OCR
Oxford Cambridge and RSA

Tuesday 19 June 2018 – Afternoon

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 Use Simpson's rule with four strips to find an approximation to

$$\int_1^5 e^{\frac{2}{x}}\,dx.$$ **[3]**

2 Solve the inequality $|4x+3| < |x-8|$, showing all your working. **[5]**

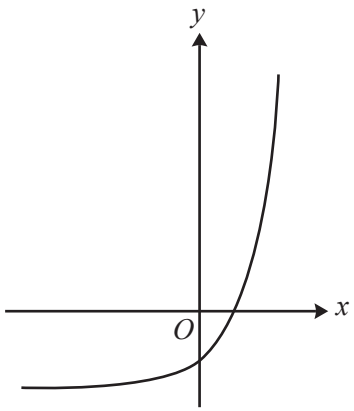
3 A curve has equation $y = 3\ln(2x-a)$, where a is a positive constant. The curve crosses the x-axis at the point P.

(i) Sketch the curve and determine the x-coordinate of P in terms of a. **[3]**

(ii) Find an equation of the tangent to the curve at P. **[3]**

4 A curve has equation $y = \dfrac{2x^2+1}{x^4+30}$. Find $\dfrac{dy}{dx}$ and hence determine the exact coordinates of the stationary points on the curve. **[7]**

5



The diagram shows the curve $y = f(x)$, where f is the function defined for all real values of x by $f(x) = e^{2x} - 3$.

(i) State the range of f. **[1]**

(ii) Find an expression for $f^{-1}(x)$. **[2]**

(iii) The curve $y = e^x$ can be transformed to the curve $y = f^{-1}(x)$ by means of a stretch, a translation and a reflection in that order. Give details of these three transformations. **[3]**

(iv) Sketch the curve $y = |f(x)|$. Given that the equation $|f(x)| = k$ has two distinct roots, determine the set of possible values of the constant k. **[3]**

6 **(a)** A reservoir is being filled with water at a constant rate of 15 cubic metres per minute. At the instant when the depth of the water is x metres, the volume of water in the reservoir is V cubic metres where

$$V = 2(5+2x)^3 - 250.$$

Find the rate at which the depth of the water is increasing at the instant when $x = 1.6$. **[4]**

(b) In an experiment, the mass of a substance is increasing exponentially. At a time t hours after the start of the experiment, the mass, m grams, of the substance is given by

$$m = A\mathrm{e}^{\lambda t},$$

where A and λ are constants. It is given that, at the instant when $t = 15$, the mass is 48 grams and the rate at which the mass is increasing is 1.2 grams per hour.

(i) Find the values of A and λ. **[4]**

(ii) Find the value of t for which the mass is 70 grams. **[2]**

7 It is given that there is exactly one value of x, where $0 < x < \pi$, that satisfies the equation

$$3\tan 2x - 8\tan x = 4.$$

(i) Show that $t = \sqrt[3]{\frac{1}{2} + \frac{1}{4}t - \frac{1}{2}t^2}$, where $t = \tan x$. **[3]**

(ii) Show by calculation that the value of t satisfying the equation in part **(i)** lies between 0.7 and 0.8. **[2]**

(iii) Use an iterative process based on the equation in part **(i)** to find the value of t correct to 4 significant figures. Use a starting value of 0.75 and show the result of each iteration. **[3]**

(iv) Solve the equation $3\tan 4y - 8\tan 2y = 4$ for $0 < y < \frac{1}{2}\pi$. **[2]**

8 **(a)** Given that α satisfies the equation

$$3\sin(\alpha + 60°) - 3\cos(\alpha + 30°) = \mathrm{cosec}^2\alpha,$$

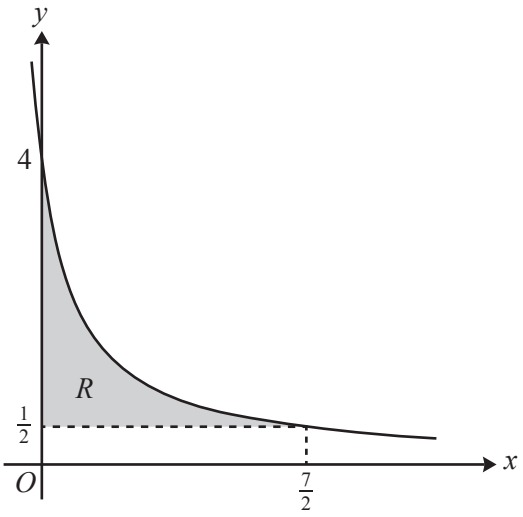
find the exact value of $\sin\alpha$. **[4]**

(b) It is given that β satisfies the equation

$$\sin 4\beta \sec^3\beta = 8\sin\beta + 2.$$

By first expressing $\sin 4\beta$ in terms of $\sin 2\beta$ and $\cos 2\beta$, find the exact value of $\sin\beta$. **[7]**

9



The diagram shows part of the curve $y = \dfrac{4}{2x+1}$. The shaded region R is enclosed by the curve and the lines $x = 0$ and $y = \frac{1}{2}$.

(i) Find the exact area of R, giving your answer in the form $a \ln 2 + b$ where a and b are constants. **[4]**

(ii) The region R is rotated completely about the ***y*-axis**. Find the exact volume of the solid produced, giving your answer in the form $c \ln 2 + d$ where c and d are constants. **[7]**

END OF QUESTION PAPER