OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4723/01

MATHEMATICS

Core Mathematics 3

TUESDAY 19 JUNE 2018:
Afternoon

DURATION: 1 hour 30 minutes
plus your additional time allowance
MODIFIED ENLARGED 36pt

Candidates answer on the Printed Answer Book sent with the standard paper or any suitable paper supplied by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4723/01 sent with the standard paper

List of Formulae (MF1) sent with the standard paper

OTHER MATERIALS REQUIRED:
Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (LK/TP) 172289/1

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INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer ALL the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer ALL the questions.

1 Use Simpson's rule with four strips to find an approximation to

$$\int_1^5 e^{\frac{2}{x}}\,dx.$$ **[3]**

2 Solve the inequality $|4x+3| < |x-8|$, showing all your working. **[5]**

3 A curve has equation $y = 3\ln(2x-a)$, where a is a positive constant. The curve crosses the x-axis at the point P.

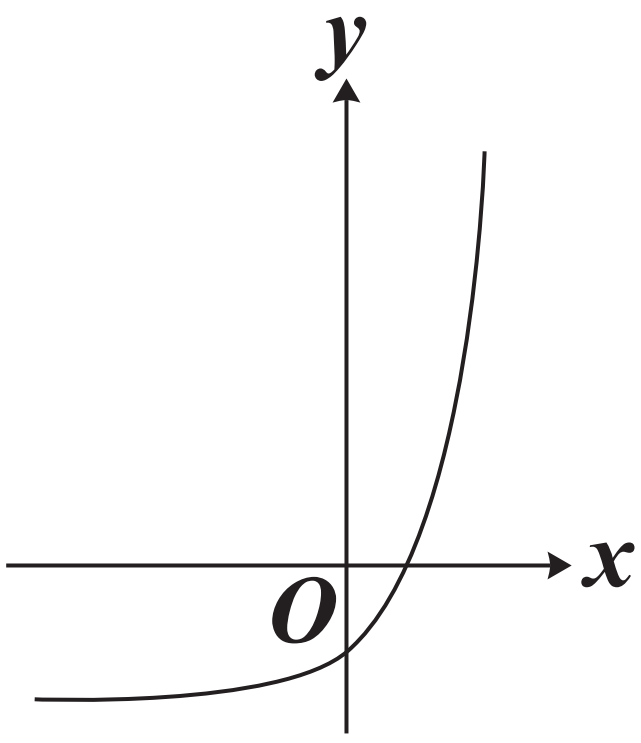
(i) Sketch the curve and determine the x-coordinate of P in terms of a. **[3]**

(ii) Find an equation of the tangent to the curve at P. **[3]**

4 A curve has equation $y = \dfrac{2x^2 + 1}{x^4 + 30}$.

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and hence determine the exact coordinates of the stationary points on the curve. [7]

5 The diagram below shows the curve $y = \mathrm{f}(x)$, where f is the function defined for all real values of x by $\mathrm{f}(x) = \mathrm{e}^{2x} - 3$.



(i) State the range of f. [1]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(iii) The curve $y = \mathrm{e}^{x}$ can be transformed to the curve $y = \mathrm{f}^{-1}(x)$ by means of a stretch, a translation and a reflection in that order. Give details of these three transformations. [3]

(iv) Sketch the curve $y = \left|\mathrm{f}(x)\right|$. Given that the equation $\left|\mathrm{f}(x)\right| = k$ has two distinct roots, determine the set of possible values of the constant k. [3]

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6 (a) A reservoir is being filled with water at a constant rate of 15 cubic metres per minute. At the instant when the depth of the water is x metres, the volume of water in the reservoir is V cubic metres where

$$V = 2(5 + 2x)^3 - 250.$$

Find the rate at which the depth of the water is increasing at the instant when $x = 1.6$. [4]

(b) In an experiment, the mass of a substance is increasing exponentially. At a time t hours after the start of the experiment, the mass, m grams, of the substance is given by

$$m = A\mathrm{e}^{\lambda t},$$

where A and λ are constants. It is given that, at the instant when $t = 15$, the mass is 48 grams and the rate at which the mass is increasing is 1.2 grams per hour.

(i) Find the values of A and λ. [4]

(ii) Find the value of t for which the mass is 70 grams. [2]

7 It is given that there is exactly one value of x, where $0 < x < \pi$, that satisfies the equation

$$3\tan 2x - 8\tan x = 4.$$

(i) Show that $t = \sqrt[3]{\frac{1}{2} + \frac{1}{4}t - \frac{1}{2}t^2}$, where $t = \tan x$. [3]

(ii) Show by calculation that the value of t satisfying the equation in part (i) lies between 0.7 and 0.8. [2]

(iii) Use an iterative process based on the equation in part (i) to find the value of t correct to 4 significant figures. Use a starting value of 0.75 and show the result of each iteration. [3]

(iv) Solve the equation $3\tan 4y - 8\tan 2y = 4$ for $0 < y < \frac{1}{2}\pi$. [2]

8 (a) Given that α satisfies the equation

$$3\sin(\alpha+60°)-3\cos(\alpha+30°)=\operatorname{cosec}^2\alpha,$$

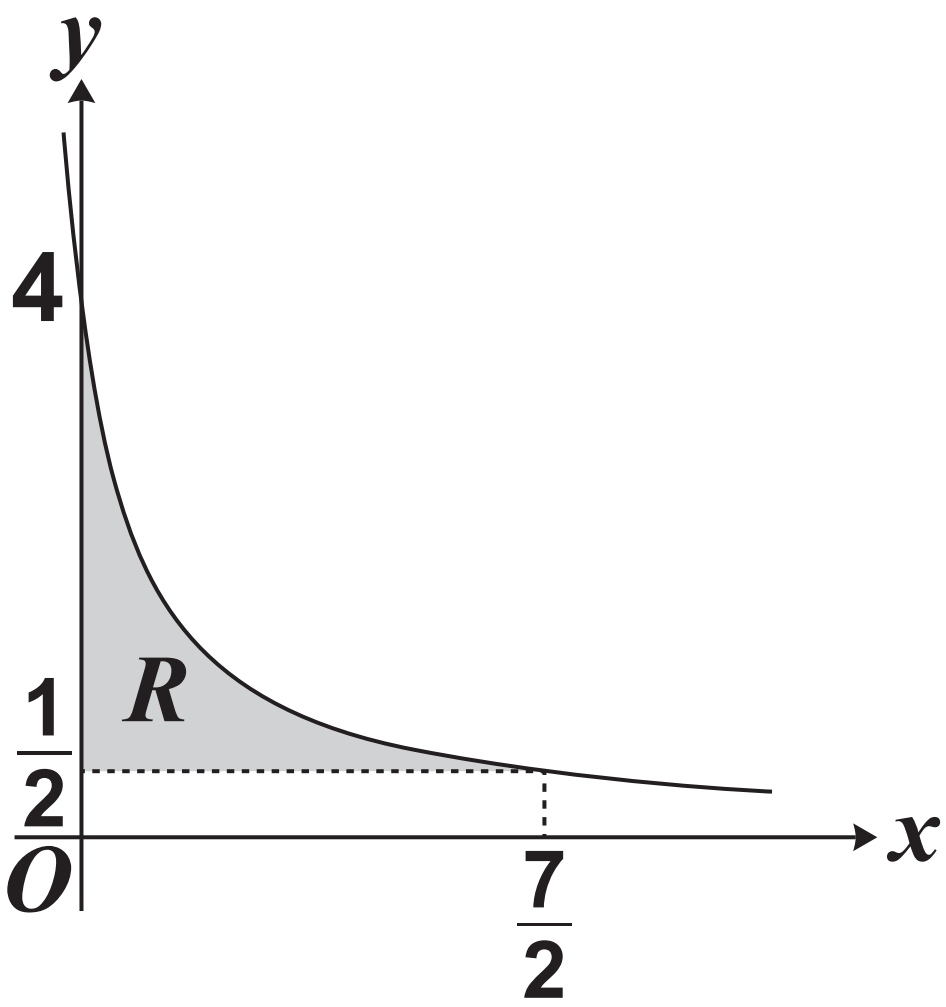
find the exact value of $\sin\alpha$. [4]

(b) It is given that β satisfies the equation

$$\sin 4\beta\sec^3\beta=8\sin\beta+2.$$

By first expressing $\sin 4\beta$ in terms of $\sin 2\beta$ and $\cos 2\beta$, find the exact value of $\sin\beta$. [7]

9 The diagram below shows part of the curve $y = \frac{4}{2x+1}$. The shaded region R is enclosed by the curve and the lines $x = 0$ and $y = \frac{1}{2}$.



(i) Find the exact area of R, giving your answer in the form $a\ln 2 + b$ where a and b are constants. [4]

(ii) The region R is rotated completely about the *y*-axis. Find the exact volume of the solid produced, giving your answer in the form $c\ln 2 + d$ where c and d are constants. [7]

END OF QUESTION PAPER