OCR
RECOGNISING ACHIEVEMENT

LEVEL 3 CERTIFICATE

MATHEMATICS FOR ENGINEERING

H860/02

Paper 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Insert (inserted)
- List of Formulae (MF1)

Other Materials Required:

- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

* H 8 6 0 0 2 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- The total number of marks for this paper is **40**.
- This document consists of **4** pages. Any blank pages are indicated.

Turn over

1 Use the information given in Table 1 and Table 2 of the pre-release material to answer this question.

(a) State, with reasons, whether the terminal velocity has been approximately reached for each parachute just before the canister returns to the ground. **[5]**

(b) Using the equation of motion for the falling canister

$$\frac{d^2x}{dt^2} = g - kS\frac{dx}{dt},$$

determine the value of k. **[4]**

2 **(a)** For the motion of the canister during its ascent, derive formulae, in terms of t and its initial upward velocity V_0, for

(i) its acceleration,

(ii) its velocity,

(iii) its height. **[5]**

(b) Calculate the initial velocity required for the canister to reach a maximum height of 200 m. **[5]**

3 **(a)** **(i)** Show that the differential equation

$$\frac{d^2x}{dt^2} = g - kS\frac{dx}{dt}$$

can be written as

$$\frac{dv}{dt} = g - kSv.$$

(ii) Solve the differential equation in part **(i)** to show that

$$v = \frac{g}{kS}\left(1 - e^{-kSt}\right).$$ **[8]**

(b) Hence show that, t s after the canister reaches its maximum height,

$$x = \frac{g}{kS}\left(t + \frac{1}{kS}\left(e^{-kSt} - 1\right)\right).$$ **[4]**

4 **(a)** Show that the terminal velocity of the canister is $\frac{g}{kS}$. **[1]**

(b) In a particular experiment, $k = 0.25$ and the terminal velocity of the canister is $5\,\mathrm{m\,s^{-1}}$.

(i) Calculate the diameter, d, as defined in the pre-release material.

(ii) Calculate the time elapsed from the canister being at its maximum height to reaching half its terminal velocity. **[8]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

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- You must **not** bring your previous copy of the pre-release material into the examination.

Testing small parachutes

A parachute is a device designed to reduce the velocity of a falling object to an acceptable value.

An object freely falling to Earth in a vacuum experiences a constant acceleration of approximately $9.8\,\mathrm{m\,s^{-2}}$, regardless of its shape or mass. When an object falls through air, it experiences an upward drag which causes the acceleration to decrease towards zero. When this happens, the object approaches a constant velocity referred to as its terminal velocity. For an object of low density, such as a feather, the drag is relatively large and so the terminal velocity will be low. For small dense objects, such as stones, there will be minimal drag and so the terminal velocity will be higher.

In the case of a person, the terminal velocity should be about $5\,\mathrm{m\,s^{-1}}$ or less in order to minimise possible injury on landing. For other falling objects, such as food parcels or medical supplies, the acceptable terminal velocity may be different. By opening a parachute attached to a falling object the drag is increased; this will make the terminal velocity lower than it would have been.

Engineers have devised an experiment to test the properties of a new parachute used to drop a particular canister gently. They are interested in the vertical acceleration, velocity and height of the canister during its journey. The engineers enclose the parachute in a tightly closed bag and attach this to the canister. Rather than using expensive aeroplane flights, the engineers project the canister, together with the attached bag, vertically into the air using a high-powered catapult. When the canister reaches its maximum height, the bag opens automatically and the parachute is immediately deployed. The canister then falls to the ground under the combined influence of gravity and the drag created by the parachute. For the purpose of the model it is assumed that there is no horizontal movement of the parachute.

When deployed, the parachute opens as a canopy. The base of the canopy is circular with diameter d m and area $S\,\mathrm{m}^2$ as shown in Fig. 1. The drag now created by the parachute is directly proportional to S.

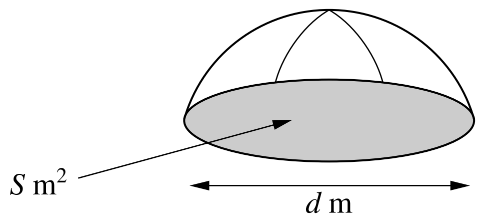

Fig. 1

While the canister and bag are travelling vertically upwards it is assumed that there is zero drag and the only force acting is that of gravity. When the canister begins to fall, its motion is modelled on the basis that its acceleration is equal to gravitational acceleration less an amount that is proportional to the product of its velocity, $v\,\mathrm{m\,s^{-1}}$ and the area $S\,\mathrm{m}^2$.

The motion of the falling canister is modelled by the equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = g - kS\frac{\mathrm{d}x}{\mathrm{d}t},$$

where

k is a positive constant,
$g = 9.8\,\mathrm{m\,s^{-2}}$,
x is the distance travelled, in metres, measured from the initial maximum height,
t is the time, in seconds, measured from the time at which the canister begins its descent.

Fig. 2 shows the canister and parachute in descent.

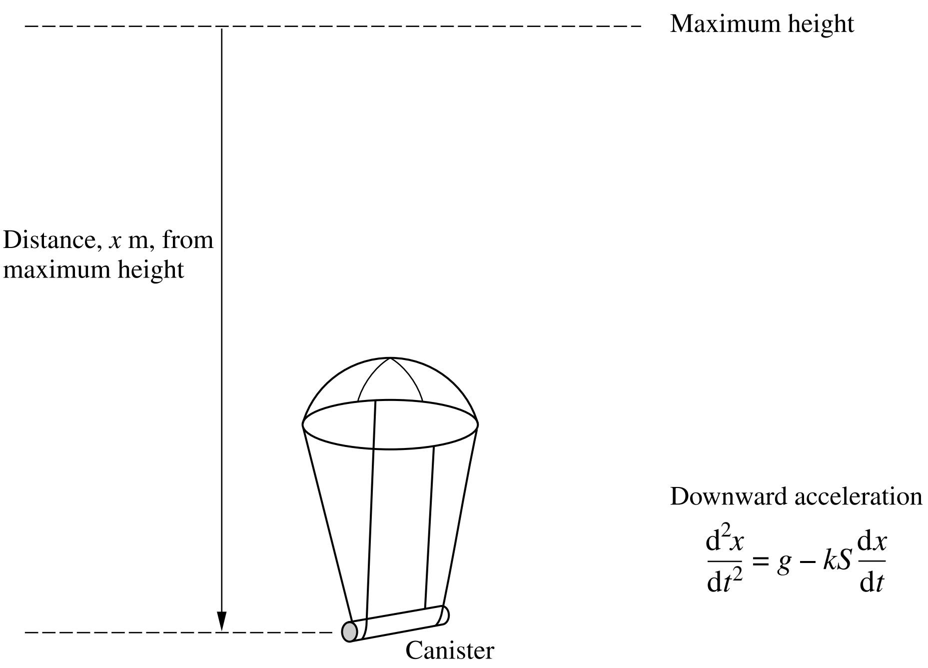

Fig. 2

Three parachute sizes, with diameters 1 m, 1.5 m and 2 m, are tested. For each of these the canister is projected vertically upwards so that it reaches a height of 200 m. The time of descent and the velocity of the canister just before reaching the ground are recorded in Table 1.

Parachute diameter (d m)	1	1.5	2
Time of descent (t s)	7.77	9.96	14.41
Velocity just before reaching the ground (v m s^{-1})	43.66	26.90	15.59

Table 1

The same three parachutes are tested again but this time the canister is projected so that it reaches a maximum height of 400 m. The same quantities are recorded and are shown in Table 2.

Parachute diameter (d m)	1	1.5	2
Time of descent (t s)	11.78	17.25	27.24
Velocity just before reaching the ground (v m s^{-1})	52.58	27.66	15.59

Table 2

These results can be used to design new parachutes that would allow similar canisters to be dropped without damage.

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