OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE

4736/01

MATHEMATICS

Decision Mathematics 1

QUESTION PAPER

WEDNESDAY 13 JUNE 2018: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED 36pt

Candidates answer on the Printed Answer Book.

OCR SUPPLIED MATERIALS:
Printed Answer Book 4736/01
List of Formulae (MF1) sent with standard paper

OTHER MATERIALS REQUIRED:
Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.

WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK.
If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer ALL the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer ALL the questions.

1 Pat works at the packing depot for a company that sells shoes. Shops place orders for a number of boxes of shoes and Pat needs to pack the boxes into vans to make the deliveries to the shops. The order from any shop must all be in the same van.

Each van can contain at most 1000 boxes.

Pat has orders for the following numbers of boxes.

Shop	A	B	C	D	E	F	G	H
Boxes	500	400	600	300	300	400	300	200

(i) Demonstrate the use of first-fit to determine a way to pack the orders. Show, using the letters for the shops, which shop's orders are in which van. [2]

(ii) Demonstrate the use of first-fit decreasing to determine a way to pack the orders. [2]

(iii) Find a packing that uses fewer vans. [1]

(iv) Why might a packing not be practical? [1]

2 **An algorithm involves the following steps.**

Step 1 **Input positive integers M, N and P.**

Step 2 **If M is odd, replace P by $P + N$.**

Step 3 **If M=1 go to step 7.**

Step 4 **Replace N by $2N$.**

Step 5 **If M is even, replace M by $M \div 2$, otherwise replace M by $(M-1) \div 2$.**

Step 6 **Go to step 2.**

Step 7 **Output the value of P.**

(i) **Show the use of the algorithm for the inputs M=8, N=10 and P=3. State the output value of P. [4]**

(ii) For inputs M=13, N=n and P=p, what is the output of P in terms of n and p? What is the relationship between the output of P and the original inputs for M, N and P? [4]

3 **A problem to maximise P as a function of x and y (where x and y are both $\geqslant 0$) is represented by the initial Simplex tableau below.**

P	x	y	s	t	u	RHS
1	–2	4	0	0	0	0
0	4	–12	1	0	0	12
0	7	–19	0	1	0	35
0	–3	15	0	0	1	0

(i) **Write down P as a function of x and y. [1]**

(ii) **Write down the constraints, apart from non-negativity, as inequalities involving x and y. [3]**

(iii) **Identify which element should be used as the initial pivot and then carry out one iteration of the Simplex algorithm. [4]**

After a second iteration of the Simplex algorithm the tableau is as given below.

P	x	y	s	t	u	RHS
1	0	0	$\frac{3}{4}$	0	$\frac{1}{3}$	9
0	1	0	$\frac{5}{8}$	0	$\frac{1}{2}$	$7\frac{1}{2}$
0	0	0	-2	1	$-\frac{1}{3}$	11
0	0	1	$\frac{1}{8}$	0	$\frac{1}{6}$	$1\frac{1}{2}$

(iv) Write down the values of P, x, y, s, t and u after this second iteration. [2]

(v) Show that the values of x and y from part (iv) satisfy the constraints from part (ii) and interpret the values of s, t and u. [4]

4 The running costs (in £million) of 10 bus routes between six locations in a city are shown in the table below. A blank indicates that there is no bus route between the locations. The route between the university and the museum is new and therefore its running costs are currently unknown, this is labelled as x.

		M	N	P	R	S	U
Museum	M			6		7	x
Nautical centre	N				5	4	8
Park and ride	P	6			3	2	
Railway station	R		5	3		4	8
Shopping centre	S	7	4	2	4		
University	U	x	8		8		

To save money the bus company are planning to cut some of the bus routes. Assume that there is no change in the running costs of the remaining bus routes.

(i) **What is the minimum number of bus routes that must operate so that it is still possible to travel between the six locations? [1]**

(ii) **Apply Prim's algorithm to the reduced table for which the row and column representing the university have been removed. Start by crossing out the row for M and choosing an entry from the column for M. Write down the arcs used in the order that they are chosen. Draw the resulting minimum spanning tree and give its total weight. [5]**

(iii) **Give the value of x for which the spanning tree for all six locations has total weight 21. [1]**

(iv) **Give the value of x for which a most expensive bus route would be included in the minimum spanning tree for all six locations. [1]**

(v) **Describe in detail what happens to the total weight of the minimum spanning tree for all six locations as the value of x increases. [2]**

(vi) **Assume that x is less than the value from part (iv). Use the nearest neighbour method, starting at P to find, in terms of x, an upper bound for the least cost cycle that connects the six locations. [2]**

(vii) **Use the information in the table to draw a graph to show which locations are connected by bus routes. [1]**

(viii) **Draw a subgraph of your answer to part (vii) that is a tree connecting all six locations in which no location is more than two bus routes from the university. [1]**

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5 Jimmy has made a batch of marmalade to sell to raise money for charity. He wants to raise as much money as possible.

He has enough marmalade to fill 40 small jars or 30 large jars. He has enough jars so that he could use all small jars or all large jars or a mixture of small and large jars.

Jimmy knows that small jars are more popular than large jars so he wants to fill at least as many small jars as large jars. He can only carry at most 36 jars in total.

He intends to charge £2 for each small jar and £3 for each large jar. It costs Jimmy a total of £12 to make each batch of marmalade. He expects to be able to sell all the jars he fills.

Let x denote the number of small jars that Jimmy fills and y denote the number of large jars.

(i) Show why the constraint $3x + 4y \leqslant 120$ is needed. Write down the other constraints on the values of x and y, apart from needing to be integer valued. [4]

(ii) Write down an objective function P, in terms of x and y, where P is to be maximised. [1]

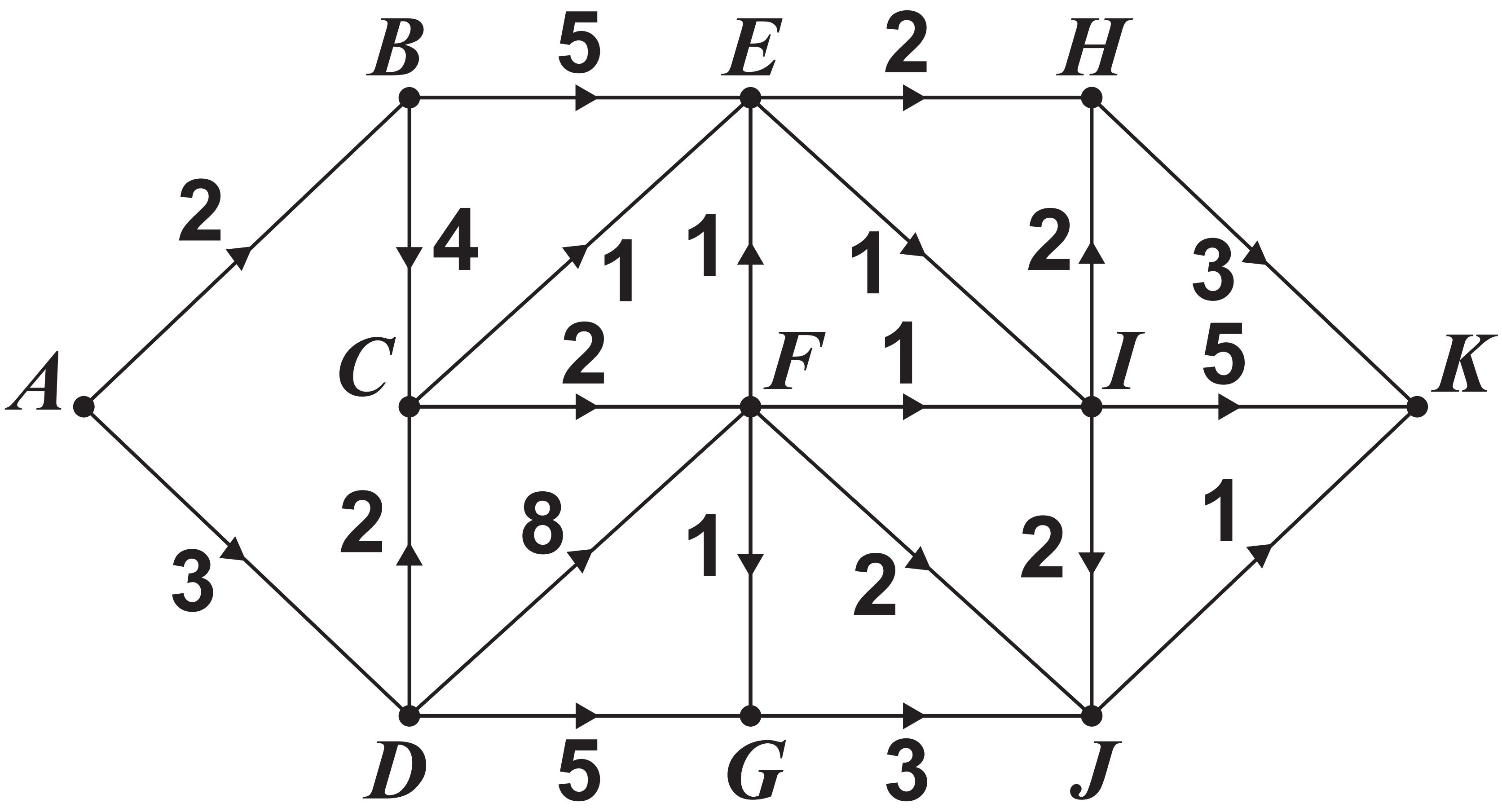
(iii) Plot the feasible region graphically. [4]

(iv) Use your graph to find how many small jars and how many large jars Jimmy should fill. [3]

(v) How much profit can Jimmy expect to make? [1]

6 (a) The network in Fig. 1 represents a system of one-way streets in a city centre. The vertices represent roundabouts, the arcs represent roads and the weights show distances in units of 100 m.

FIG. 1



(i) Apply Dijkstra's algorithm to the network, starting at A, to find the shortest distance from A to K (in units of 100 m) and write down the route(s) of the shortest path(s). [6]

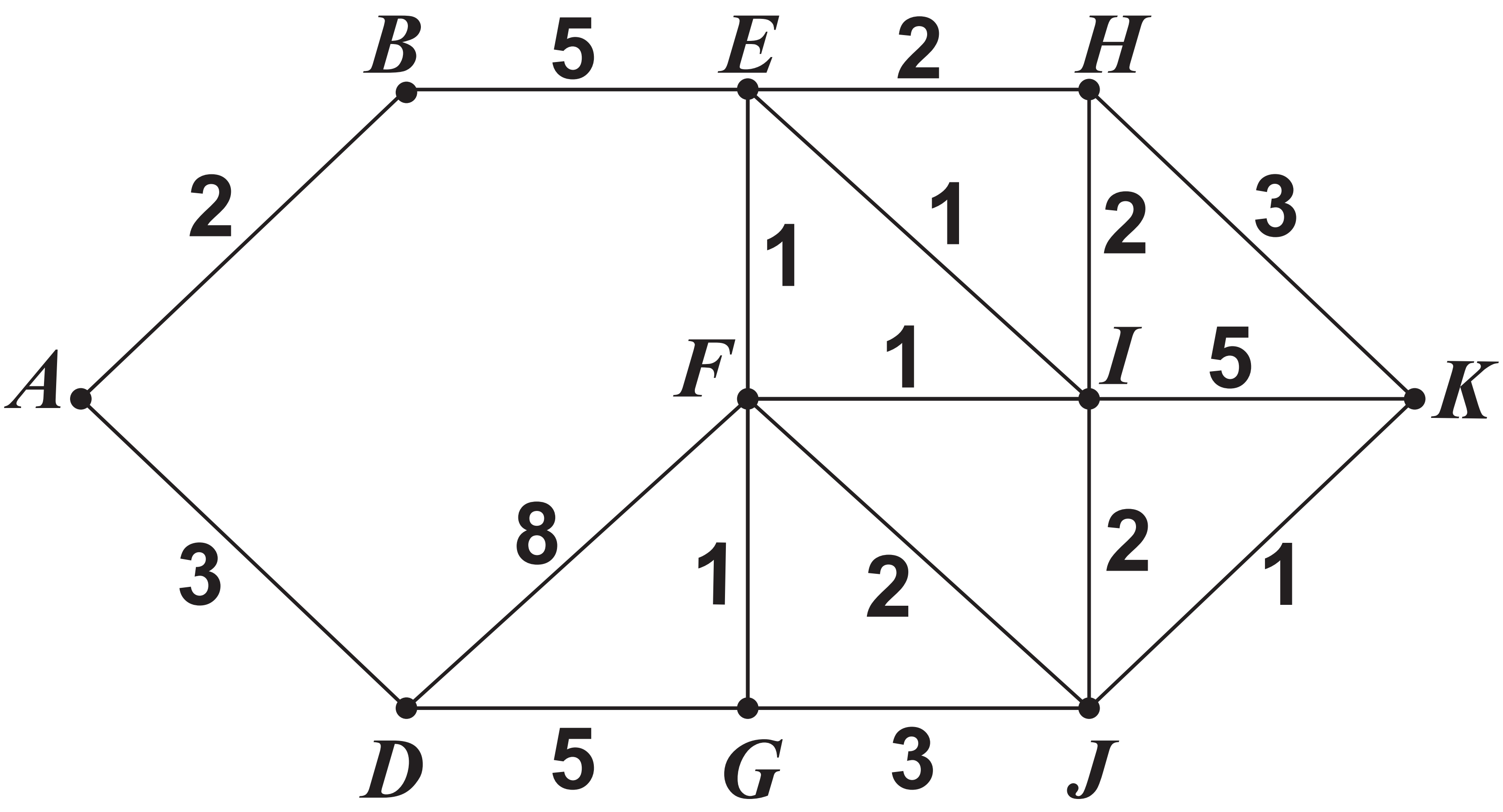
Roadworks mean that the roundabout at C is not available.

(ii) By considering the shortest distance from A to each of E, F and G, when vertex C is not available, find the shortest distance from A to K (in units of 100 m) and write down the route(s) of the shortest path(s). [You do not need to use Dijkstra's algorithm to find the shortest distances.] [4]

(b) **Cycle lanes allow cyclists to travel in either direction on the roads.**

While the roundabout at *C* is not available the undirected network is as shown in Fig. 2.
The total weight of the arcs shown in Fig. 2 is 47.

FIG. 2



(i) A cyclist wants to travel along every road at least once in a continuous route that starts and ends at A. Given that arc DG must be travelled exactly twice, apply the route inspection algorithm, showing your working, to find the minimum distance that the cyclist would need to travel. Write down all the arcs that represent roads that would be repeated in this minimum route. [4]

The roadworks at C finish, so C and all the arcs connected to C are available again.

(ii) Find the minimum distance that a cyclist would need to travel to pass along every road at least once in a continuous route that starts at B and ends at E. How many times does this route travel on the roundabout represented by vertex F? [3]

END OF QUESTION PAPER