

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE
4735/01

MATHEMATICS
Probability & Statistics 4
QUESTION PAPER

TUESDAY 19 JUNE 2018: Afternoon
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book sent with the standard paper or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4735/01 sent with the standard paper
List of Formulae (MF1) sent with the standard paper
Insert for Question 5(i)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF



INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer ALL the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer ALL the questions.

- 1** A Wilcoxon signed-rank test is carried out at the 5% level of significance on a random sample of size 32. The hypotheses are $H_0: m = m_0$, $H_1: m < m_0$ where m is the population median and m_0 is a specific numerical value. The value obtained for the test statistic T is 162. Find the outcome of the test. [5]
- 2** The distances from home to work, in km, of 8 men and 5 women were recorded and are given below. The workers were chosen at random.

Men	4	7	10	13	16	17	20	21
Women	1	2	14	18	22			

Carry out a Wilcoxon rank-sum test at the 5% significance level to test whether there is a significant difference between the distances from home to work between men and women. [8]

- 3** Events A and B are such that $P(A) = 0.6$, $P(B) = 0.4$ and $P(A \cup B) = 0.8$.

(i) Find $P(A \cap B)$. [2]

(ii) Find $P(A \cap B')$. [1]

(iii) Find $P(A|B)$. [2]

Events A and B are as above and a third event C is such that $P(A \cup B \cup C) = 1$, $P(A \cap B \cap C) = 0.05$, $P(A \cap C) = P(B \cap C)$ and $P(A \cap B' \cap C') = 3P(A' \cap B \cap C')$.

(iv) Find $P(C)$. [5]

- 4 The random variable X has a χ^2 distribution with v degrees of freedom. The moment generating function of X is**

$$M_X(t) = (1 - 2t)^{-\frac{1}{2}v}.$$

- (i) Show that $E(X) = v$. [3]**
- (ii) Find $\text{Var}(X)$. [3]**
- (iii) Obtain the moment generating function of the sum Y of two independent χ^2 random variables, one with 6 degrees of freedom and the other with 8 degrees of freedom. [2]**
- (iv) Identify the distribution of Y . [2]**

- 5 The independent discrete random variables U and V can each take the values 1, 2 and 3, all with probability $\frac{1}{3}$. The random variables X and Y are defined as follows:**

$$X = |U - V|, Y = U + V.$$

- (i) In the Printed Answer Book or on the insert complete the table showing the joint probability distribution of X and Y . [3]**
- (ii) Find $\text{Cov}(X, Y)$. [4]**
- (iii) State with a reason whether X and Y are independent. [2]**
- (iv) Find $P(Y = 3|X = 1)$. [2]**

6 In each round of a quiz a contestant can answer up to three questions. Each correct answer scores 1 point and allows the contestant to go on to the next question. A wrong answer scores 0 points and the contestant is allowed no further question in that round. If all 3 questions are answered correctly 1 bonus point is scored, making a total score of 4 for the round. For a certain contestant, A , the probability of giving a correct answer is $\frac{3}{4}$, independently of any other question. The random variable X_r is the number of points scored by A during the r^{th} round.

- (i) Find the probability generating function of X_r . [4]**
- (ii) Use the probability generating function found in part (i) to find the mean and variance of X_r . [6]**
- (iii) Write down an expression for the probability generating function of $X_1 + X_2$ and find the probability that A has a total score of 4 at the end of two rounds. [3]**

- 7 Two independent observations X_1 and X_2 are made of a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{\theta} & 0 \leq x \leq \theta, \\ 0 & \text{otherwise,} \end{cases}$$

where θ is a parameter whose value is to be estimated.

- (i) Find $E(X)$. [1]
- (ii) Show that $S_1 = X_1 + X_2$ is an unbiased estimator of θ . [1]
- L is the larger of X_1 and X_2 , or their common value if they are equal.
- (iii) Show that the probability density function of L is $\frac{2l}{\theta^2}$ for $0 \leq l \leq \theta$. [4]
- (iv) Find $E(L)$. [1]
- (v) Find an unbiased estimator S_2 of θ , based on L . [1]
- (vi) Determine which of the two estimators S_1 and S_2 is the more efficient. [7]

END OF QUESTION PAPER

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