OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4734/01

MATHEMATICS

Probability & Statistics 3

QUESTION PAPER

TUESDAY 26 JUNE 2018: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book sent with the standard paper, or any suitable paper provided by the centre. The centre may enlarge the Printed Answer Book.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4734/01 sent with standard paper
List of Formulae (MF1) sent with standard paper

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

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INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer ALL the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Answer ALL the questions.

1 The random variables X and Y have independent Poisson distributions with parameters 2 and 3 respectively, and $Z = 3X + 4Y$. Find $\mathrm{P}(Z = \mathrm{E}(Z))$. [5]

2 A 95% confidence interval for the mean μ of a certain population, based on a sample of size 35, is [6.0061, 7.9939]. Find the minimum sample size such that the width of a 95% confidence interval for μ is less than 1. [4]

3 The cumulative density function of the continuous random variable X is given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 1, \\ \dfrac{2x-2}{x+3} & 1 \leqslant x \leqslant 5, \\ 1 & x > 5. \end{cases}$$

Given that $Y = 2X - 3$, find the probability density function of Y. [6]

4 The mean number of matches in a box is claimed to be 48. A random sample of 7 boxes is taken and the number of matches in each box is counted. The results are shown below.

48 45 46 47 47 46 48

Stating a necessary assumption, test at the 2.5% significance level whether the mean number of matches in a box is less than 48. [8]

5 A certain brand of ice cream is sold in cartons of different sizes. Large cartons contain ice cream whose mass is normally distributed with mean 412 g and standard deviation 10 g. Small cartons contain ice cream whose mass is normally distributed with mean 112 g and standard deviation 8 g.

(i) Find the probability that the total mass of ice cream in two randomly chosen large cartons and two randomly chosen small cartons is greater than 1 kg. [4]

(ii) Find the probability that the mass of ice cream in a randomly chosen large carton is greater than 4 times the mass of ice cream in a randomly chosen small carton. [4]

6 The continuous random variable X has probability density function

$$\mathrm{f}(x) = \begin{cases} k\cos x & 0 \leqslant x < \frac{\pi}{4}, \\ k\sin x & \frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{2}, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $k = \frac{1}{\sqrt{2}}$. [3]

(ii) Find $\mathrm{P}(X \leqslant 1)$. [3]

(iii) Find the upper quartile of X. [3]

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7 Greyhound racing in England involves six dogs racing over distances of approximately 500 m. The dogs wear jackets numbered from 1 to 6. A researcher observes that the winning dog in exactly 24 of 80 randomly chosen races wore jacket number 3.

(i) The probability that a randomly chosen race is won by the dog wearing jacket number 3 is denoted by p. Calculate an approximate 99% confidence interval for p. [3]

(ii) Explain whether your result from part (i) is consistent with p taking the value 0.2. [1]

Greyhound racing meetings in England usually consist of 12 races. The researcher chooses 100 such meetings at random and, at each meeting, he records how many of the 12 races were won by the dog wearing jacket number 3. The results for the 100 meetings are given in the table below.

Number of races won by the dog wearing jacket number 3	0	1	2	3	4	5	6	7	8 or more
Observed frequency	9	18	33	17	14	6	2	1	0

(iii) Show that the proportion of races won by the dog wearing jacket number 3 at the 100 meetings was 0.2. [2]

The expected frequencies using a binomial distribution with $n = 12$ are given in the table below.

Number of races won by the dog wearing jacket number 3	Expected frequency
0	6.87
1	20.62
2	28.35
3	23.62
4	13.29
5	5.32
6	1.55
7	0.33
8 or more	0.05

(iv) Show how for the case where 4 races were won by dog number 3 the expected frequency of 13.29 was calculated. [2]

(v) Carry out a χ^2 test, at the 5% significance level, to test the null hypothesis that the data can be well modelled by a binomial distribution with $n = 12$. [6]

8 The numbers of hours of sunshine observed on 8 randomly chosen days in London were as follows.

9.3 3.9 11.8 5.0 10.6 0.0 6.1 2.7

The numbers of hours of sunshine observed on 6 randomly chosen days in Berlin were as follows.

7.3 2.5 6.0 9.8 12.1 4.6

It may be assumed that the number of hours of sunshine in each city is normally distributed.

(i) Carry out a suitable t-test, at the 1% significance level, to test whether the mean daily number of hours of sunshine in Berlin is higher than the mean daily number of hours of sunshine in London. [9]

(ii) Explain how the observations could be modified so as to use a paired-sample t-test, and explain why this would be preferable. [2]

9 The results of an examination are represented in the contingency table below, in which x and y are integers.

	Pass	Fail	Total
Females	x	y	50
Males	y	x	50
Totals	50	50	100

The null hypothesis, that there is no association between examination result and gender, was rejected at the 5% significance level. Given that $x > y$, find the smallest possible value of x. [7]

END OF QUESTION PAPER

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