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Contents

Introduction 3
Paper 4726/01 series overview 4
Question 1(i) 5
Question 1(ii) 5
Question 1(iii) 5
Question 2(i) 5
Question 2(ii) 6
Question 3 6
Question 4(i) 6
Question 4(ii) 6
Question 4(iii) 6
Question 5 7
Question 6(i) 7
Question 6(ii) 7
Question 6(iii) 7
Question 7(i) 8
Question 7(ii) 8
Question 7(iii) 8
Question 8(i) 9
Question 8(ii) 9
Question 8(iii) 9
Question 9(i) 9
Question 9(ii) 10
Question 9(iii) 10

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4726/01 series overview

There was a spread of marks for this paper, with a good proportion of candidates obtaining high marks.

As with the report of last year, one concern remains the inability of candidates to produce good sketches. It may be that candidates are entering the required function into their graphical calculator, but if the window is not appropriate then the resulting sketch may miss some important features.

Another main concern is the lack of attention to the requirement on accuracy. If a question asks for an answer correct to 4 decimal places then candidates will be penalised if they fail to do this. In this context, over accuracy is treated the same way as under accuracy.

Apart from resit candidates in 2019, this is the last series for this specification. Centres should be aware that the comments made in this report on specific questions will be applicable in the new A level Further Mathematics qualification (H245) for the pure papers Y540 and Y541.

Question 1(i)

1 The equation of a curve is $y = \dfrac{x^2+1}{(x-1)^2}$.

(i) Write down the equations of the asymptotes. **[2]**

For most candidates this tended to be an easy starter, though some had to engage in a fair amount of algebra to deduce the equations.

Question 1(ii)

(ii) Find the coordinates of the stationary point on the curve and hence determine the range of y. **[6]**

The majority of candidates began their response by differentiating the given function, usually using the Quotient rule and equating to zero; most achieved the turning point and many stated that $y \geq \frac{1}{2}$. Very few candidates adopting this approach attempted to show that this point was a minimum. Any of the standard methods to determine the nature of a stationary point were acceptable, but usually omitted. The alternative approach was to form a quadratic in *x* and then use the discriminant either to find the values for which *y* exists or when a horizontal line formed a tangent at the turning point; candidates using this method tended to be more confident in their approach and it was pleasing to see some excellent entirely correct solutions.

Question 1(iii)

(iii) Sketch the curve. Indicate the asymptotes in your sketch. **[2]**

Those who did not find the correct vertical asymptote were unable to achieve marks in this part, but it was only a tiny minority. Asymptotes should be clearly labelled and curves should approach the asymptotes. A common error is for the curve to begin to approach the asymptote, but then to curve away from it.

Key points such as axis intercepts and turning point(s) should be shown on such sketches.

Question 2(i)

2 **(i)** Using the definition of $\cosh x$ in terms of e^x and e^{-x}, show that $\cosh 2x = 2\cosh^2 x - 1$. **[2]**

This was a standard proof and the majority of candidates found no difficulty with it. Candidates should remember to write out their working in full and ensure that all algebra and arithmetic are correct.

Question 2(ii)

(ii) Find $\int_0^1 \cosh^2 3x \, dx$, giving your answer in the form $A + B\sinh C$, where A, B and C are constants to be determined. **[3]**

Most candidates were able to score full marks on this question, though some did not take the hint of part (i) and wrote down an incorrect double angle formula.

Question 3

3 The equation of a curve is $y = \cosh x - 2\sinh 2x$.

Show that the curve has no turning points. **[5]**

Most candidates started the question well and the majority used the double angle formula again successfully to obtain a quadratic in sinh*x*, which they were able correctly to demonstrate that there were no real roots.

The candidates who turned the equation for the gradient function into exponentials were less successful. The result is a quartic equation in e^x, which candidates could not demonstrate had no roots.

Candidates should be aware that such questions require a full algebraic proof and that they should not simply appeal to their calculator.

Question 4(i)

4 It is given that $I_n = \int_0^1 x^n e^{x-1} \, dx$ for $n \geqslant 1$.

(i) Show that $I_n = 1 - nI_{n-1}$ for $n \geqslant 2$. **[3]**

A very simple reduction formula found through integration by parts provided easy marks for almost the entire cohort.

Question 4(ii)

(ii) Find the exact value of I_4. **[4]**

Many candidates did not appreciate that what had been proved in part (i) was valid for $n > 0$ and so to use I_0 as a starting value was inadmissible.

Question 4(iii)

(iii) For the curve $y = x^n e^{x-1}$, where $n \geqslant 1$, find $\frac{dy}{dx}$ and sketch the curve for $0 \leqslant x \leqslant 1$.

Deduce that $0 < I_n < 1$ for all $n \geqslant 1$. **[5]**

The gradient function was usually found, but candidates did not realise that the question asked for it for a purpose; subsequent attempts to prove that the value of the integral lay between 0 and 1 were often unsuccessful. Candidates who used the gradient function and made some deductions about its value in the given range managed this part well.

Question 5

5 By using the substitution $t = \tan\frac{1}{2}x$, find the exact value of $\int_0^{\frac{\pi}{2}} \frac{1}{\sin x + 1}\,dx$. **[5]**

For the vast majority of candidates this question provided a rich source of marks. Where candidates were not successful, the problem usually lay in lower demand algebraic work.

Question 6(i)

6 It is given that $y = \tan^{-1}3x$.

(i) Using the derivative of $\tan^{-1}x$ given in the List of Formulae (MF1), find $\frac{dy}{dx}$. **[2]**

This was a simple application of the chain rule and the majority of candidates obtained the correct result.

Question 6(ii)

(ii) Hence show that $(1+9x^2)\frac{d^3y}{dx^3} + 36x\frac{d^2y}{dx^2} + 18\frac{dy}{dx} = 0$. **[4]**

Candidates who wrote $(1+9x^2)\frac{dy}{dx} = 3$ were then able to differentiate easily to the required result and a very simple 4 marks were gained in a very few lines. The majority, however, proceeded by finding the second and third differentials through product or quotient rule and then substituting every term into the given expression; in doing this the risk of making numerical or algebraic errors is considerable, but the time taken is also a significant penalty. Questions such as this have been common and the former approach is highlighted as being far more efficient overall. Many candidates taking this approach spread their work over two or even three pages and in many cases the result was 'fudged' so could not be awarded full marks.

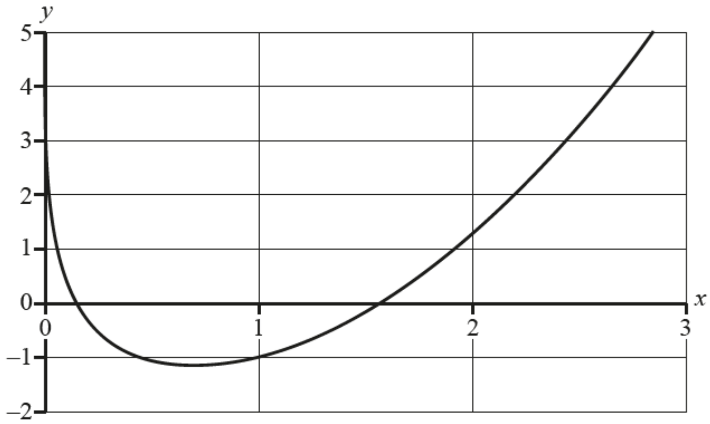
Question 6(iii)

(iii) Find the Maclaurin series for y up to and including the term in x^3. **[3]**

Even if candidates were unsuccessful in part (ii), the result was given and so it should still have been possible to obtain this expansion. Most candidates did indeed give the correct two terms, but a large proportion of candidates did not achieve that.

Question 7(i)

7 You are given that the equation $x^2 - \ln x - 2 = 0$ has two roots, α and β, where $0 < \alpha < 1$. The diagram shows a sketch of the curve $y = x^2 - \ln x - 2$.



(i) The Newton-Raphson method is to be used to find α.
Explain what happens when the initial values $x_1 = 0.5$ and $x_1 = 1$ are used. **[2]**

The word *explain* seemed to be missed by many candidates. In this context the Newton-Raphson method is founded upon the behaviour of the tangent; this is therefore the primary focus required in the answer and candidates who did not draw the tangents could not therefore gain any marks. The tangent at *x* = 0.5 crosses the *x*-axis at a negative value, which causes difficulties with ln x_2 and the process fails as a result. The tangent at *x* = 1 yielded a more successful process converging to the other root β. A number of explanations were far too brief; comments such as "it does not find α", are simply too vague at this level. The common error was to assume via a poor sketch of the tangent that this starting value gave a converging sequence to α. Numerical work here gained no credit.

Question 7(ii)

(ii) Show that the Newton-Raphson iterative formula for this equation can be written in the form

$$x_{r+1} = \frac{x_r(x_r^2 + \ln x_r + 1)}{(2x_r^2 - 1)}$$ **[3]**

This was a familiar question and it was pleasing to see a good proportion of detailed and fully correct responses. The main reason for losing marks was a tendency to move too quickly to the final answer without showing the required steps; others provided perfect algebraic explanations, but omitted the subscripts in the final answer.

Question 7(iii)

(iii) Using the initial value $x_1 = 0.1$, find x_2 and x_3. Find α correct to 5 decimal places. **[2]**

Most candidates had no difficulty with this part, though many candidates ignored the demands on accuracy.

Question 8(i)

8 You are given that the equation $x^3 + x - 4 = 0$ has a root, α, where $1 < \alpha < 2$.

The iterative formula $x_{r+1} = \sqrt[3]{(4 - x_r)}$ with $x_1 = 1.4$ is to be used to find α.

(i) Demonstrate that the iterative values converge to α by sketching a cobweb diagram. **[2]**

The main error in this problem was appreciating that the cobweb is based on the graph of $y = \sqrt[3]{(4 - x)}$ and $y = x$, rather than the original cubic. Providing the iteration begins on the curve, the cobweb follows very easily. Some candidates had difficulty in providing a clear diagram and the sketch of the curve was often incorrect.

Question 8(ii)

(ii) Use the iterative formula to find α correct to 4 decimal places. Show the result of each step of the iteration correct to 5 decimal places. **[3]**

This was another question where the accuracy stated in the question was not followed. Candidates should take care to read the question carefully.

Question 8(iii)

(iii) The error, e_n is defined by $e_n = \alpha - x_n$.

Using the value of α found in part **(ii)**, evaluate $\frac{e_3}{e_2}$ and $\frac{e_4}{e_3}$.

Comment on the values of $\frac{e_3}{e_2}$ and $\frac{e_4}{e_3}$ in relation to the gradient of the curve $y = \sqrt[3]{(4 - x)}$ at $x = \alpha$. **[3]**

It was expected that candidates would find F'(x) at the root and comment on the similarity of values. A significant number did not do this, merely making general comments.

Question 9(i)

9 A curve has polar equation $r = 2\cos\theta - 1$ for $-\frac{\pi}{3} \leqslant \theta \leqslant \frac{\pi}{3}$.

(i) Show that the line $\theta = 0$ is a line of symmetry. **[2]**

There were many full and detailed responses to this question. The main error was to calculate the values of r at $\theta = \frac{-\pi}{3}$ and $\frac{\pi}{3}$ then assume that because these were the same then they would be the same for all values of θ.

Question 9(ii)

(ii) Sketch the curve, indicating

- the point on the curve where $\theta = 0$,
- the tangents at the pole. **[4]**

There were many correct, detailed and well drawn sketches here, but many lost marks through the usual inability to sketch well including the important features of the graph. Candidates should also know that the specification states that the usual notation $r \geqslant 0$ is used, but that calculators will usually include the whole graph.

Question 9(iii)

(iii) Find the area enclosed by the curve. **[5]**

This is a standard question and most candidates obtained full marks. In questions where there is symmetry it is obviously acceptable to find half the area and to double it. However, when this is done, candidates should state what they are doing so that it is clear that they know that the formula includes ½, but that by doubling this disappears.

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