OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4737/01

MATHEMATICS

Decision Mathematics 2

QUESTION PAPER

WEDNESDAY 23 MAY 2018: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED 24pt

Candidates answer on the Printed Answer Book.

OCR SUPPLIED MATERIALS:
Printed Answer Book 4737/01
List of Formulae (MF1) sent with the standard paper

OTHER MATERIALS REQUIRED:
Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (LEG/TP) 168294/1

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.

WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer ALL the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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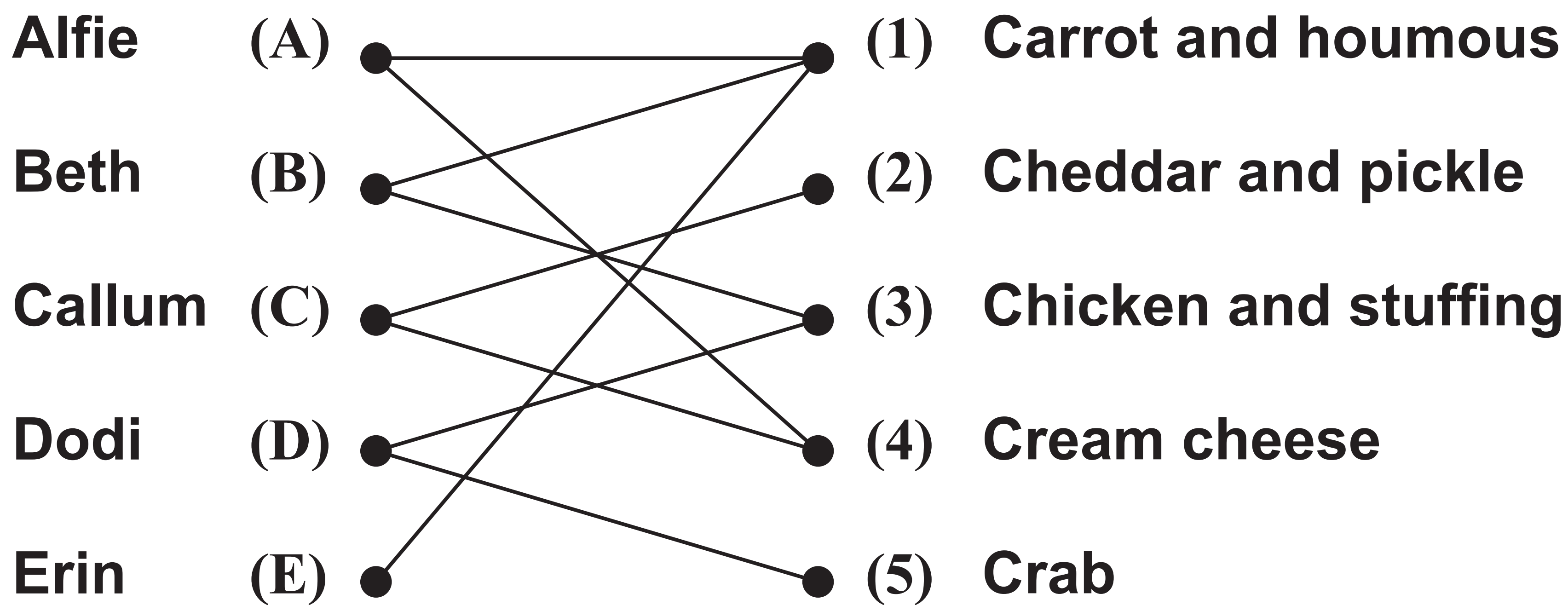
Answer ALL the questions.

1 Five students have bought a five pack of sandwiches for their lunch. The bipartite graph below shows which student likes which sandwich.



Initially Alfie chooses carrot and houmous, Callum chooses cream cheese and Dodi chooses chicken and stuffing.

(i) Draw this initial incomplete matching. [1]

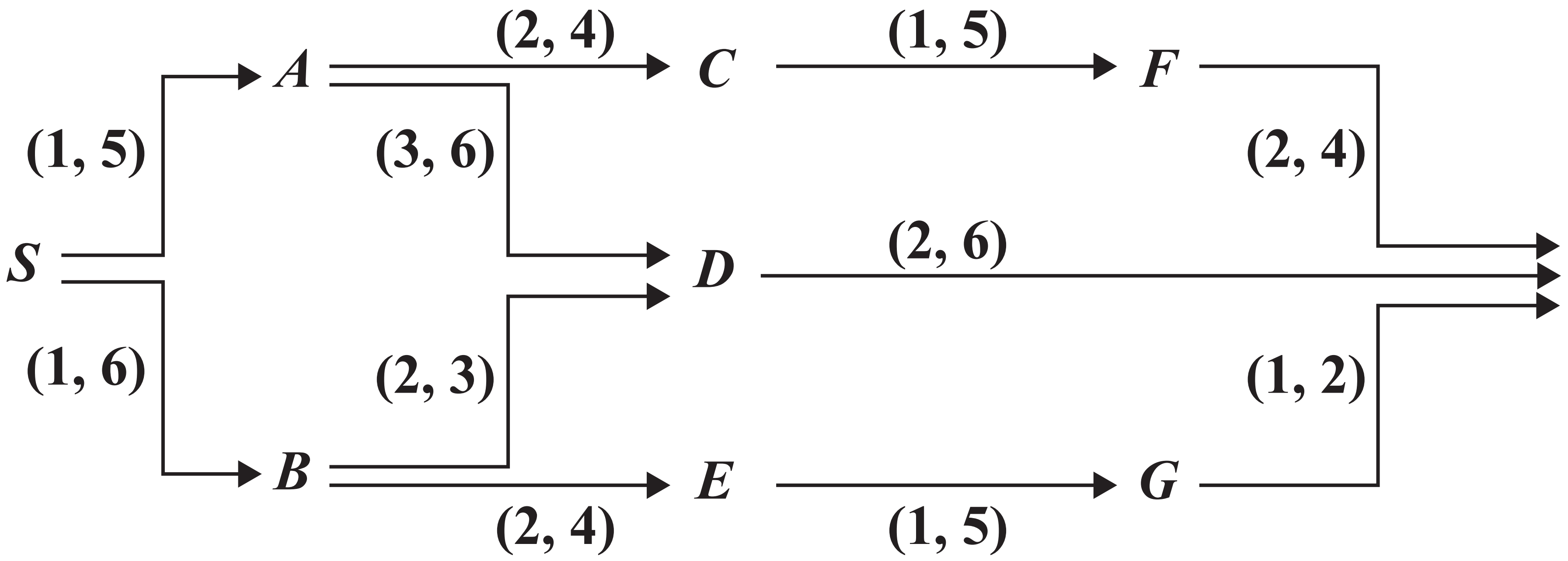
(ii) Construct a shortest alternating path starting from Erin (E).
Draw the matching that this gives. [2]

(iii) Construct a shortest alternating path starting from Crab (5) to improve the incomplete matching from part (ii).
List the complete matching that this gives. [2]

Suppose that, in addition to the preferences shown in the bipartite graph above, Erin had also liked crab sandwiches.

(iv) Draw a bipartite graph to show a complete matching in which Erin chooses the crab sandwich. [1]

2 The flow of oil through an engine is modelled in the network below. The arcs represent components of the engine. The weights on the arcs show the minimum and maximum rate of flow, in cl per second, around each component. All flows are directed as shown. From T the oil is passed back to S and topped up, if necessary, so that the oil can pass through the engine continuously.



(i) Explain why the flow in arc SA is at its maximum value. [1]

(ii) Explain why the flow in arc FT is at its minimum value. [1]

(iii) (a) Show a flow of 9 cl per second from S to T. [1]

(b) Label the arrows in the diagram in the Printed Answer Book to show the excess capacities and potential backflows. [2]

(c) Use the labelling procedure to augment the flow to 10 cl per second. Write down the route that has been augmented. [2]

(d) Find a cut that shows that 10 cl per second is the maximum flow. [1]

3 The publicity team at a college want to advertise four events. The events are for different age groups, so nobody will attend more than one event and each event will be advertised in a different way.

For each method of advertising, the number of people who are expected to turn up is shown in the table below. The team want to know which advertising method to use for which event to maximise the total number of people who attend the four events.

	Advertising method			
	newspaper	posters	emails	on buses
Event 1	1000	600	200	200
Event 2	800	500	300	200
Event 3	500	400	300	100
Event 4	1000	300	400	200

(i) Convert this into a suitable form so that the problem can be solved using the Hungarian algorithm with single digit entries in each cell of the initial matrix. [2]

(ii) Use the Hungarian algorithm, reducing rows first, to find an optimal allocation. Give a brief description of what you have done to form each table. [5]

(iii) Which method should be used to advertise each event and what is the total number of people who are expected to turn up to the four events? [2]

Advertising on buses is too expensive for the returns it gives, so the team decide to use a leaflet drop instead. The number of people who are expected to turn up for each event if it is advertised in this way are:

Event 1 = 120 Event 2 = 120 Event 3 = 20 Event 4 = 120

(iv) Explain why the events being advertised using newspaper adverts, posters and emails are the same as in part (iii). You do not need to carry out the full Hungarian algorithm again. [1]

4 Leo and Maya play a card game. The game involves each player being dealt three cards from a set of eight cards (labelled A to H). Each player chooses one of their cards. The players then simultaneously show their choices and deduce how many points they have won or lost using the table below.

The table shows the number of points won by the player ON ROWS for each pair of cards. A negative entry means that the player loses that number of points. The game is zero-sum.

	A	B	C	D	E	F	G	H
A		–1	–1	–2	–2	–3	–3	6
B	1		–1	–1	–2	–2	4	5
C	1	1		0	0	–3	0	5
D	2	1	0		–1	–2	–3	–4
E	2	2	0	1		–1	–2	–4
F	3	2	3	2	1		–6	–6
G	3	–4	0	3	2	6		–6
H	–6	–5	–5	4	4	6	6	

(i) In the first game Leo is dealt cards A, D and E. Maya is dealt cards B, F and H. Each player knows which cards the other has.

(a) Write out the pay-off matrix for Leo, with Leo on rows, when they play with these six cards.
Hence find the play-safe choice(s) for each player. [4]

(b) If each player plays safe how many points would Maya win? [1]

(c) If Leo knows that Maya will play safe, which card should he choose to maximise the points that he wins? [1]

(ii) In the second game Leo is again dealt cards A, D and E. This time Leo does not know which cards Maya has, although she knows which cards he has.

(a) Explain why this makes no difference to Leo's play-safe choice. [1]

(b) If Maya knows that Leo will play safe, which are the worst three cards for Maya to have if she wants to maximise the points that she wins? [1]

(iii) In the third game Maya is dealt cards B, F and H. This time neither player knows which cards the other has. Maya wants to maximise the points that she wins.

Give a reason, based on the values in the table, why Maya might choose each of her cards. [3]

5 The table below shows an incomplete dynamic programming tabulation to solve a maximin problem.

Stage	State	Action	Working	Suboptimal maximin
3	0	0	4	4
	1	0	3	3
2	0	0	min (2,) =	
		1	min (4,) =	
	1	0	min (3,) =	
		1	min (2,) =	
	2	0	min (5,) =	
		1	min (2,) =	
1	0	0	min (2,) =	
		1	min (5,) =	
	1	1	min (2,) =	
		2	min (2,) =	
0	0	0	min (4,) =	
		1	min (3,) =	

(i) Complete the working and suboptimal maximin columns on the copy of the table in your Printed Answer Book. Write down, using (stage; state) variables, the corresponding maximin route from stage 0 to stage 4. [6]

(ii) Record the weight of each of the arcs in the maximin route. Use these weights to explain what a maximin route is. [3]

Sally solves a different problem on the same network and produces the following table.

Stage	State	Action	Working	Suboptimal
3	0	0	4	4
	1	0	3	3
2	0	0	2 + 4 = 6	
		1	4 + 3 = 7	7
	1	0	3 + 4 = 7	7
		1	2 + 3 = 5	
	2	0	5 + 4 = 9	9
		1	2 + 3 = 5	
1	0	0	2 + 7 = 9	
		1	5 + 7 = 12	12
	1	1	2 + 7 = 9	
		2	2 + 9 = 11	11
0	0	0	4 + 12 = 16	16
		1	3 + 11 = 14	

(iii) (a) What problem has Sally solved? [1]

(b) Write down the route that is given by Sally's tabulation. [1]

(c) Use the arc weights to explain how Sally's route solves her problem. [1]

6 (i) State three properties that a game must possess to be a two-person zero-sum game. [3]

Usha, Val and Wesley are in a quiz team. They have been practising for the championship final.

The final will consist of 100 questions from the four topics shown in the table below.

The table shows the number of points, out of 10, that each person can expect to score on a question from each topic that could come up.

	Topic			
	Famous faces	Films	Food	Football
Usha	5	3	7	3
Val	2	4	1	4
Wesley	3	5	2	6

Only two of the team can play in the final. Either of them can be chosen to answer each question, but the team have to choose who will answer each question before they know the topic for the question.

Usha is the team captain, so she will play in the final. She has to reject either Val or Wesley.

(ii) Who should not play in the final? Explain why that person should be rejected. [2]

Usha decides to use random numbers to choose who will answer each question.
Let p be the probability that Usha answers a question, where p is the same for each question.

(iii) Write down an expression of the form $a + bp$, where a and b are constants, for the expected number of points won if each topic comes up. [3]

(iv) **Use a graphical method to calculate the value of p that Usha should use to maximise the minimum expected number of points won. [4]**

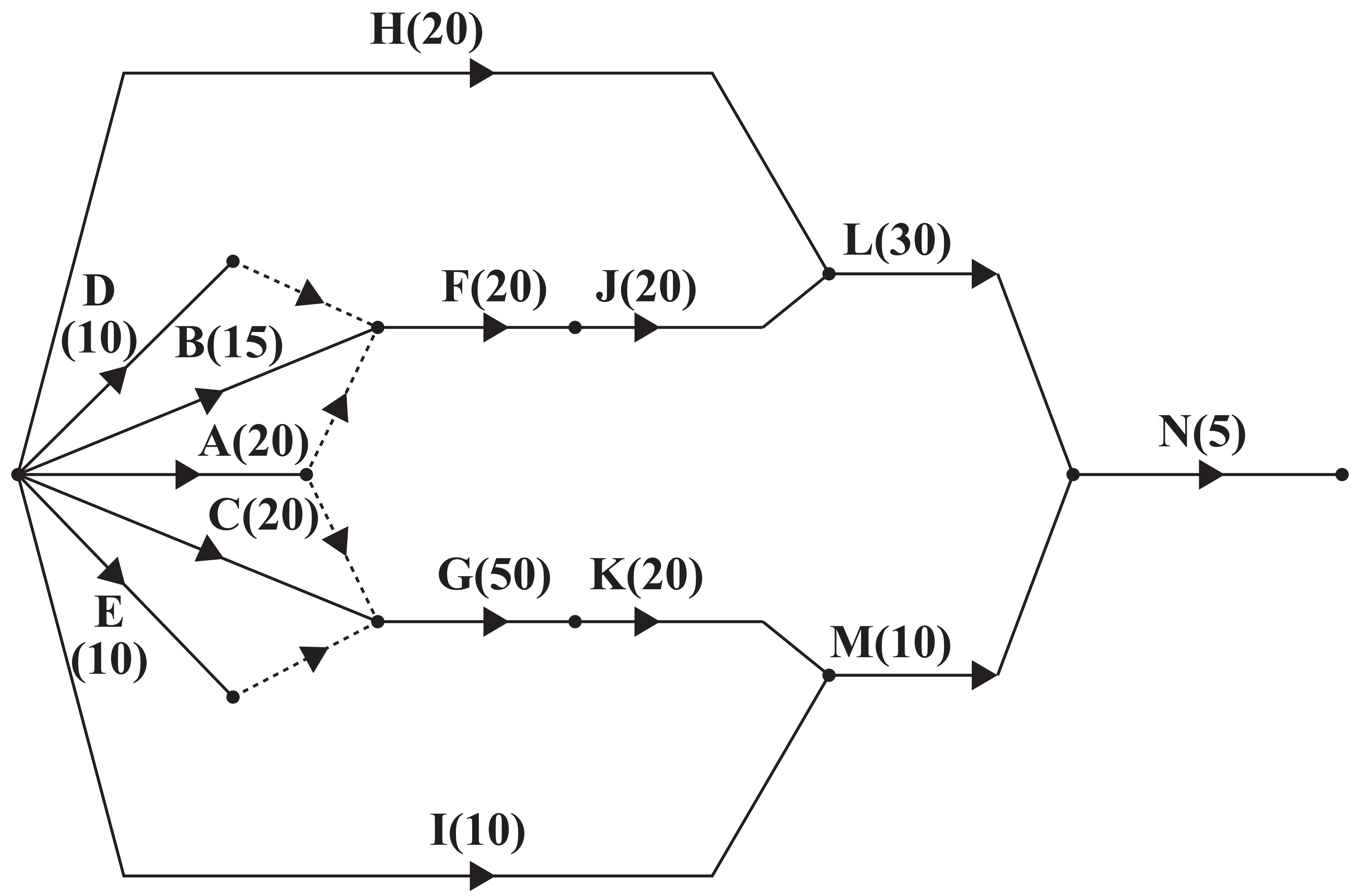
Suppose instead that all three team members can play in the final. Let p be the probability that Usha answers a question, with q and r being the corresponding probabilities for Val and Wesley, respectively.

(v) **Set up an initial Simplex tableau for the problem of choosing the optimal values for p, q and r to maximise the minimum expected number of points won. You are not required to solve your LP. [4]**

7 The table lists the activities involved in making cupcakes and carrot cake slices for a bake sale, their durations (in minutes) and their immediate predecessors.

ACTIVITY		DURATION (MINS)	IMMEDIATE PREDECESSORS
Heat oven	A	20	
Mix cupcakes	B	15	
Mix carrot cakes	C	20	
Line cupcake tins with paper cases	D	10	
Prepare cake tins for carrot cakes	E	10	
Bake cupcakes	F	20	A, B, D
Bake carrot cakes	G	50	A, C, E
Make topping for cupcakes	H	20	
Make topping for carrot cakes	I	10	
Let cupcakes cool	J	20	F
Let carrot cakes cool and slice	K	20	G
Decorate cupcakes	L	30	H, J
Decorate carrot cake slices	M	10	I, K
Pack cupcakes and carrot cake slices into boxes	N	5	L, M

This information is represented in the activity network below.



(i) Carry out a forward pass and a backward pass through the activity network. Show the early event times and the late event times at each vertex on the copy of the activity network in the Printed Answer Book. [3]

Assume that activities A, F, G, J and K require no people. All other activities require one person.
The cupcakes cannot be baked in the same oven as the carrot cakes and there is only one oven available, so activities F and G cannot happen at the same time.

(ii) Construct a schedule to show how two people can complete the activities in 125 minutes. [4]

Now suppose that there is only one person available. Two possible sets of activity start times are listed below.

Version (1): bake cupcakes first

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Start time	0	0	25	15	45	25	55	55	75	45	105	85	125	135

Project completion time 140 minutes

Version (2): bake carrot cakes first

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Start time	0	30	0	45	20	80	30	55	75	100	80	120	100	150

Project completion time 155 minutes

(iii) Find the project completion time for each of these versions if two ovens were available instead of one. [2]