

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS  
AS GCE  
4725/01  
MATHEMATICS  
Further Pure Mathematics 1  
QUESTION PAPER  
MONDAY 14 MAY 2018: Afternoon  
DURATION: 1 hour 30 minutes  
plus your additional time allowance  
MODIFIED ENLARGED 24pt**

**Candidates answer on the Printed Answer Book sent with the standard paper, or any suitable paper provided by the centre. The centre may enlarge the Printed Answer Book.**

**OCR SUPPLIED MATERIALS:**

**Printed Answer Book 4725/01 sent with the standard paper**

**List of Formulae (MF1) sent with the standard paper**

**OTHER MATERIALS REQUIRED:**

**Scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**



**Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.**

**IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED.**

**Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**

**Use black ink. HB pencil may be used for graphs and diagrams only.**

**Answer ALL the questions.**

**Read each question carefully. Make sure you know what you have to do before starting your answer.**

**You are permitted to use a scientific or graphical calculator in this paper.**

**Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

## **INFORMATION FOR CANDIDATES**

**The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**

**YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**

**The total number of marks for this paper is 72.**

## **INSTRUCTION TO EXAMS OFFICER / INVIGILATOR**

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**Answer ALL the questions.**

**1 The matrices A, B and C are given by**

$$A = \begin{pmatrix} 5a \\ 2 \end{pmatrix}, B = \begin{pmatrix} 7b \\ -3 \end{pmatrix} \text{ and } C = \begin{pmatrix} 3 & 6 \end{pmatrix}.$$

**Find**

**(i)  $5A - 4B$ , [2]**

**(ii)  $BC$ . [2]**

**2 The complex number  $w$  has modulus 6 and argument  $\frac{2\pi}{3}$ .**

**Find  $\frac{\sqrt{3} + 2i}{w}$ , giving your answer in the form  $x + iy$ , where  $x$  and  $y$  are exact real numbers. [5]**

**3 The matrix D is given by  $D = \begin{pmatrix} d & 0 \\ 0 & 1 \end{pmatrix}$ , where  $d \neq 0$ .**

**(i) Find  $D^{-1}$ . [2]**

**Matrix D represents the transformation P.**

**(ii) Describe fully the transformation P. [2]**

**The transformation T is represented by the matrix  $\begin{pmatrix} 0 & 1 \\ -d & 0 \end{pmatrix}$  and is equivalent to the transformation P followed by the transformation Q.**

**(iii) Find the matrix that represents the transformation Q and describe fully the transformation Q. [4]**

- 4 The loci  $L_1$ ,  $L_2$  and  $L_3$  are given by  $|z - 3 - 4i| = 2$ ,  $\arg(z - 3 - 4i) = \frac{\pi}{3}$  and  $|z| = |z - 12|$  respectively.
- (i) Sketch on a single Argand diagram the loci  $L_1$ ,  $L_2$  and  $L_3$ . [6]
- (ii) Indicate, by shading, the region of the Argand diagram for which  $|z - 3 - 4i| \geq 2$ ,  $0 \leq \arg(z - 3 - 4i) \leq \frac{\pi}{3}$  and  $|z| \leq |z - 12|$ . [3]
- 5 The cubic equation  $x^3 + 2x^2 + 3x + 4 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .
- (i) Use the substitution  $x = \frac{1}{u+1}$  to obtain a cubic equation in  $u$  with integer coefficients. [4]
- (ii) Hence, or otherwise, find the value of  $\left(\frac{1}{\alpha} - 1\right)\left(\frac{1}{\beta} - 1\right)\left(\frac{1}{\gamma} - 1\right)$ . [3]
- 6 (i) Find  $\sum_{r=1}^n r(r^2 + r - 7)$ , giving your answer in a fully factorised form. [5]

A sequence  $u_0, u_1, u_2, \dots$  is defined by  $u_0 = 5$ ,

$$u_n = u_{n-1} + n^3 + n^2 - 7n \text{ for } n \geq 1.$$

- (ii) By considering  $\sum_{r=1}^n (u_r - u_{r-1})$ , find a formula for  $u_n$  in terms of  $n$ . [3]

[You do not need to factorise your answer.]

- 7 The complex number  $a + 3i$  is a root of the quadratic equation  $z^2 - (7 + i)z + 16 + ki = 0$ ,

where  $a$  and  $k$  are positive real numbers.

(i) Find the value of  $a$  and the value of  $k$ . [7]

(ii) Hence find the other root of the quadratic equation. [2]

- 8 The matrix  $A$  is given by  $A = \begin{pmatrix} a & 1 & -2 \\ -1 & a & 0 \\ 2a & 3 & 1 \end{pmatrix}$ ,  
where  $a$  is a real constant.

(i) Show that  $A$  is non-singular. [4]

(ii) Find  $A^{-1}$ . [4]

(iii) Hence solve the three simultaneous equations given below. [3]

$$\begin{aligned} ax + y - 2z &= 2 \\ -x + ay &= 1 \\ 2ax + 3y + z &= 0 \end{aligned}$$

(iv) Explain briefly why these equations have a unique solution. [1]

**9 The matrix  $M$  is given by  $M = \begin{pmatrix} m & m \\ 0 & 1 \end{pmatrix}$ ,**

**where  $m$  is a positive constant.**

**(i) Find  $M^2$  and  $M^3$  in terms of  $m$ . [4]**

**(ii) Hence suggest a suitable form for the matrix  $M^n$ , where  $n$  is a positive integer,  $n \geq 2$ . [2]**

**(iii) Use induction to prove that your answer to part (ii) is correct. [4]**

**END OF QUESTION PAPER**

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