OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4726/01

MATHEMATICS

Further Pure Mathematics 2

QUESTION PAPER

MONDAY 25 JUNE 2018: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book sent with the standard paper, or any suitable paper supplied by the centre.
The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4726/01 sent with the standard paper
List of Formulae (MF1) sent with the standard paper
Insert for Question 7(i)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (LK/SW) 167760/2

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INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer ALL the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer ALL the questions.

1 The equation of a curve is $y = \dfrac{x^2 + 1}{(x-1)^2}$.

(i) Write down the equations of the asymptotes. [2]

(ii) Find the coordinates of the stationary point on the curve and hence determine the range of y. [6]

(iii) Sketch the curve. Indicate the asymptotes in your sketch. [2]

2 (i) Using the definition of cosh x in terms of e^x and e^{-x}, show that $\cosh 2x = 2\cosh^2 x - 1$. [2]

(ii) Find $\int_0^1 \cosh^2 3x \, dx$, giving your answer in the form $A + B\sinh C$, where A, B and C are constants to be determined. [3]

3 The equation of a curve is $y = \cosh x - 2\sinh 2x$.

Show that the curve has no turning points. [5]

4 It is given that $I_n = \int_0^1 x^n e^{x-1} \, dx$ for $n \geqslant 1$.

(i) Show that $I_n = 1 - nI_{n-1}$ for $n \geqslant 2$. [3]

(ii) Find the exact value of I_4. [4]

(iii) For the curve $y = x^n e^{x-1}$, where $n \geqslant 1$, find $\frac{dy}{dx}$ and sketch the curve for $0 \leqslant x \leqslant 1$.

Deduce that $0 < I_n < 1$ for all $n \geqslant 1$. [5]

5 By using the substitution $t = \tan \frac{1}{2}x$, find the exact value of $\int_0^{\frac{\pi}{2}} \frac{1}{\sin x + 1} \, dx$. [5]

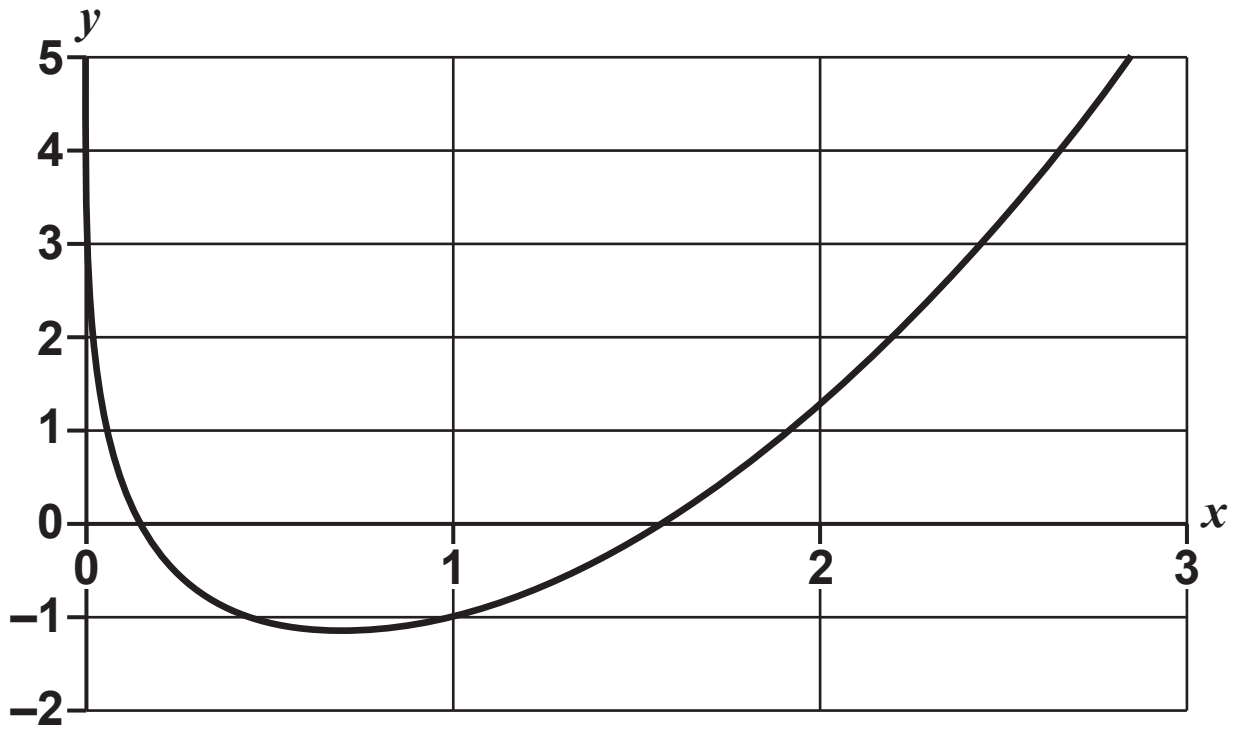
6 It is given that $y = \tan^{-1} 3x$.

(i) Using the derivative of $\tan^{-1} x$ given in the List of Formulae (MF1), find $\frac{dy}{dx}$. [2]

(ii) Hence show that $(1 + 9x^2)\frac{d^3y}{dx^3} + 36x\frac{d^2y}{dx^2} + 18\frac{dy}{dx} = 0$. [4]

(iii) Find the Maclaurin series for y up to and including the term in x^3. [3]

7 You are given that the equation $x^2 - \ln x - 2 = 0$ has two roots, α and β, where $0 < \alpha < 1$. The diagram shows a sketch of the curve $y = x^2 - \ln x - 2$.



(i) The Newton-Raphson method is to be used to find α.
Explain what happens when the initial values $x_1 = 0.5$ and $x_1 = 1$ are used. You may use the Printed Answer Book or the Insert provided. [2]

(ii) Show that the Newton-Raphson iterative formula for this equation can be written in the form

$$x_{r+1} = \frac{x_r(x_r^2 + \ln x_r + 1)}{(2x_r^2 - 1)}$$ [3]

(iii) Using the initial value $x_1 = 0.1$, find x_2 and x_3.
Find α correct to 5 decimal places. [2]

8 You are given that the equation $x^3 + x - 4 = 0$ has a root, α, where $1 < \alpha < 2$.

The iterative formula $x_{r+1} = \sqrt[3]{(4 - x_r)}$ with $x_1 = 1.4$ is to be used to find α.

(i) Demonstrate that the iterative values converge to α by sketching a cobweb diagram. [2]

(ii) Use the iterative formula to find α correct to 4 decimal places. Show the result of each step of the iteration correct to 5 decimal places. [3]

(iii) The error, e_n is defined by $e_n = \alpha - x_n$.

Using the value of α found in part (ii), evaluate $\frac{e_3}{e_2}$ and $\frac{e_4}{e_3}$.

Comment on the values of $\frac{e_3}{e_2}$ and $\frac{e_4}{e_3}$ in relation to the gradient of the curve $y = \sqrt[3]{(4 - x)}$ at $x = \alpha$. [3]

9 A curve has polar equation $r = 2\cos\theta - 1$ for $-\frac{\pi}{3} \leqslant \theta \leqslant \frac{\pi}{3}$.

(i) Show that the line $\theta = 0$ is a line of symmetry. [2]

(ii) Sketch the curve, indicating
the point on the curve where $\theta = 0$,
the tangents at the pole. [4]

(iii) Find the area enclosed by the curve. [5]

END OF QUESTION PAPER