**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**A2 GCE**

# 4727/01

# MATHEMATICS

## Further Pure Mathematics 3

**QUESTION PAPER**

**FRIDAY 15 JUNE 2018: Afternoon**

**DURATION: 1 hour 30 minutes**
**plus your additional time allowance**

**MODIFIED ENLARGED**

**Candidates answer on the Printed Answer Book sent with the standard paper or any suitable paper supplied by the centre. The Printed Answer Book may be enlarged by the centre.**

**OCR SUPPLIED MATERIALS:**

**Printed Answer Book 4727/01 sent with the standard paper**
**List of Formulae (MF1) sent with the standard paper**

**OTHER MATERIALS REQUIRED:**

**Scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**

DC (LK) 166328/1

## INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer ALL the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

## INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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**Answer ALL the questions.**

**1 (i) Find the shortest distance from the point $(3, -1, -2)$ to the plane with equation $x - 2y + 4z = 11$. [2]**

**(ii) Find a cartesian equation of the plane which passes through the point $(3, -1, -2)$ and is parallel to the plane $x - 2y + 4z = 11$. [2]**

**2 A multiplicative group $G$ consists of the elements $\{1, z, z^2, z^3, z^4, z^5\}$.**

**(i) State the order of the element $z^4$. [1]**

**(ii) List all the subgroups of $G$. [3]**

**The group $H$ consists of the set $\{1, 2, 3, 4, 5, 6\}$ with the operation of multiplication modulo 7.**

**(iii) Determine whether $G$ is isomorphic to $H$. [2]**

**3 It is given that the differential equation**

$$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 10\mathrm{e}^{-x}$$

**has a particular integral of the form $axe^{-x}$, where $a$ is a constant. Solve the differential equation subject to the conditions $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{9}{2}$ when $x = 0$. [10]**

4 The operation $*$ is defined by $x * y = xy + k(x+y) + 12$, where $x$ and $y$ are real numbers and $k$ is a real constant. It is given that the operation $*$ is associative.

(i) Show that there are two possible values for $k$, one of which is 4. [4]

(ii) In the case where $k = 4$, determine whether the set of real numbers, under the operation $*$, forms a group. [4]

5 The differential equation

$$\frac{dy}{dx} + \frac{2y}{1-x} = 4(1-x^2)\sqrt{y}$$

is to be solved for $x < 1$. Use the substitution $u = \sqrt{y}$ to find the general solution of the differential equation, expressing your answer in the form $y = f(x)$.

[8]

6 (i) Use de Moivre's theorem to find an expression for $\cot 7\theta$ in terms of $\cot\theta$ and hence find the exact roots of the equation $u^6 - 21u^4 + 35u^2 - 7 = 0$. [7]

(ii) State the exact roots of the equation $v^3 - 21v^2 + 35v - 7 = 0$, justifying your answer. Hence find the exact value of

$$\frac{\cot^2\left(\frac{1}{14}\pi\right)\cot^2\left(\frac{3}{14}\pi\right) + \cot^2\left(\frac{3}{14}\pi\right)\cot^2\left(\frac{5}{14}\pi\right) + \cot^2\left(\frac{5}{14}\pi\right)\cot^2\left(\frac{1}{14}\pi\right)}{\cot\left(\frac{1}{14}\pi\right)\cot\left(\frac{3}{14}\pi\right)\cot\left(\frac{5}{14}\pi\right)}.$$

[4]

**7** The plane $\Pi_1$ passes through the points (5, 2, −2), (4, 0, −1) and (2, 1, −3).

(i) Find a cartesian equation of the plane $\Pi_1$. [5]

The line $l_1$ has equation $\frac{x}{2} = \frac{y-4}{-1} = \frac{z+3}{3}$.

(ii) Find the acute angle between $\Pi_1$ and $l_1$. [3]

The line $l_2$ has equation $\mathbf{r} = \begin{pmatrix} p \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} q \\ -6 \\ 12 \end{pmatrix}$ and lies in $\Pi_1$.

(iii) Find the value of $p$ and show that $q = 12$. [3]

The plane $\Pi_2$ is perpendicular to $\Pi_1$ and $l_2$ lies in $\Pi_2$.

(iv) Find an equation of $\Pi_2$, giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = d$. [3]

**8** **(i)** Show that, if $z \neq \pm 1$ and $z \neq 0$,

$$\sum_{r=1}^{n} z^{2r-1} = \frac{1 - z^{2n}}{z^{-1} - z}. \quad [2]$$

**(ii)** Hence show that, if $\sin\theta \neq 0$,

$$\sum_{r=1}^{n} \sin(2r-1)\theta = \frac{\sin^2 n\theta}{\sin\theta}. \quad [6]$$

**(iii)** Hence find the exact value of

$$\int_0^{\frac{1}{6}\pi} \frac{\sin^2 3\theta}{\sin\theta}\,d\theta. \quad [3]$$

**END OF QUESTION PAPER**