } \mathbf{r} = (\mu-1)\mathbf{i} + (5-\mu)\mathbf{j} + (2-5\mu)\mathbf{k}$$

are parallel, intersect or are skew. [6]

4 The equation of a curve is $y = \cos 2x + 2\sin x$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and hence find the coordinates of the stationary points on the curve for $0 < x < \pi$. [6]

5 (i) Show that $\dfrac{1}{1-\tan x} - \dfrac{1}{1+\tan x} \equiv \tan 2x$. [2]

(ii) Hence evaluate $\displaystyle\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi} \left(\frac{1}{1-\tan x} - \frac{1}{1+\tan x} \right) \mathrm{d}x$, giving your answer in the form $a \ln b$. [5]

6 Use the substitution $u = 1 + \ln x$ to find $\displaystyle\int \frac{\ln x}{x(1+\ln x)^2}\,\mathrm{d}x$. [6]

7 For this question you are given the following formula.
The volume of a pyramid is
$V = \frac{1}{3} \times$ base area $\times$ perpendicular height.

Points A (2, 2, 5), B (1, −1, −4), C (3, 3, 10) and D (8, 6, 3) are the vertices of a pyramid with a triangular base.

(i) Calculate the lengths AB and AC, and the angle BAC. [4]

(ii) Show that $\overrightarrow{AD}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [3]

(iii) Calculate the volume of the pyramid $ABCD$. [3]

8 For this question you are given the following formula.
The volume of a sphere is $V = \frac{4}{3}\pi r^3$.

At time t seconds, the radius of a spherical balloon is r cm. The balloon is being inflated so that the rate of increase of its radius is inversely proportional to the square root of its radius. When $t = 5$, $r = 9$ and, at this instant, the radius is increasing at 1.08 cm s^{-1}.

(i) Write down a differential equation to model this situation, and solve it to express r in terms of t. [7]

(ii) How much air is in the balloon initially? [2]

9 A curve has parametric equations $x = \frac{1}{t} - 1$ and $y = 2t + \frac{1}{t^2}$.

(i) Find $\frac{dy}{dx}$ in terms of t, simplifying your answer. [3]

(ii) Find the coordinates of the stationary point and, by considering the gradient of the curve on either side of this point, determine its nature. [4]

(iii) Find a cartesian equation of the curve. [2]

10 **(i)** Show that $\frac{x}{(1-x)^3} \approx x + 3x^2 + 6x^3$ for small values of x. [2]

(ii) Use this result, together with a suitable value of x, to obtain a decimal estimate of the value of $\frac{100}{729}$. [2]

(iii) Show that $\frac{x}{(1-x)^3} = -\frac{1}{x^2}\left(1 - \frac{1}{x}\right)^{-3}$. Hence find the first three terms of the binomial expansion of $\frac{x}{(1-x)^3}$ in powers of $\frac{1}{x}$. [4]

(iv) Comment on the suitability of substituting the same value of x as used in part (ii) in the expansion in part (iii) to estimate the value of $\frac{100}{729}$. [1]

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