OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE

4725/01

MATHEMATICS

Further Pure Mathematics 1

QUESTION PAPER

MONDAY 14 MAY 2018: Afternoon

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED 24pt

Candidates answer on the Printed Answer Book sent with the standard paper, or any suitable paper provided by the centre. The centre may enlarge the Printed Answer Book.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4725/01 sent with the standard paper

List of Formulae (MF1) sent with the standard paper

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (LK) 165392/1

7851660678

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer ALL the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Answer ALL the questions.

1 The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by

$\mathbf{A} = \begin{pmatrix} 5a \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 7b \\ -3 \end{pmatrix}$ and $\mathbf{C} = (3 \quad 6)$.

Find

(i) $5\mathbf{A} - 4\mathbf{B}$, [2]

(ii) $\mathbf{BC}$. [2]

2 The complex number w has modulus 6 and argument $\frac{2\pi}{3}$.
Find $\frac{\sqrt{3} + 2\mathrm{i}}{w}$, giving your answer in the form $x + \mathrm{i}y$, where x and y are exact real numbers. [5]

3 The matrix $\mathbf{D}$ is given by $\mathbf{D} = \begin{pmatrix} d & 0 \\ 0 & 1 \end{pmatrix}$, where $d \neq 0$.

(i) Find $\mathbf{D}^{-1}$. [2]

Matrix $\mathbf{D}$ represents the transformation $\mathbf{P}$.

(ii) Describe fully the transformation $\mathbf{P}$. [2]

The transformation $\mathbf{T}$ is represented by the matrix $\begin{pmatrix} 0 & 1 \\ -d & 0 \end{pmatrix}$ and is equivalent to the transformation $\mathbf{P}$ followed by the transformation $\mathbf{Q}$.

(iii) Find the matrix that represents the transformation $\mathbf{Q}$ and describe fully the transformation $\mathbf{Q}$. [4]

4 The loci L_1, L_2 and L_3 are given by $|z-3-4\mathrm{i}|=2$, $\arg(z-3-4\mathrm{i})=\frac{\pi}{3}$ and $|z|=|z-12|$ respectively.

(i) Sketch on a single Argand diagram the loci L_1, L_2 and L_3. [6]

(ii) Indicate, by shading, the region of the Argand diagram for which $|z-3-4\mathrm{i}|\geqslant 2$, $0\leqslant \arg(z-3-4\mathrm{i})\leqslant\frac{\pi}{3}$ and $|z|\leqslant|z-12|$. [3]

5 The cubic equation $x^3+2x^2+3x+4=0$ has roots α, β and γ.

(i) Use the substitution $x=\frac{1}{u+1}$ to obtain a cubic equation in u with integer coefficients. [4]

(ii) Hence, or otherwise, find the value of $\left(\frac{1}{\alpha}-1\right)\left(\frac{1}{\beta}-1\right)\left(\frac{1}{\gamma}-1\right)$. [3]

6 (i) Find $\sum_{r=1}^{n} r(r^2+r-7)$, giving your answer in a fully factorised form. [5]

A sequence u_0, u_1, u_2, . . . is defined by $u_0=5$, $u_n=u_{n-1}+n^3+n^2-7n$ for $n\geqslant 1$.

(ii) By considering $\sum_{r=1}^{n}(u_r-u_{r-1})$, find a formula for u_n in terms of n. [3]

[You do not need to factorise your answer.]

7 The complex number $a + 3\mathrm{i}$ is a root of the quadratic equation $z^2 - (7 + \mathrm{i})z + 16 + k\mathrm{i} = 0$,

where a and k are positive real numbers.

(i) Find the value of a and the value of k. [7]

(ii) Hence find the other root of the quadratic equation. [2]

8 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 1 & -2 \\ -1 & a & 0 \\ 2a & 3 & 1 \end{pmatrix}$,

where a is a real constant.

(i) Show that A is non-singular. [4]

(ii) Find $\mathbf{A}^{-1}$. [4]

(iii) Hence solve the three simultaneous equations given below. [3]

$$\begin{aligned} ax + y - 2z &= 2 \\ -x + ay \quad\quad &= 1 \\ 2ax + 3y + z &= 0 \end{aligned}$$

(iv) Explain briefly why these equations have a unique solution. [1]

9 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} m & m \\ 0 & 1 \end{pmatrix}$,

where m is a positive constant.

(i) Find $\mathbf{M}^2$ and $\mathbf{M}^3$ in terms of m. [4]

(ii) Hence suggest a suitable form for the matrix $\mathbf{M}^n$, where n is a positive integer, $n \geqslant 2$. [2]

(iii) Use induction to prove that your answer to part (ii) is correct. [4]

END OF QUESTION PAPER