

GCE

Mathematics

Unit **4727**: Further Pure Mathematics 3

Advanced GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
 and 	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0 M1	Method mark awarded 0, 1
A0 A1	Accuracy mark awarded 0, 1
B0 B1	Independent mark awarded 0, 1
SC	Special case
	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance	
1 (i)	$\frac{ 3 - 2(-1) + 4(-2) - 11 }{\sqrt{1^2 + (-2)^2 + 4^2}}$ $= \frac{14}{\sqrt{21}}$	M1 A1 [2]	Condone lack of modulus signs for M1, oe, isw	$3.05(5\dots),$ $\frac{2}{3}\sqrt{21}$
	<p>Alternative 1: $1(3 + \lambda) - 2(-1 - 2\lambda) + 4(-2 + 4\lambda) = 11$ $\lambda = \frac{2}{3}$ Distance = $\frac{2}{3}\sqrt{1^2 + (-2)^2 + 4^2}$</p> <p>Alternative 2: use eg $\frac{\begin{pmatrix} 11-3 \\ 0-(-1) \\ 0-(-2) \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}}{\sqrt{1^2 + (-2)^2 + 4^2}}$ where (11,0,0) is a point in the plane</p>	M1 M1	For complete method with calculation errors	
	<p>Alternative 3: Use equation from (ii), distance = $\frac{11-3}{\sqrt{1^2 + (-2)^2 + 4^2}}$</p>	M1		
1 (ii)	$x - 2y + 4z = d$ where $d = 3 - 2(-1) + 4(-2)$ $x - 2y + 4z = -3$	M1 A1 [2]	Complete method	M1 for vector form only
2 (i)	Order of z^4 is 3	B1 [1]		
2 (ii)	$\{1, z^3\}, \{1, z^2, z^4\}$ $\{1\}, G$	B1 B1 B1 [3]	For Proper subsets, B1 for one correct and at most 1 incorrect, second B mark for both correct with no additional subsets B1 for both trivial subgroups	

Question	Answer	Marks	Guidance	
2 (iii)	<p>(z or z^5 is a generator for G and)</p> <p>3 (or 5) is a generator for H</p> <p>So both groups are cyclic therefore the two groups are isomorphic to each other</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Or G,H Abelian M1 and only 1 abelian Group of order 6 so Isomorphic A1</p> <p>Or orders of elements G,H correctly shown M1 and only 1 such Group of order 6 so Isomorphic A1</p> <p>Or States a correct isomorphism M1 ...so Isomorphic A1</p>	
3	<p>AE: $2\lambda^2 - \lambda - 3 = 0$</p> <p>$\lambda = -1, \frac{3}{2}$</p> <p>CF: $Ae^{\frac{3}{2}x} + Be^{-x}$</p> <p>PI: $y = axe^{-x}$</p> <p>$y' = ae^{-x} - axe^{-x}$</p> <p>$y'' = -2ae^{-x} + axe^{-x}$</p> <p>$2(ax - 2a) - (a - ax) - 3ax = 10 \Rightarrow a = \dots$</p> <p>GS: $y = Ae^{\frac{3}{2}x} + (B - 2x)e^{-x}$</p> <p>$x = 0, y = 0 \Rightarrow A + B = 0$</p> <p>$\frac{dy}{dx} = \frac{3}{2}Ae^{\frac{3}{2}x} + (B - 2x)(-e^{-x}) - 2e^{-x}$</p> <p>$\frac{dy}{dx} = -\frac{9}{2}, x = 0 \Rightarrow 3A - 2B = -5$</p> <p>$A = -1, B = 1$</p> <p>$y = -e^{\frac{3}{2}x} + (1 - 2x)e^{-x}$</p>	<p>B1</p> <p>B1ft</p> <p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[10]</p>	<p>Correct from their solution to auxiliary equation</p> <p>Differentiates twice using product rule</p> <p>Substitutes and attempts to solve for a</p> <p>From GS</p> <p>Differentiate their GS of correct form (from 2 real roots to auxiliary equation)</p> <p>Correctly substitutes into derivative of their GS and forms equation in A and B</p> <p>Must have “y = “</p>	<p>$a = -2$</p> <p>Allow one error</p>

Question	Answer	Marks	Guidance	
4 (i)	$(a*b)*c = abc + kc(a+b) + 12c + \dots$ $\dots k(ab + k(a+b) + 12 + c) + 12$ $a*(b*c) = abc + ka(b+c) + 12a + \dots$ $\dots k(a+bc + k(b+c) + 12) + 12$ $k^2 - k - 12 = 0 \Rightarrow k = \dots$ $k_1 = 4 \text{ and } k_2 = -3$	M1* A1 M1dep* A1 [4]	Attempts to expand either $a*(b*c)$ or $(a*b)*c$ Both correct Forms quadratic in k and attempts to solve Sc1 for correct solution from numerical values	Note that k_1 is AG
4 (ii)	$x*e = x \Rightarrow xe + 4(x+e) + 12 = x$ $e = \frac{-3x-12}{4+x} = -3$ $x*x^{-1} = -3 \Rightarrow xx^{-1} + 4(x+x^{-1}) + 12 = -3$ $x^{-1} = -\frac{15+4x}{4+x}$ <p>So there is no inverse element for $x = -4$ and so therefore the set of real numbers under the operation $*$ does not form a group</p>	M1* A1 M1dep* A1 [4]	Attempts to find identity element Sc1* for $e = -3$ without workings, but both marks if verifies result Uses their e in an attempt to find a general inverse element For clearly demonstrating that the element -4 does not have an inverse and correct conclusion	Must reach $e =$
5	$u = y^{1/2} \Rightarrow \frac{du}{dx} = \frac{1}{2u} \frac{dy}{dx}$ $\Rightarrow \frac{du}{dx} + \left(\frac{1}{1-x}\right)u = 2(1-x^2)$	M1* A1	Forms correct relationship between $\frac{dy}{dx}$ and $\frac{du}{dx}$ oe	Or $\frac{du}{dx} = \frac{1}{2} y^{-\frac{1}{2}} \frac{dy}{dx}$

Question	Answer	Marks	Guidance	
	$I = \exp\left(\int \frac{1}{1-x} dx\right) = e^{-\ln(1-x)}$ $= \frac{1}{1-x}$ $\frac{d}{dx}\left(\frac{u}{1-x}\right) = 2\left(\frac{1-x^2}{1-x}\right)$ $\frac{u}{1-x} = 2 \int \frac{1-x^2}{1-x} dx$ $\frac{u}{1-x} = 2\left(x + \frac{1}{2}x^2 + c\right)$ $u = (1-x)(2x + x^2 + A) \Rightarrow y = \dots$ $y = (1-x)^2(2x + x^2 + A)^2$	<p>M1*</p> <p>A1</p> <p>M1*</p> <p>A1</p> <p>M1dep*</p> <p>A1</p> <p>[8]</p>	<p>Attempts to find I by correctly integrating their P from form $\frac{du}{dx} + Pu = Q$ Allow $e^{\ln(1-x)}$</p> <p>Attempt to re-arrange and substitutes for u to get $y = \dots$</p>	<p>Allow inspection</p> <p>Must include constant of integration</p>

Question	Answer	Marks	Guidance	
6 (i)	$\cos 7\theta + i \sin 7\theta = (\cos \theta + i \sin \theta)^7$ $= c^7 + 7ic^6s - 21c^5s^2 - 35ic^4s^3 + 35c^3s^4 + \dots$ $\dots - 21ic^2s^5 - 7cs^6 - is^7$ $\cos 7\theta = c^7 - 21c^5s^2 + 35c^3s^4 - 7cs^6$ $\sin 7\theta = 7c^6s - 35c^4s^3 + 21c^2s^5 - s^7$ $\cot 7\theta = \frac{\cot^7 \theta - 21\cot^5 \theta + 35\cot^3 \theta - 7\cot \theta}{7\cot^6 \theta - 35\cot^4 \theta + 21\cot^2 \theta - 1}$ $\cot 7\theta = 0 \Rightarrow u^6 - 21u^4 + 35u^2 - 7 = 0$ <p>where $u = \cot \theta$</p> $7\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}$ $u = \cot\left(\frac{(2r-1)\pi}{14}\right), r = 1, 2, 3, 5, 6, 7 \text{ oe}$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A2</p> <p>[7]</p>	<p>soi by at least $\cot 7\theta = \frac{\operatorname{Re}\left((\cos \theta + i \sin \theta)^7\right)}{\operatorname{Im}\left((\cos \theta + i \sin \theta)^7\right)}$</p> <p>Take real and imaginary parts</p> <p>Sets $\cot 7\theta = 0$ and attempts to solve in terms of 7θ (dependent on correct numerator for $\cot 7\theta$)</p> <p>All correct with no extras (A1 for any three correct or for all 6 angles correct)</p>	<p>Condone CiS shorthand</p>
6 (ii)	$v^3 - 21v^2 + 35v - 7 = 0 \text{ where } v = u^2$ <p>has roots $\cot^2\left(\frac{\pi}{14}\right), \cot^2\left(\frac{3\pi}{14}\right)$ and $\cot^2\left(\frac{5\pi}{14}\right)$</p> <p>because $\cot\left(\frac{\pi}{14}\right) = -\cot\left(\frac{13\pi}{14}\right)$, etc.</p> <p>Given expression is equivalent to $\frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\sqrt{\alpha\beta\gamma}}$ where α, β, γ are the roots of the cubic in v</p> $\text{Given expression} = \frac{35}{\sqrt{7}}$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Recognises that numerator is $\sum \alpha\beta$ and denominator is $\sqrt{\alpha\beta\gamma}$</p> <p>If M0 A0, then Sc1 for 35/7 www</p>	

Question	Answer	Marks	Guidance	
7 (i)	<p>Vectors in plane $\begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$ and</p> $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$ <p>r. $\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$</p> $3x - 4y - 5z = 17$	<p>M1*</p> <p>M1dep* A1</p> <p>M1</p> <p>A1 [5]</p>	<p>Or $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ Or multiple(s)</p> <p>For M1, method shown or 2 correct elements</p> <p>Substituting any point on plane</p> <p>AEF (cartesian)</p>	<p>Allow 1 sign error, or method shown</p> <p>Check axb not bxa M0</p>
7 (ii)	$\cos \alpha = \frac{\begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}}{\sqrt{3^2 + (-4)^2 + (-5)^2} \sqrt{2^2 + (-1)^2 + 3^2}} = \frac{-5}{\sqrt{50}\sqrt{14}}$ <p>$\theta = \alpha - \frac{1}{2}\pi$</p> <p>$\theta \approx 10.9^\circ$ or 0.190</p>	<p>M1*</p> <p>M1dep* A1 [3]</p>	<p>Method seen or 1 calc error in denominator ft from (i)</p> <p>Can use $\sin \theta$</p>	<p>Implied by unsimplified cosine rule plus 79.10... or 100.8... or 1.380... or 1.760...</p>

Question	Answer	Marks	Guidance	
7 (iii)	$3(p + \lambda q) - 4(2 - 6\lambda) - 5(4 + 12\lambda) = 17$ $3p - 8 - 20 = 17 \text{ and } 3q + 24 - 60 = 0$ $p = 15, q = 12$	<p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>[3]</p>	<p>Substitutes line into their plane, or two points</p> <p>or 1 point and uses (their n). $\begin{pmatrix} q \\ -6 \\ 12 \end{pmatrix} = 0$</p> <p>Obtain 2 equations and attempt to solve for both p and q</p> <p>If zero scored, SC1 for $p = 15$</p>	<p>NB q given</p>
7 (iv)	$\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} = \begin{pmatrix} 13 \\ 16 \\ -5 \end{pmatrix} \text{ or multiple}$ $\mathbf{r} \cdot \begin{pmatrix} 13 \\ 16 \\ -5 \end{pmatrix} = \begin{pmatrix} 15 \\ 2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 13 \\ 16 \\ -5 \end{pmatrix}$ $\mathbf{r} \cdot \begin{pmatrix} 13 \\ 16 \\ -5 \end{pmatrix} = 207$	<p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>[3]</p>	<p>Method shown or 2 correct elements Ft their n</p> <p>Uses a point on the line Ft their $\begin{pmatrix} p \\ 2 \\ 4 \end{pmatrix}$</p> <p>oe (vector)</p>	

Question	Answer	Marks	Guidance	
8 (i)	$\sum_{r=1}^n z^{2r-1} = z + z^3 + z^5 + \dots + z^{2n-1} = \frac{z(1-(z^2)^n)}{1-z^2}$ $\frac{z(1-(z^2)^n)}{1-z^2} = z \left(\frac{1-z^{2n}}{1-z^2} \right) = \frac{z-z^{2n+1}}{1-z^2} \text{ divide through by } z$ $= \frac{1-z^{2n}}{z^{-1}-z}$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>for correct use of correct GP formula.</p> <p>AG Complete argument to include either “$a = z, r = z^2$” or explicit “divide through by z”</p>	<p>“$a = 1$” scores zero unless z extracted initially. $a = z^{-1}$ can score M1.</p> <p>Accept final answer with z^{-1} written as $1/z$</p>
8 (ii)	$\sum_{r=1}^n \sin(2r-1)\theta = \text{Im} \left[\frac{1-z^{2n}}{z^{-1}-z} \right] \text{ with } z = \cos\theta + i\sin\theta$ $\frac{1-z^{2n}}{z^{-1}-z} = \frac{1-(\cos 2n\theta + i\sin 2n\theta)}{(\cos\theta - i\sin\theta) - (\cos\theta + i\sin\theta)}$ $= \frac{i - i\cos(2n\theta) + \sin(2n\theta)}{2\sin\theta}$ $\sum_{r=1}^n \sin(2r-1)\theta = \frac{1-\cos(2n\theta)}{2\sin\theta}$ $\sin^2(n\theta) = \frac{1}{2}(1-\cos(2n\theta))$ $\sum_{r=1}^n \sin(2r-1)\theta = \frac{2\sin^2(n\theta)}{2\sin\theta} = \frac{\sin^2(n\theta)}{\sin\theta}$	<p>B1</p> <p>M1</p> <p>M1*</p> <p>A1</p> <p>M1dep*</p> <p>A1</p> <p>[6]</p>	<p>Seen or implied</p> <p>Substitutes and uses de Moivre</p> <p>Rationalise with imaginary terms only in the numerator</p> <p>Equating imaginary part</p> <p>NB AG – simplify using trig. identity for $\sin^2(n\theta)$ and at least one intermediate step</p>	

Question	Answer	Marks	Guidance
	<p>Alternative 1</p> $\frac{1-z^{2n}}{z^{-1}-z} = \left(\frac{z^n - z^{-n}}{z - z^{-1}} \right) z^n \quad \text{M1,} = \left(\frac{\sin n\theta}{\sin \theta} \right) (\cos n\theta + i \sin n\theta)$ <p>M1A1</p> $\sum_1^n \sin(2r-1)\theta = \text{Im(previous)} \quad \text{M1,} = \frac{\sin^2 n\theta}{\sin \theta} \quad \text{A1 .}$		B1 as above
	<p>Alternative 2</p> $\frac{1-z^{2n}}{z^{-1}-z} = \frac{1-(\cos n\theta + i \sin n\theta)^2}{-2i \sin \theta} = \frac{1-\cos^2 n\theta + \sin^2 n\theta - 2i \cos n\theta \sin n\theta}{-2i \sin \theta}$ <p>M1</p> $= \frac{i(1-\cos^2 n\theta + \sin^2 n\theta) + 2 \cos n\theta \sin n\theta}{2 \sin \theta} \quad \text{M1A1}$ $\sum_1^n \sin(2r-1)\theta = \text{Im(previous)} \quad \text{M1,} = \frac{\sin^2 n\theta}{\sin \theta} \quad \text{A1}$ <p>(using Pythagoras).</p>		B1 as above Note change denomiannot in first step. M1 by middle step(delete third step)
	<p>Alternative 3</p> $\sum_1^n \sin(2r-1)\theta = \frac{1}{2i} \sum_1^n (z^{2r-1} - z^{1-2r}) = \frac{1}{2i} \left(\frac{1-z^{2n}}{z^{-1}-z} - \frac{1-z^{-2n}}{z-z^{-1}} \right)$ <p>M1</p> $= \frac{1}{2i} \left(\frac{-2 + z^{2n} + z^{-2n}}{z - z^{-1}} \right) = \frac{-2 + 2 \cos 2n\theta}{-4 \sin \theta} \quad \text{M1M1A1,}$ $= \frac{\sin^2 n\theta}{\sin \theta} \quad \text{A1 .}$		B1as above M1 use sin theta formula M1 use both formulae implied by (i) M1A1 for conversion to rational trig function

Question	Answer	Marks	Guidance	
8 (iii)	$\int_0^{\frac{1}{6}\pi} \frac{\sin^2 3\theta}{\sin \theta} d\theta = \int_0^{\frac{1}{6}\pi} (\sin \theta + \sin 3\theta + \sin 5\theta) d\theta$ $= \left[-\cos \theta - \frac{1}{3} \cos 3\theta - \frac{1}{5} \cos 5\theta \right]_0^{\frac{1}{6}\pi}$ $= \left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{10} \right) + \left(1 + \frac{1}{3} + \frac{1}{5} \right)$ $= \frac{1}{15} (23 - 6\sqrt{3})$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Correct three terms plus integral sign present or implied</p> <p>Attempts integration and correct substitution</p>	<p>0.840(5)...gains 2/3</p>
Total		72		

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