OCR

RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4727/01**

MATHEMATICS

Further Pure Mathematics 3

MONDAY 18 JUNE 2007 Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 **(i)** By writing z in the form $re^{i\theta}$, show that $zz^* = |z|^2$. [1]

(ii) Given that $zz^* = 9$, describe the locus of z. [2]

2 A line l has equation $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k} + t(\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$ and a plane Π has equation $8x - 7y + 10z = 7$. Determine whether l lies in Π, is parallel to Π without intersecting it, or intersects Π at one point. [5]

3 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = e^{3x}.$$ [6]

4 Elements of the set $\{p, q, r, s, t\}$ are combined according to the operation table shown below.

	p	q	r	s	t
p	t	s	p	r	q
q	s	p	q	t	r
r	p	q	r	s	t
s	r	t	s	q	p
t	q	r	t	p	s

(i) Verify that $q(st) = (qs)t$. [2]

(ii) Assuming that the associative property holds for all elements, prove that the set $\{p, q, r, s, t\}$, with the operation table shown, forms a group G. [4]

(iii) A multiplicative group H is isomorphic to the group G. The identity element of H is e and another element is d. Write down the elements of H in terms of e and d. [2]

5 **(i)** Use de Moivre's theorem to prove that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1.$$ [4]

(ii) Hence find the largest positive root of the equation

$$64x^6 - 96x^4 + 36x^2 - 3 = 0,$$

giving your answer in trigonometrical form. [4]

6 Lines l_1 and l_2 have equations

$$\frac{x-3}{2} = \frac{y-4}{-1} = \frac{z+1}{1} \quad \text{and} \quad \frac{x-5}{4} = \frac{y-1}{3} = \frac{z-1}{2}$$

respectively.

(i) Find the equation of the plane Π_1 which contains l_1 and is parallel to l_2, giving your answer in the form $\mathbf{r}.\mathbf{n} = p$. [5]

(ii) Find the equation of the plane Π_2 which contains l_2 and is parallel to l_1, giving your answer in the form $\mathbf{r}.\mathbf{n} = p$. [2]

(iii) Find the distance between the planes Π_1 and Π_2. [2]

(iv) State the relationship between the answer to part **(iii)** and the lines l_1 and l_2. [1]

7 **(i)** Show that $(z - e^{i\phi})(z - e^{-i\phi}) \equiv z^2 - (2\cos\phi)z + 1$. [1]

(ii) Write down the seven roots of the equation $z^7 = 1$ in the form $e^{i\theta}$ and show their positions in an Argand diagram. [4]

(iii) Hence express $z^7 - 1$ as the product of one real linear factor and three real quadratic factors. [5]

8 **(i)** Find the general solution of the differential equation

$$\frac{dy}{dx} + y\tan x = \cos^3 x,$$

expressing y in terms of x in your answer. [8]

(ii) Find the particular solution for which $y = 2$ when $x = \pi$. [2]

9 The set S consists of the numbers 3^n, where $n \in \mathbb{Z}$. ($\mathbb{Z}$ denotes the set of integers $\{0, \pm1, \pm2, \ldots\}$.)

(i) Prove that the elements of S, under multiplication, form a commutative group G. (You may assume that **addition** of integers is associative and commutative.) [6]

(ii) Determine whether or not each of the following subsets of S, under multiplication, forms a subgroup of G, justifying your answers.

(a) The numbers 3^{2n}, where $n \in \mathbb{Z}$. [2]

(b) The numbers 3^n, where $n \in \mathbb{Z}$ and $n \geqslant 0$. [2]

(c) The numbers $3^{(\pm n^2)}$, where $n \in \mathbb{Z}$. [2]