RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Report on the Units

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REPORTS FOR THE UNITS

Chief Examiner's Report – Pure Mathematics

Inevitably the four Core Mathematics and three Further Pure Mathematics reports which follow concentrate on aspects of the candidates' performance where improvement is possible. However, this should not obscure the fact that a significant number of candidates recorded full marks on units and produced solutions which were a pleasure for examiners to assess. Many candidates demonstrated a most impressive level of mathematical ability and insight which enabled them to meet the various challenges posed by these papers; precision, command of correct mathematical notation and excellent presentational skills were evident on many scripts.

When asked for a sketch graph, the majority of candidates wisely used the answer booklet to provide the answer rather than laboriously plotting points on graph paper. Often the sketches were fine but graph sketching is a skill which generally merits attention. Sketches must be executed with care and, where appropriate, the following aspects should be considered.

- The sketch should show the essential shape of the curve and should extend sufficiently far for that shape to be seen.
- The sketch should be located correctly.
- Axes should be drawn with a ruler.
- A y-intercept value should be shown and, where they are already available, any x-intercept values should be shown.
- Further values on axes might be needed in certain cases. For example, a sketch of $y = 2 + \cos x$ cannot be fully assessed unless a value such as $180°$ or $360°$ is marked on the x-axis.
- The shape of the curve close to an asymptote should be correct. For example, with $y = \frac{1}{x}$, the curve should not touch either asymptote nor should it carelessly be shown to drift away from it as the absolute value of x or y increases.

4721: Core Mathematics 1

General Comments

The majority of candidates were well prepared for this paper and were able to try every question, usually working through the paper in question order. Most candidates appeared to have adequate time to finish the paper although there was evidence of a small number unable or unwilling to try even the most straightforward questions. In general, answers were neatly presented and working was easy to follow, although there were plenty of exceptions to this.

It was pleasing to note that fewer candidates used graph paper when asked to sketch graphs, although candidates were expected to draw axes with a ruler.

Most candidates solved the quadratic equations on the paper successfully although many made this harder than intended by failing to choose the simplest method. For example, candidates often tried to complete the square in Q10(i) or used the quadratic formula in Q8 (i), when factorisation would have been quicker and more straightforward.
The full range of marks from 0 to 72 was awarded.

Comments on Individual Questions

1) This opening question was tackled confidently by candidates, all but the very weakest able to square at least one, if not both, of the expressions correctly. Unfortunately, relatively few scored full marks because of an inability to deal with the signs correctly. In the case of some very high scoring candidates, the only mark that they dropped across the entire paper was the third mark of this question!

2) (a) (i) While most candidates knew the general shape of the curve $y = \frac{1}{x}$, they were often careless with their sketches, with curves parallel to or moving away from the axes. The asymptotes often looked more like the lines $x = 0.8$ and $y = 0.8$ than $x = 0$ and $y = 0$. However, it was encouraging that only a very tiny minority tried to plot the curves accurately.

(ii) Nearly all candidates were able to sketch this curve correctly, realising that it passed through the origin, although a few appeared to think that it was a vertical translation of the curve $y = x^2$.

(b) Only the strongest candidates scored both marks here. Although most candidates correctly identified the transformation as a stretch and gained one mark, they often stated that it was a stretch in the x direction, scale factor 8 or $\frac{1}{8}$, or that it was an ‘upwards’ stretch by 8 units. A few candidates thought that it was a translation or an enlargement. Others described what happened to the coordinates (e.g. ‘the y-coordinates get 8 times bigger’) which was not eligible for any marks.

3) (i) This question was well answered by the vast majority of candidates, although a small number did not attempt it at all, seeming unfamiliar with the rules for manipulation of surds. The most commonly seen error was $3(\sqrt{2}\times\sqrt{5})$ becoming $3\sqrt{2}\times 3\sqrt{5}$.

(ii) This part of the question was also answered consistently well. Apart from the few whose first line of working was $\sqrt{625}$, an error from which they were unable to recover, nearly all candidates simplified and combined the surds correctly.

4) (i) Although there was a small proportion of candidates who left this part blank, presumably not understanding the word 'discriminant', the majority were able to gain both marks, with pleasingly few instances of $\sqrt{b^2-4ac}$ seen. However, having reached the correct answer of $16-4k^2$, very many candidates continued incorrectly, either dividing the expression by 4, or setting it equal to zero and attempting to solve it. Although this subsequent working was not penalised in part (i), it could not count for part (ii).

(ii) This part of the question proved more challenging. A significant minority of candidates who knew the condition for equal roots overlooked the negative square root of 4, thus losing a mark. Others stated that the discriminant had to be positive and were then unable to gain any marks at all.

There were some serious algebraic errors seen, such as $\sqrt{16-4k^2}=4-2k$, or $4k^2=16$ simplified to $4k=4, k=1$.

A fair number of candidates, some of whom had solved $16-4k^2=0$ correctly in part (i), attempted to use the quadratic formula in some way, while others tried to factorise the original expression to find *k* by trial and improvement. Only in a handful of cases were these approaches successful.

5) (i) A good proportion of candidates gave a clear, concise demonstration of the given result, often with a helpful diagram, although a significant number made no attempt at all.

(ii) Surprisingly few candidates scored full marks on this straightforward part question. The vast majority of candidates differentiated the area expression and gained the first mark and many then stated correctly that $x = 4$. However, the final mark was lost by many as they failed to substitute their *x* value into the area formula to obtain the maximum area.

A large number of candidates worked out $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, some of them using this to justify that the turning point was a maximum, but most going on to claim that the maximum area was -4!

6) All but the very weakest candidates were able to score at least half marks on this question, with many completely correct solutions. A significant minority of candidates misinterpreted the instruction 'find the real roots' and instead stated how many real roots the equation had. Those who tried to expand the brackets and solve the quartic, completely ignoring the instruction to use a substitution, did not score any marks at all. Of those that substituted to gain a quadratic in *y*, most were able to solve it correctly but many candidates stopped here, omitting to work out the values for *x*.

As in Q4(ii), too many candidates forgot to consider the negative square root of 1, and thus found only one root. Others gained extra solutions by 'solving' $(x+2)^2=-6$, instead of rejecting their negative *y* value. A small number of otherwise excellent candidates lost marks by failing to simplify $\sqrt{1}-2$ and $-\sqrt{1}-2$ for their final answers.

7) (a) As in previous sessions, this question involving differentiation was done very well. The notation $\mathrm{f}'(x)$ was well understood by all but a very few, some of

whom evaluated f(1), while others integrated the expression. However, a substantial minority thought that x differentiated to zero, rather than 1. It was also fairly common to see $\frac{3}{x} = x^{-3}$ which was then differentiated incorrectly to produce the correct final expression. Obviously no marks were awarded when the correct answer was derived from incorrect working.

(b) The second part of the question was also done well by most, although some candidates did not realise that differentiation was required and merely substituted $x = 4$ into the given expression. There was one mark available for those who correctly evaluated $4^{\frac{5}{2}}$ as 32. Of those who differentiated, most scored well, the most common errors occurring during the substitution, with both $4^{\frac{3}{2}} = 6$ and $\frac{5}{2} \times 4^{\frac{3}{2}} = 10^{\frac{3}{2}}$ being seen quite often.

8) (i) This was one of the best answered questions on the paper, with almost every candidate who attempted it gaining all 3 marks.

(ii) Again, this question was answered well, most candidates using their expression from part (i), but a few differentiating successfully and a handful finding the roots and using the symmetry of the curve. Some candidates clearly did not understand the term 'vertex' and instead found the roots. Some candidates gave two answers, $x = -4$ and $x = -1$, with or without associated y-coordinates.

(iii) This question demonstrated the wide diversity in understanding of quadratic inequalities. Many candidates showed that they could solve a quadratic inequality with confidence; many others failed to gain even the first method mark. Some candidates who had solved a quadratic equation successfully elsewhere in the paper started by subtracting 15, after which they had little hope of finding the correct roots.

Too many of the candidates who decided to use the completed square form of the quadratic ignored the negative square root and therefore found only one root. Of those who found two roots, there were many good sketches leading to fully correct answers, although a fair number of candidates forfeited the final mark by inappropriate 'wrapping', usually $-5 > x > -3$.

9) (i) This part question proved to be one of the most difficult on the paper. Only a very few candidates managed to obtain $k = 7$ from valid working, although slightly more were able to give the correct centre. The expression $(x-3)^2$ was frequently seen but then $\left(y - \frac{k}{2}\right)^2$ also appeared which meant that the next 3 marks were lost. Far too many candidates simply changed the zero on the right hand side of the given expression to 16 (radius squared). There were also many cases of careless arithmetic (16 – 9 = 5 being seen frequently) and, as in Q4, some worrying manipulation of expressions with square roots such as $\sqrt{9+k} = 4$ followed by $3 + \sqrt{k} = 4$. A few candidates used the point (–1, 0) given later in the question to evaluate k, which was not intended, but was given credit.

(ii) Although most candidates scored some marks on this part, very many incorrect versions of the length formula were seen: candidates either reversed the + and – signs or failed to square their bracketed expressions. Some candidates who had identified the centre as (3, 0) saw straightaway that, for point B, $a = 4$, but a much larger proportion did not use the information given in the question and worked in terms of a. As noted before, poor understanding of square roots, in this case $\sqrt{16+a^2} = 4+a$, was all too commonly seen.

(iii) The equation of a straight line was well known by almost all candidates. However, as in the previous part of the question, many candidates gave their final answer in terms of a, which meant that they scored a maximum of 2 marks out of 3.

10) (i) This part of the final question was done very well by the majority, with a variety of approaches seen. Factorisation was the most successful method, as use of the formula required the correct square of 14 and square root of 256, arithmetic which proved too demanding for many. Those candidates who unwisely decided to complete the square were unlikely to make enough progress to earn any marks.

(ii) The general shape of the graph was well known by the vast majority of candidates, although a significant number contrived to make (0, -5) the minimum point, either drawing an asymmetrical curve or marking the points $\left(-\frac{1}{3}, 0\right)$ and (5, 0) as equidistant from the origin.

(iii) Candidates usually started this question in one of two ways: either they differentiated or they attempted to solve the equations simultaneously. Those who differentiated were generally more successful although a minority equated their derivative to zero instead of 4. A disappointingly high number of candidates were unable to calculate 27 – 42 – 5 correctly but, in general, good understanding of how to solve this type of problem was evident.

Those candidates who equated the 2 equations frequently did not realise that the discriminant of the resulting quadratic expression needed to be zero and so made little progress.

A minority of candidates used a mixture of both methods and they usually scored well, with some able to reason that the quadratic resulting from the simultaneous equations must be $3(x-3)^2$ and hence find c correctly.

4722: Core Mathematics 2

General Comments

This paper was accessible to the majority of candidates, and overall the standard was good. There were a number of straightforward questions where candidates who had mastered routine concepts could demonstrate their knowledge, and other questions had aspects that challenged even the most able candidates. Once again, only the most able candidates could manipulate logarithms accurately, though greater proficiency was shown in the use of logarithms to solve equations.

There are a number of formulae given in the *List of Formulae* that are useful to candidates sitting this examination. They were expected to be able to quote these accurately and no credit was given if there were errors in the formulae. It was also disappointing to see so many candidates losing marks through a lack of mastery of basic skills, such as algebraic manipulation, use of indices and solving both linear and quadratic equations.

Whilst some scripts contained clear and explicit methods, on others the presentation was poor making it difficult to follow methods used and decipher answers given. This was especially true when a candidate made a second attempt at a question. Candidates must appreciate that their final answer will be the one that is marked, unless they clearly specify otherwise.

Comments on Individual Questions

1) (i) This proved to be very straightforward first question, with the majority of candidates gaining both of the marks available. A few candidates treated it as an arithmetic progression with a common difference of 0.8.

(ii) This question was generally well done, but it was disappointing not to see more fully correct solutions. Having been told that it was a geometric progession, most candidates could attempt one of the associated formulae, but some attempted the 20^{th} term rather than the sum of 20 terms. Of those who use the correct formula, there was some uncertainty as to the values of a, r and even n. Others attempted to find the sum of an arithmetic progression, or even resorted to the Σ formulae from the *List of Formulae* – this was somewhat disappointing as the necessary formula also appears there. The weakest candidates attempted to list, and sum, all 20 terms, but this was rarely successful.

2) Most candidates could attempt the binomial expansion, though a number chose to expand the four brackets and were rarely successful. Of the latter approach, the most effective method was to square the bracket then square again. When using the binomial expansion, a few made errors such as ${}^4C_2$ becoming ${}^4/_2$ or the three components of a term being added rather than multiplied, but most could attempt a correct expansion. The most successful candidates made effective use of brackets, but this was still not always successful. The most common error was a failure to deal with powers of $2x^{-1}$ correctly. Some candidates attempted to remove a common factor but this was rarely successful. Other candidates attempted, incorrectly, to rearrange the expression within the bracket with x^{-2} being the most common error. These candidates could still get the two method marks, but no more than that. The vast majority of candidates could gain some credit on this question, but it was a minority that gained full marks.

3) It was pleasing to see how many candidates gained full marks on this question. Whilst candidates struggle to manipulate logarithms, they are becoming increasingly adept at using them to solve equations. The majority could confidently introduce logarithms on both sides, drop the powers and attempt to solve the resulting equations. There were a few slips, such as adding not subtracting 1, and some more fundamental errors such as log5 ÷ log3 becoming log($^5/_3$). In some candidates' solutions it was unclear whether they were taking log of a number or whether it was intended as a suffix – it is important that candidates make their methods clear if they wish to gain credit for the work done. It was also interesting to see the number of candidates who could successfully use their calculators to work to bases other than 10 – in this question, a number chose to use base 3 instead.

4) (i) Whilst most candidates seemed familiar with the trapezium rule and could attempt the question, it was disappointing to see so few fully correct solutions. There were the usual mistakes of using x-coordinates not y-coordinates, using an incorrect number of ordinates and using an incorrect value for h. A surprising number did not evaluate the integral between the requested limits; the common error was starting at $x = 0$, possibly from y_0 appearing in the formula. Some omitted brackets from the formula, and others either repeated y values, or placed them incorrectly. If the answer is required to three significant figures, candidates must appreciate the need for their working to be more accurate than this – the best solutions substituted into the rule using surds not decimals. Some candidates calculated the area of each trapezium individually before summing, and this was usually successful. The trapezium rule is a routine topic and candidates should be able to apply it successfully, especially in a straightforward question such as this.

(ii) Whilst most candidates could gain one mark from identifying that it was an under-estimate, gaining the second mark required a convincing reason. The best solutions referred to the tops of the trapezia being below the curve, supported by a sketch graph showing several trapezia, all with their vertices on the curve. Vague explanations referring just to the trapezia being below the curve, or references to the curve being concave gained no credit.

5) (i) Most candidates recognised the need to use $\cos^2\theta + \sin^2\theta \equiv 1$, but there were some slips on substitution, with $3\cos^2\theta$ becoming $1 - 3\sin^2\theta$. When the answer is given, candidates must ensure that their working is convincing.

(ii) This question was generally well answered. Having been given the correct quadratic most could then solve it accurately, though a surprising number resorted to using the quadratic formula. Even having identified the equation as a quadratic, some weaker candidates could not attempt an appropriate method to solve it. Whilst there were few sign errors, most could obtain the correct two roots. Candidates were usually successful in finding the two solutions resulting from $\sin\theta = {}^2/_3$, though extra solutions were sometimes seen. The final solution of 270° was sometimes omitted as the principal solution of –90° was discarded as being out of range rather than being used to find another valid solution.

6) (a)(i) This question was done well, and proved to be a valuable source of marks to candidates who had been struggling elsewhere on the paper. Virtually all candidates appreciated the need to expand the brackets first and could then attempt integration. This was usually correct, though some failed to simplify the coefficient in the second term. A number of candidates failed to gain an easy mark by omitting the constant of integration. A minority attempted differentiation not integration.

(ii) This was generally well answered, though there were some numerical slips. Most candidates used the limits in the correct order, and attempted to subtract, though a lack of brackets caused some inaccuracies.

(b) Most candidates appreciated the need to rewrite the function before attempting integration. The more able candidates could do this successfully, but a number struggled to obtain the correct index. Integrating x^{-3} often resulted in an answer involving x^{-4}. The weaker candidates simply attempted integration of the denominator without first rearranging. The constant of integration was often omitted from this part, even by candidates who had previously included it in part (a).

7) (a) This question was generally answered well, with most candidates obtaining the correct solution of $d = 5$, though a number of candidates used the formula for the n^{th} term of an arithmetic progression instead. The first method mark was available for attempting to use the correct formula for the sum. Whilst a slip when substituting was condoned, examiners had to be certain that the correct formula was being used, hence the need for full details of the method used to be shown. There was a disappointing number of candidates who could not solve the ensuing linear equation, with $24 + 69d$ becoming $93d$.

(b) This proved to be a challenging question, even for the most able candidates. Some candidates struggled even to state two correct equations, with ${}^{-4}/_{(1-r)} = 9$ being the most common error. The basic algebraic manipulation was disappointing, with many candidates attempting to solve the equations simultaneously but failing to do so accurately. Dealing with the denominator caused problems for many, and there were several sign errors. Having reached two correct solutions, a number of candidates either failed to select one as a final answer, or rejected one on somewhat spurious grounds.

8) (i) The whole of this question was generally very well done, with many candidates gaining full marks. The majority of candidates could correctly quote the formula for the length of an arc, and then solve the resulting equation.

(ii) Again, this was generally well done with most candidates successfully using $\frac{1}{2}ab\sin C$. As in previous sessions, some candidates were reluctant to work in radians and converted the angle to degrees, usually successfully, before proceeding. A few candidates calculated the height of the triangle separately, before using this to find AC. Some of the weaker candidates used an inappropriate formula such as $\frac{1}{2}r^2\theta$, $\frac{1}{2}r^2\sin\theta$ or $\frac{1}{2}bh$ with $h=6$.

(iii) Candidates were generally competent in using the cosine rule, though a number made numerical slips when evaluating the expression (and others used sine not cosine in the rule). Follow-through marks were available for those who got an incorrect value for AC in part (ii). Most candidates could then successfully find the length of the arc BD, and then use this in an attempt at the perimeter of the required region. However, some candidates struggled to identify the three sides needed for the perimeter, with some using the chord BD and others using AC not DC. When undertaking a question requiring a number of steps, it is important that candidates do not prematurely round values throughout as this can lead to inaccuracy in the final answer.

9) (i) (a) Many candidates gained this mark through an application of the factor theorem, but a number failed to use brackets, and did not make their intention clear. Others chose to use more protracted methods such as division or coefficient matching. Even if these were successful, candidates still had to draw attention to the lack of a remainder in order to gain the mark.

(b) Most candidates could make a good attempt at this question, and the majority gained the first three marks through a variety of methods. It was interesting to note that candidates have become more proficient at using algebraic long division. There was usually an attempt to solve the quadratic factor, and correct exact roots were often seen, though these were usually followed by decimal equivalents, which examiners did not penalise. A number of candidates solved the equation by attempting to factorise into two brackets, despite the hint in the question of 'exact roots'. Stating $x = -1$ was frequently omitted.

(ii)(a) As in previous sessions, candidates struggled with manipulating logarithms and very few fully correct solutions were seen. Some gained a mark for combining algebraic logarithms, or using the power rule, but many gained no credit at all. It was disappointing to see a number of candidates attempting to 'expand' the logarithm, thus demonstrating a lack of understanding of the topic. This continues to be an area of weakness for all but the most able candidates.

(b) This question proved to be challenging for all but the most able candidates. Some simply referred to a negative solution not being possible, but a more detailed explanation of why was required. Even those who identified the reason as being to do with the logarithms often struggled to give a correct reason. Examiners expected to see reference to logarithms only being defined for positive values of x. Some candidates made comments about logarithms not being negative, but this was not specific enough and gained no credit. The second mark for correctly identifying the only real root was dependent on the first mark for the explanation having been gained.

4723: Core Mathematics 3

General Comments

This paper contained several accessible and routine questions and most candidates were able to obtain a respectable total mark for the paper. However, Qs 2 and 3(iii), though apparently familiar requests, did trouble more candidates than had been expected. Qs 8 and 9 contained aspects which proved suitably challenging to all candidates but examiners were delighted to be able to assess the work of some very capable candidates, who met all the challenges with assurance and insight.

Two questions asked for sketches to be drawn and performance here was often not very convincing. Candidates should, in general, provide sufficient information on a sketch so that it is clear that the sketch has been correctly located. Thus, for the sketch of $y = 3 + \sqrt{x}$, an indication of 3 on the y-axis was expected and, for the graph of $y = \sec x$ in Q7, indications of -1 and 1 on the y-axis and of 2π on the x-axis were expected.

Examiners continue to have concerns about the quality of algebra demonstrated by a significant number of candidates. Too often, careless – and surely avoidable – algebraic slips marred solutions which otherwise showed considerable merit.

Comments on Individual Questions

1) Most candidates found no difficulty in answering these two tests of differentiation technique accurately although carelessness was evident in some cases. In part (i), a minority of candidates did not recognise the need to use the product rule and offered an answer such as $15x^2(x+1)^4$. Likewise, in part (ii), some candidates did not use the chain rule and presented answers such as $\frac{1}{2}(3x^4+1)^{-\frac{1}{2}}$. In part (i), some candidates continued with their solution to present the answer in a fully factorised form. Impressive as the algebra often was, it was not needed on this occasion; the mark allocation of two and the absence of any instruction to do more than differentiate should have indicated this.

2) This was a question of a routine nature and the expectation was that the majority of candidates would answer it competently. Of course, there were plenty of correct solutions but, generally, the response was disappointing with many candidates showing uncertainty about how to deal with the inequality. Almost all candidates had a strategy for finding the two critical values of $\frac{1}{3}$ and 2. Some squared both sides whilst others considered two linear equations or inequalities but algebraic slips were quite common.

   Three marks were available for producing the two critical values. The subsequent two marks were for determining the solution of the inequality and it was here that lack of understanding was widespread. Some candidates went no further than $x = \frac{1}{3}, x = 2$. Others followed a quadratic statement such as $(3x-1)(x-2) < 0$ by $x < \frac{1}{3}, x < 2$. Those candidates adopting an approach involving statements such as $4x - 3 < 2x + 1$ and $-4x + 3 < 2x + 1$ were often unsure how to conclude with an appropriate statement. Careful sketches were helpful but their presence was seen on relatively few scripts.

3) Part (i) was answered very well and the only problem to appear with any regularity was a misunderstanding of the symbol $\sqrt{x}$. This means the positive square root of x and so $\sqrt{169}$ is 13, and certainly not also -13. Part (ii) was also answered competently with most candidates showing a well-practised routine for finding an inverse function. A few

candidates misinterpreted $f^{-1}(x)$ as $\frac{1}{f(x)}$ and a few others as the derivative $f'(x)$.

Part (iii) was not done well, a variety of reasons contributing to the fact that all three marks were not awarded very often. It was rare for examiners to be wholly convinced by the graph of $y = f(x)$; the point 3 on the *y*-axis was expected with a curve showing essentially the correct shape. Sometimes the attempt at $y = f(x)$ showed a complete parabola, presumably due to the misunderstanding about the symbol $\sqrt{x}$ mentioned above. It was common for the graph of $y = f^{-1}(x)$ to be a complete parabola instead of just the part for which $x \geq 3$. The geometrical relation between the graphs of $y = f(x)$ and $y = f^{-1}(x)$ was widely known. The mark was available for reference to reflection in $y = x$ even when this was plainly not the case with the sketches produced, but candidates doing no more than drawing the line $y = x$ on their diagram did not earn the mark.

4) The integration in part (i) was generally carried out well. Not all obtained the correct coefficient of $\frac{3}{8}$ and a few made errors when substituting the limits but the vast majority earned all four marks without difficulty.

Part (ii) presented more problems. It was apparent that many candidates were not familiar with Simpson's rule using just two strips and they had difficulty adapting the formula given in the *List of Formulae* to this situation. For candidates with the correct form of the rule, some had difficulty with the value of *h* and others used values of $\frac{3}{8}(2x+1)^{\frac{4}{3}}$ or *x* values instead of values of $(2x+1)^{\frac{1}{3}}$.

Some candidates showed little faith in the ability of Simpson's rule to provide a reasonable approximation. Very seldom did such widely differing answers as 30 in part (i) with 176 in part (ii) or 120 in part (i) with 29.6 in part (ii) prompt any check of the working.

5) Familiarity with the idea of exponential decay is a specification item and so candidates should have been aware of the fact that the request in part (i) was equivalent to solving the equation $e^{-0.04t} = 0.5$. It does not matter whether the time is found for the mass to decrease from 240g to 120g or from 100g to 50g or from 2g to 1g, the answer is the same. Many candidates were unsure how to proceed; many solved the equation $\frac{1}{2} = 240e^{-0.04t}$ and others solved $\frac{1}{2}m = 240e^{-0.04t}$, presenting an answer still involving *m*.

Part (ii) prompted a much better response. Differentiation was handled well and the introduction of natural logarithms to solve the equation was generally a well-known technique. A lack of precision with signs was evident on many scripts but examiners adopted a tolerant view of situations where awkward minus signs discreetly vanished. A few candidates presented the answer as 38 seconds but this slip with the units was not penalised.

6) Many candidates answered this question impeccably and a total of nine marks was often awarded. A few candidates integrated incorrectly, $12e^{2x}$ or $\frac{1}{2}(6e^{2x} + x)^2$ being the usual errors, but the vast majority of candidates reached $3e^{2a} + \frac{1}{2}a^2 = 45$ without difficulty. Some care was then needed and examiners expected to see a sound process for reaching the result given in the question. Too often, however, a statement such as $e^{2a} + \frac{1}{6}a^2 = 15$ was followed by $2a + \ln(\frac{1}{6}a^2) = \ln 15$. The fact that the required

$a = \frac{1}{2}\ln(15 - \frac{1}{6}a^2)$ inevitably followed did not impress examiners!

The iteration process in part (ii) was carried out very well. There were a few slips, such as starting with a value other than 1 or dealing with an equation such as $a = \frac{1}{2}\ln(15 - \frac{1}{6}a)$. Candidates generally provided sufficient evidence of the method and concluded appropriately. The only error to occur with any frequency was a conclusion of 1.343 following several correct iterates, the result of truncating rather than rounding.

7) For those candidates with a sound knowledge of secant and cosecant and a readiness to use radian measure, this was an accessible question and full marks were easily earned. Such candidates though were in a minority and many candidates struggled to make much progress. The graph of $y = \sec x$ was not well known and, indeed, part (i) was omitted by some candidates. In part (ii), most candidates realised that the equation to be solved was $\cos x = \frac{1}{3}$ but many then provided answers in degrees or provided only one answer.

Part (iii) also proved challenging to many. Even many of those candidates who successfully reached $\tan\theta = 5$ were unable to conclude with the correct values, working in degrees or offering a second value in the wrong quadrant.

In both parts (ii) and (iii), a frequently-seen approach involved squaring, leading to equations such as $\tan^2 x = 8$ in part (ii) and $\cos^2\theta = \frac{1}{26}$ or $\tan^2\theta = 25$ in part (iii). Those taking this approach were seldom able to conclude correctly; the need to reject extra spurious answers was not appreciated.

8) Candidates appreciated the need to use the quotient rule in part (i) but clear and precise solutions were not so common. A significant number of candidates imagined the presence of non-existent brackets and provided derivatives such as $\dfrac{4}{x-3}$. Other solutions involved incorrect steps with a numerator such as $\dfrac{4}{x}(4\ln x + 3 - 4\ln x - 3)$.

Part (ii) presented further problems. Some candidates tried to find a value of x from the expression for the derivative. Others followed the correct $\dfrac{4\ln x - 3}{4\ln x + 3} = 0$ with the incorrect $4\ln x - 3 = 4\ln x + 3$, an error that surely should not be made at this level. Of those who correctly identified the crucial value of x as $\mathrm{e}^{\frac{3}{4}}$, too many resorted to decimal values and lost the final two marks.

The majority of candidates could earn no more in part (iii) than the mark for a correct statement of the expression for finding the volume. Most candidates totally failed to note the link with part (i) and made no sensible progress. A few, faced with $\displaystyle\int \frac{4\pi}{x(4\ln x + 3)^2}\,\mathrm{d}x$, showed impressive awareness by noting that the integral could be written as $-\pi(4\ln x + 3)^{-1}$.

9) Part (i) presented no problem to the majority of candidates. The appropriate identities were used accurately and the necessary simplification carried out competently. For a few, some slips of sign crept in and others seemed to believe that multiplication of the two rational expressions required the use of a common denominator.

The key to success in part (ii) was use of the identity $\sec^2\theta \equiv 1 + \tan^2\theta$ to produce an equation in $\tan\theta$. Use of the identity from part (i) together with careful simplification led to $\tan^4\theta = \frac{1}{3}$. A number of candidates managed this but, in many other cases, errors were common. For those candidates reaching $\tan^4\theta = \frac{1}{3}$, most failed to appreciate that a second value of θ followed from $\tan\theta = -\sqrt[4]{\frac{1}{3}}$. Some candidates converted to an equation involving only $\sec\theta$ and occasionally succeeded with the solution. Others embarked on unwieldy and protracted attempts involving $\sin\theta$ and $\cos\theta$ but success eluded them.

Only a few candidates were equal to the challenge of part (iii), expressing $\tan^2\theta$ in terms of k and providing a convincing justification for the existence of the two roots. Many candidates had no idea how to tackle this part, whilst others responded to the reference to ‘two roots’ by considering the discriminant of an equation without realising that this, by itself, was insufficient.

4724: Core Mathematics 4

General Comments

This paper produced a very wide range of responses; many candidates produced a fully correct paper but there were also many obtaining fewer than 10 marks. Time did not seem to be a problem for the majority; a few who appeared to be rushing towards the end had generally given insufficient thought earlier in the paper and had produced convoluted solutions or made difficulties which were unnecessary. There were fewer occasions where the answer was given but it must be stressed that, in such situations, every aspect of working is carefully scrutinised. Q8(i) was a very obvious case; as might be expected, the answer was nearly always produced but probably more than half of the candidates failed to convince the examiner.

Weaker candidates tend to feel that questions are deliberately set to make life difficult for them but there are ways in which they can improve their performance. One is to tackle questions in simple stages with suitable mathematical symbols; for example in Q2(i), with several stages to pursue, there was a proliferation of negative signs and brackets were often missing – if each part had been done carefully, many more would have obtained the correct answer. Candidates often seem to be conditioned to perform certain techniques in a standard order; for example in Q6, having differentiated the given equation, many automatically decided to find the general form of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and then the version suitable for the normal before substituting (2,3) for (x, y); how much easier it would have been if they had substituted (2,3) for (x, y) as soon as the differentiation had been completed, giving the numerical value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and hence the gradient of the normal. This method would also have prevented the use of the general (instead of the numerical) normal gradient in the equation of the normal.

It would be helpful if candidates would also give the briefest of explanations; in Q9(i), candidates had to decide which part of each equation to use – it would have helped if they had said "We need to use $\begin{pmatrix}-6\\8\\-2\end{pmatrix}$ and $\begin{pmatrix}1\\3\\2\end{pmatrix}$" , but in so many cases all that was visible was, for example,. $15+8$, $\sqrt{36+64+4}$ or $\sqrt{25+1+4}$ followed by an angle – the marking scheme was designed to be as fair as possible testing which vectors were used, the processes for finding scalar products and moduli, and the method for finding the angle. It should be obvious to candidates that vector questions are prime examples for mis-copying and much greater care taken in transferring numbers to their answer booklets.

Comments on Individual Questions

1) (i) Most got off to a good start here. A few failed to give the format $\frac{A}{x+2}+\frac{B}{x-3}$ before producing the identity $A(x-3)+B(x+2)\equiv 3x+1$ and often gave the result as $\frac{2}{x+2}+\frac{1}{x-3}$. As usual at the beginning of an examination, nerves/carelessness had a part to play and it was by no means uncommon to see $-5A=-5$ followed by $A=-1$. The cover-up rule was used with good effect.

(ii) In this part, there were many instances of $\ln(x+2)$ and $\ln(x-3)$ appearing; was this carelessness in differentiating or are candidates programmed to expect integration after partial fractions? Explanations concerning the negative gradient were very varied; some tried using specific values of x to prove that it was true generally but many realised the non-negative aspect of each denominator was a vital situation.

2) Almost everyone realised this was testing integration by parts, certainly at the first stage, but it was surprising that many did not realise that the second stage, involving $\int -2x\,\mathrm{e}^x\,\mathrm{d}x$, was also a similar case. As mentioned in the "General Comments", there were several negative signs involved and only the better candidates managed to progress to the stage of, or imply, $(x^2 - 2x + 2)\mathrm{e}^x$ before substituting the limits.

3) The volume was usually quoted as $\pi\int_0^\pi \sin^2 x\,\mathrm{d}x$ although 2π and 1 both appeared in place of the correct π; for a significant number, this was the only mark that was scored as the integral was frequently shown as $\frac{1}{3}\sin^3 x$. Those who did know what to do mostly went on to score well although all versions of $+/-\left(\frac{1}{2}\text{ or }2\right)(1+/-\cos 2x)$ were seen – and the integral of $\cos 2x$ was not always $\frac{1}{2}\sin 2x$. Most of the errors still led to the correct answer of $\frac{1}{2}\pi^2$ but all working was closely scrutinised.

4) Most candidates realised they needed to convert the first term inside the bracket to 1 before they attempted the expansion – but there were very frequent errors in achieving this with factors outside the bracket being seen as 2, 4 or $\frac{1}{2}$ almost as frequently as the correct factor $\frac{1}{4}$. Consequently the final expansion was often incorrect although the intermediate stage for $\left(1+\frac{1}{2}x\right)^{-2}$ was usually correct. The validity range was often omitted, probably just forgotten in the heat of the moment. Candidates were most successful in part (ii), either getting the correct answer or following-through correctly from an incorrect answer in part (i). However a mis-read of $(1+x)^2$ instead of the given $(1+x^2)$ was seen too often, and omission of brackets in $1+x^2(2+x)^{-2}$ was slap-dash. A total mark above 5 was uncommon..

5) A careless mis-read was again in evidence with the use of $y = 3 + 2\cos t$ instead of the given $y = 3 + 2\cos 2t$ but, irrespective of that, this was probably the least well answered question on the paper. Most candidates scored the first 2 marks but did not think of simplifying their obtained expression; hence their attempts, often substitutions of specific values of t, to prove that the gradient could not exceed 8 were doomed to failure. Part (ii) was relatively easy but candidates did not sit back and plan an attack and so their efforts were often very convoluted and badly explained. In part (iii), most obtained a rough indication of a parabola but, in most cases, were unable to identify *C*.

6) This question was commented on at the early stages of this Report but, on the whole, candidates were pleasingly successful. Most of the errors occurred in differentiating the $3xy$ term and the constant 58, although the equation of the tangent instead of the normal was not uncommon. It was good to see that, although "$\frac{\mathrm{d}y}{\mathrm{d}x} =$" often appeared at the

beginning of the differentiation, it was rarely brought into operation. Most candidates scored at least 6 of the 8 marks.

7) The first part was well done, either using algebraic division or by means of equating coefficients – but a number of other approaches were seen (some best described as 'home-spun') with varying degrees of success. Although mentioned in an earlier Report, the terms 'quotient' and 'remainder' are still not fully appreciated by candidates; part (ii), which was designed to provide a bridge for candidates to the final part, often seemed to confuse candidates and a surprising number, having obtained the right answer to part (i) now reversed the parts or, in some cases, used an entirely new remainder e.g. $9x+12$. This often made it difficult or impossible for candidates to fully integrate the function in part (iii) and consequently 2 marks was a common score.

8) Most candidates were able to separate the variables or, in a few cases, invert each side but many stated that $\int \frac{1}{6-h}\,\mathrm{d}h$ was $\ln(6-h)$ and a surprising number, considering the additional information, failed to show a constant of integration. The rest of the question was answered well, particularly part (iii) which required candidates to perform a fair amount of algebraic manipulation.

9) This was a very successful question for candidates. Most knew how to find the angle between two vectors although, predictably, some used the position vectors rather than the direction vectors. Again, most knew how to deal with part (ii) and, although explanations were often rather obscure, the correct answer was generally seen. Part (iii) was very well done with only a few errors being made in solving the simultaneous equations.

4725: Further Pure Mathematics 1

General Comments

Most candidates were able to make an attempt at all the questions and there was no evidence of candidates being under time pressure. Most candidates worked sequentially through the paper and the presentation of work was usually of a high standard.

A good number of candidates were able to score high marks on this paper, but a significant minority lost a large proportion of the marks, sometimes on more than one question, by making algebraic errors that careful checking should have highlighted.

Although it has been mentioned in previous reports, sketches often included no indication of scales. Candidates should be made aware that this may lead to a significant loss of marks.

Comments on Individual Questions

1) Candidates who use trigonometry usually obtained the correct answers. Those who derived two equations and then solved them, often obtained two solutions and failed to appreciate that one of their answers had a negative argument. A common error was to have the equation $a^2 + b^2 = 4$ rather than $a^2 + b^2 = 16$.

2) Most candidates managed to verify that the result was valid when $n = 1$. A significant number did not add the correct term to the given result. Too many candidates failed to provide sufficient, or the correct, algebraic steps to justify the validity of the result for $n = k + 1$. The induction conclusion was often omitted or not clearly explained.

3) Most candidates scored full marks on this question. The main error was to use the third term as 1 instead of n. Most candidates gave sufficient working to justify the given answer.

4) (i) Most answered this part correctly, the omission of the determinant being the most common error.

   (ii) A considerable number of candidates thought that $(\mathbf{AB})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1}$. Many candidates tried to find **B**, **AB** and then $(\mathbf{AB})^{-1}$ and most made an arithmetic error in the process.

5) (i) The given result was usually derived correctly.

   (ii) Most candidates gave a clear demonstration of the difference method and derived a correct form for the required sum.

   (iii) This part proved quite demanding. A considerable number thought that the required sum was just the value of the sum to infinity, i.e. 1, and so gained little credit.

6) (i) (a) Most candidates wrote down the correct values. The most frequent error was to omit dividing by 3.

   (b) Most candidates knew, or derived, the correct identity to use in this part of the question.

   (ii)(a) Most candidates used the given substitution, but very many then failed to simplify their equation into a cubic equation, as requested. This usually meant that part (b) scored no marks.

(b) A significant number of candidates did not understand how to use part (ii) (a) to write down the value required in terms of the coefficients of the derived cubic equation.

7) (i) Most candidates knew a valid method for finding the determinant of the matrix, with only minor arithmetic slips occurring. Some thought that the reciprocal was the value they required, confusing the inverse matrix computation with the required value, but this was not heavily penalised.

(ii) Those who found a determinant invariably deduced singularity or not correctly from the value of the determinant when $a = 2$. Some thought that **M** was non-singular because det **M** > 0.

(iii) The majority of candidates scored only 1 mark for this part by stating that det **M** = 0 implies there are no solutions, without attempting to solve properly to distinguish between no solutions and an infinite number of solutions.

8) (i) Lack of an indication of scales on the axes led to marks being lost by a number of candidates who otherwise had a correct sketch. Common errors were to have the centre of the circle at the origin or on the y-axis and for the line to pass through the origin and not be a half-line. Sometimes the two sketches were not labelled as C_1 and C_2, a small detail that would have been helpful to examiners when the sketch was not totally correct.

(ii) The required region was usually shown correctly, with the most common error being to include the area inside C_1 below the x-axis.

9) (i) This was answered correctly by almost all candidates.

(ii) This part was not answered well. Many candidates had combinations of shears, stretches, enlargements and rotations. Some recognised that it was a single rotation and could usually write down most or all of the correct features. Very few candidates drew a sensible sketch of the unit square and its image under the given transformation, which would have shown clearly the transformation represented by the given matrix.

(iii) Provided that part (i) was answered correctly, few failed to answer this part correctly.

(iv) Again, scales were missing from many sketches. Often (1, 1) was shown to map to (2, 0) but was plotted on the sketch at (0, 2).

(v) As in Q7 there were confusions between the value of the determinant and its reciprocal. A good proportion of candidates knew that the determinant was the scale factor for area, but quite a number gave it as a scale factor for enlargement, not indicating that it was area rather than length.

10) (i) Most candidates knew the correct method for finding the square roots, but sign errors in simplifying to a quadratic in x^2 or y^2, meant that the final answers were incorrect and few candidates showed any sign of checking that they had the correct answers.

(ii) A good number of candidates completed the square to derive the correct answers very easily. Those who used the quadratic formula often made sign errors and so could not see the relevance of part (i).

4726: Further Pure Mathematics 2

General Comments

Most candidates found the examination accessible, answering the questions in the order set. There was no evidence that candidates had any problems with timing, and no question proved particularly difficult. The early questions gave candidates a good start, although a significant number of attempts at differentiating and integrating in Qs 3, 4 and 5 were disappointing, even on the use of the basic chain and product rules. As in previous years, there was some evidence of poor algebraic manipulation and simplification, but candidates in general appeared to be well-prepared for the range of questions asked. Problems arose when overlong methods were selected, particularly in question 7, and when the contents of the *List of Formulae* were not well-known.

It has been reported before that candidates should be aware of the importance of full and detailed responses to questions in which the answer to be proved is given. It is expected that such an answer should be fully justified, with little seen as "obvious". Similarly, in questions such as Q6(i) and (ii), candidates should expect to explain fully and not merely write down what is given to them to explain. Nevertheless, most candidates were able to demonstrate their knowledge and to produce good, well-presented scripts. There were few poor scripts. Overall, candidates performed well, though original solutions were in short supply this year.

Comments on Individual Questions

1) Most candidates produced the correct formula for area and attempted to rewrite the angle using a form of the double-angle formulae. Even those candidates who made numeric or sign errors in their formulae were able to pick up three marks, and this question provided a good start for the majority. The (very few) candidates who successfully applied integration by parts are to be congratulated, even if they were penalised by time constraints.

2) (i) This was the first question in which the "answer given" appeared, and it was dealt with well by most candidates. Candidates who thought the answer "obvious" usually lost one mark, as it was expected that $\cos\frac{1}{4}\pi = \sin\frac{1}{4}\pi = \frac{1}{2}\sqrt{2}$ should be clearly seen or implied before the final factorisation to the answer provided.

   (ii) The majority of candidates used the standard expansions for $\cos 2x$ and $\sin 2x$, and there were many correct answers. It was disappointing (though accepted in this case) to see how many candidates failed to simplify their coefficients, and final answers involving factorials appeared too frequently. Marks were not lost if the bracket was not multiplied out. The minority of candidates who opted to derive Maclaurin by repeated differentiation and substituting $x = 0$ were often equally successful, and such candidates could gain a minimum of two marks even if basic errors in differentiating were seen. Whichever method was used, it was surprising how many candidates went beyond the first four terms requested, in some cases far beyond!

3) (i) Most candidates successfully produced the correct partial fractions from correct working, often by equating coefficients rather than substituting values of x. Candidates using only $A/(x^2 + 9)$ were heavily penalised, although such candidates (and others who made errors using the correct partial fractions) could still pick up both marks in part (ii).

   (ii) Although many candidates gained both marks, a significant number did not recognise the integral of $9/(x^2 + 9)$ as one requiring a trigonometric substitution or as a standard integral to be found in the *List of Formulae*. There were many attempts using parts or quoting answers such as $9 \ln(x^2 + 9)/2x$.

4) (i) Problems arose in the application of the chain and product rules in the first part of the expression. Three marks were available for the correct differentiation without simplification, but many candidates scored only two marks because of basic errors, often involving signs. Even candidates getting $(2 - 2x^2)/\sqrt{(1 - x^2)}$ failed to see the relevance of their answer to part (ii).

(ii) It was apparent that candidates not getting the full simplification to part (i) were generally unprepared to look for "appropriate trigonometric or hyperbolic substitutions". The candidates who used $x = \sin y$ or $x = \cos y$ were usually as successful as those who had completed part (i), but there was a significant number of candidates who thought they could write down the answer or who used inappropriate substitutions.

5) This question was answered well by most candidates.

(i) Care had to be taken by candidates; with the answer being given, the derivation of both e and nI_{n-1} had to be clear. The use of both limits had to be demonstrated to justify e. Some candidates used an incorrect derivation $\mathrm{d}/\mathrm{d}x(\ln x)^n = n(\ln x)^{n-1}$ to produce I_{n-1}.

(ii) Apart from the few candidates who left n in their answers or who confused the meaning of I_0, it was only minor errors that prevented candidates gaining full marks.

6) (i) The best explanations covered a method for getting the area of a rectangle, the summation of the areas of all the rectangles and a comparison with the area under the graph, preferably with the limits noted for both the rectangles and the area under the graph. The worst explanations stated "Area (singular) of rectangles > Area of graph". There were many other explanations in between. Candidates should expect, for the marks awarded, to explain fully, deriving both sides of the inequality, defining the inequality and considering the end values or limits. Some leeway was allowed in this case, with failure to mention the limits not being penalised.

(ii) The extra mark was for an indication of the new set of rectangles, either using a "left shift by one along the x-axis", a description or, preferably a diagram. If the diagram also showed the limits clearly, the candidate could expect full marks. The changes between part (i) and part (ii) were expected to be highlighted. In general, explanations were not as full as expected, with comments such as "It is obvious that…" used. Even so, there was often sufficient detail to gain a minimum of three marks overall.

(iii) This part was generally well done. Candidates who could not derive the $2 - {}^1/n$ often used the wrong limits to produce the answer, with both 2 and ½ being seen as bottom limits. Others produced the extra 1 required by referring to areas that had been omitted instead of the ${}^1/_1{}^2$ needed for the summation. Others added an apparently arbitrary 1. The majority of candidates gained at least half marks.

(iv) Although there was some indiscriminate use of ∞ in solutions, most candidates gained full marks.

7) (i) Most candidates scored well. The main errors were ½ x ½ = ½ and $e^x.e^y = e^{xy}$.

(ii) Again, candidates were largely successful, the majority going from $\cosh(x - y) = 1$ to get the answer given. Again, justification was required. Most candidates used $x - y = \cosh^{-1}1 = 0$, whist others opted for $x - y = \cosh^{-1}1 = \ln(1+\sqrt{(1^2 - 1)}) = 0$. Those candidates who resorted to the exponential definitions often wasted some time in setting up quadratics in $e^{(x-y)}$ or in rewriting this in other ways such as $e^{2x} - 2e^xe^y + e^{2y} = 0$. However, with care, such candidates scored full marks. Those who used $e^{x-y} + e^{y-x} = 2$ and merely inserted $x = y$ to show $x = y$ was a possible solution gained only one mark.

(iii) Candidates who realised $\cosh x = 3$ and $\sinh x = \pm 2\sqrt{2}$ usually gained at least three marks. The quickest solutions came from using the *List of Formulae*, although it was surprising how often only one solution was found. It was also surprising how many candidates resorted to the exponential definitions to produce quadratics in e^x or even in e^{2x}. There were some original solutions involving both the use of $\cosh 2x$ and the neat method of noting $e^x = \cosh x + \sinh x = 3 \pm 2\sqrt{2}$. Many candidates failed to answer the question set and to give solutions for both x and y.

8) (i) Most candidates used the iteration correctly, usually working to a minimum of four decimal places throughout. Minor errors were accepted if candidates recovered to the correct answer. This was expected to be given in the form α=. . ., and not as x_i.. A minority of candidates used the Newton-Raphson method for which no credit was given.

(ii) The answer $f'(x) = -2/(x+2)$ was often seen. Candidates who obtained $-2/(x+2)^3$ were often loathe to substitute their value of α and show numerically that their derivative was non-zero. Statements such as "$f'(x) = 0$ makes $x = -2$ which it is not" were surprisingly common. Candidates using the general $f'(x)$ and showing $f'(x) \neq 0$ for any x gained full marks.

(iii) The mark was usually gained, even if candidates had the incorrect x_4 and x_3.

(iv) Very few candidates could write down $\delta_{10} \approx [f'(\alpha)]^7.\delta_3$ and use their earlier results. Candidates resorted to a variety of strategies, such as using $\delta_2 = f'(\alpha).\delta_1$ and building up term by term to δ_{10}. Any reasonable starting point, together with their value of $f'(\alpha)$, was considered acceptable.

9) (i) Most candidates could write down the asymptote $x = a$. Various attempts were seen for obtaining the oblique asymptote, with division being the most successful. Other methods such as equating coefficients were less successful, although candidates noting the numerator could be written as $(x - a)^2 - a^2$ quickly arrived at the answer. Minor errors were accepted if the correct second asymptote was seen, but the approximation $y = x$ gained only a maximum of two marks.

(ii) It was often difficult to follow the logic that candidates used. Most set up the quadratic in x and then attempted to do something with the discriminant. If an inequality was attempted at this stage, for no reason given or the wrong reason given, marks were awarded. With no inequality or just a consideration of $b^2 - 4ac$, it required more work later to pick up the marks. Many candidates arrived at $y^2 + 4a^2 \geq 0$ (or > 0) and then attempted to solve it or made no comment at all. Nevertheless, most candidates picked up at least three marks. The minority of candidates who looked for turning points often gained half marks, as they stopped when they believed they had shown there were no turning points.

(iii) As usual, candidates should expect to mark in asymptotes (from part (i)), approaches to these asymptotes and obvious points. The obvious points were often missing. They would have helped many candidates who produced maximum and/or minimum turning points on their graphs, particularly as such points left gaps in the graph, despite the result from part (ii). Approaches to asymptotes were usually good, but candidates should ensure that the curve approaches the asymptotes and does not wander off.

4727: Further Pure Mathematics 3

General Comments

This paper was found accessible by most candidates; it had more questions that were found straightforward than has sometimes been the case. Nevertheless, careful and accurate working was needed, and this was seen on many of the scripts. The best answered questions were Qs 3, 4 and 8, with Qs 1 and 9 being the least successful. Presentation of answers was usually quite good, although in a few cases candidates' writing was so poor as to be almost illegible. A small but pleasing point was the use by nearly all candidates of the written $\cancel{z}$. There did not appear to be any problems about the length of the paper, and almost all candidates reached the end, even when the last question was found demanding.

Comments on Individual Questions

1) (i) Although this was intended to be an easy start to the paper, a surprising minority of candidates did not obtain the mark. In particular, it was quite common for z^* to be written as $-r\mathrm{e}^{\mathrm{i}\theta}$ or for the expression for zz^* to be left as r^2, without any indication that $r = |z|$ or $r^2 = |z|^2$. Answers which proved the result using the cartesian form of complex numbers were not allowed.

   (ii) The request to "describe the locus" should have been a clear indication that a geometrical description was wanted, but many answers went no further than $|z| = 3$. Those candidates who then attempted to describe this equation often gave answers such as "z has a magnitude of 3 and angle θ" or "a line of length 3". When a circle was given as part of the answer it was quite common for the centre not to be stated.

2) This question was quite well answered. In many solutions the components of a general point on l were correctly substituted into the equation of the plane and a contradiction was obtained with the terms in t cancelling out. However, only the better candidates realised that there was no more to be done, apart from stating that l is parallel to Π without intersecting it. Most others then found the scalar product of the directions of l and the normal to the plane, showing again that l is parallel to Π. There was, of course, no penalty for doing extra work. The minority who found the same scalar product and substituted a single point of l into the equation of the plane usually gave the correct conclusion. Solutions which involved solving simultaneously the equations of l and Π were sometimes attempted, and these also led correctly to a contradiction.

3) This was a very straightforward differential equation and full marks were often earned. The complementary function was almost always found correctly, and the correct form for a particular integral was usually used, leading to the general solution. It should be noted that the solution of a differential equation includes "$y =$ " and is not just the expression on the right-hand side.

4) (i) The verification of the associative property in this particular case was almost always correct, both sides being equal to s.

   (ii) The other three group properties were usually checked correctly, but the closure property was not always stated sufficiently clearly by reference to the given operation table. In most cases the inverses of the elements were stated, but the alternative of noting that the identity element occurred in each row or column was sometimes seen.

(iii) This part tested candidates' knowledge of the structure of the group of order 5. It was only necessary to write down the elements of the cyclic group H, which many did correctly, and the correspondence between elements was not required on this occasion.

5) (i) This part was a standard piece of bookwork which was usually carried out accurately. It was encouraging to find many candidates using the abbreviated notation of c and s for $\cos\theta$ and $\sin\theta$ making both their work and the examiners' checking easier.

(ii) In previous papers the concept of certain types of algebraic equations having trigonometric solutions has not been understood very well. On this occasion it was pleasing to find that most candidates did manage to find a value for $\cos 6\theta$ and in many cases they expressed x as the cosine of an appropriate value of θ. However, in many solutions there was no appreciation of the fact that $\cos 6\theta = \frac{1}{2}$ has more than one root, or that more work needed to be done to establish that the "obvious" root of $\cos\frac{1}{18}\pi$ was indeed the largest positive root. To earn all four marks it was necessary to state a sufficient number of other values of θ, or to use the general solution, or, best of all, to realise that the shape of the cosine function requires the smallest positive value of θ for the largest value of x.

6) (i) This part was done very well: some thinking had to be done to establish that the required normal vector $\mathbf{n}$ was perpendicular to both lines, but once this was done it was a straightforward matter to find the appropriate vector product and to substitute a point. The work was carried out accurately in nearly all cases, although occasionally answers were given in cartesian form. Other methods were sometimes seen, but they all involved more work, usually dealing with simultaneous equations.

(ii) Although this part appeared to require the same working as part (i), the marks allocated were fewer, and examiners were pleased to find that the majority of candidates correctly used the same vector $\mathbf{n}$ that they had found in part (i), realising that all they had to do was substitute a different point into the left-hand side.

(iii) It had been expected that, having found the equations of the planes in the form $\mathbf{r}.\mathbf{n} = p$, candidates would simply find the difference between the two values of p and divide by the modulus of $\mathbf{n}$, and so 2 marks were considered appropriate. However, this method was seen infrequently and, when it was seen, there was often no division by $|\mathbf{n}|$. Instead, most answers used the standard method for finding the distance between two skew lines, thus anticipating part (iv). As is customary, there was no mark penalty for using a longer method, but candidates inevitably had more chance of making an error.

(iv) Most candidates knew the relationship between the distance between the planes and the lines, but many wrote simply that it was the "distance" between the lines. As the lines are skew this statement is meaningless, and it was "shortest distance" or "perpendicular distance" that earned the mark.

7) (i) This was a small piece of complex number work which was established correctly by many candidates, but in some cases algebraic or complex number errors were made.

(ii) Familiarity with the complex roots of unity in exponential form was evident in most scripts, and the seven roots were usually correctly listed, although some omitted i or included θ in the powers of e. The Argand diagrams seen varied in detail, in accuracy and in tidiness: the features which earned the marks were that there were seven points equally spaced round the unit circle, that one point was on the positive real axis and that the other six points were in approximately the right positions. Of these, the identification of the circle having unit radius was most often omitted.

(iii) Examiners were impressed that a substantial number of candidates answered this part correctly. Of course, part (i) was a broad hint, but many realised the need to link together pairs of complex conjugate factors from the seven linear factors. They were able to do this quite easily whether they had used $k = 0$ to 6, or $k = 0$ and $k = \pm 1, \pm 2, \pm 3$, in part (ii). Occasionally the linear factor $(z-1)$ was missing or wrong, but many answers were entirely correct, sometimes with very little working being seen. However, other candidates had no idea of how to start and quickly abandoned any attempt.

8) (i) This second differential equation in the paper was also done very well, and full marks were often obtained. Few candidates failed to obtain the integrating factor, usually simplified to $\sec x$, and the process for multiplying by the integrating factor and obtaining an integral on the right-hand side was usually carried out correctly. The majority of those who reached $\int \cos^2 x \, dx$ then attempted to use the appropriate formula for $\cos 2x$. There were occasional sign errors in the formula, but most proceeded to the solution, inserting $+c$ at the right stage and giving the final solution in the form "$y =$ " as required. Final solutions such as $y = \dfrac{2x + \sin 2x + c}{\frac{4}{\cos x}}$ which ought to have been simplified were not penalised.

(ii) The method for finding the particular solution was almost always done correctly. Howlers such as $2 + \frac{1}{2}\pi = \frac{5}{2}\pi$ were seen occasionally.

9) (i) Although there were some excellent solutions to this question, many candidates had a very sketchy knowledge of what was expected. Usually closure was the first property to be checked, but it was quite common for a statement such as "Closed because $3^n \in \mathbb{Z}$ " to be made. Credit was given only for using $3^n \times 3^m$, and answers which used numerical values of n, or which used special cases like $3^n \times 3^n$ or $3^n \times 3^{n+1}$ were not allowed. Associativity was usually checked correctly by those who realised that a proof was needed, except that again some used $3^n \times \left(3^{n+1} \times 3^{n+2}\right)$ or numerical values. However, some answers claimed incorrectly that the assumption about addition of integers being associative meant that nothing had to be done. The identity element was usually correct, indeed many candidates scored only this mark. The inverse also had to be done properly, quoting or obtaining 3^{-n}. It was quite common for commutativity to be overlooked, but a number of other candidates proved only this property, perhaps thinking that the other group properties were to be assumed.

(ii) In the three cases to be considered, precise reasons were again required. Correct answers to parts (a) and (c) were seen only in scripts from the better candidates.

(a) Closure was the key to this, but proper consideration of $3^{2n} \times 3^{2m}$ had to be made. It was also necessary to verify that the identity and inverse elements were in the subset, but associativity carried through from the group itself.

(b) This part was quite often done correctly, although a clear statement about why the inverse of 3^n was not in the subset was expected.

(c) There were some good answers to this part. Consideration of $3^{n^2} \times 3^{m^2}$ was needed, either in general or in a specific case. Most correct answers quoted an appropriate counter-example such as $3^{1^2} \times 3^{2^2} = 3^5$ and concluded with $5 \notin \{n^2\}$, so that it is not a subgroup. In this instance $3^{n^2} \times 3^{n^2} = 3^{2n^2}$ can be correctly used as a counter-example, and some other explanations, such as a reference to Pythagorean triples, were also seen.

Chief Examiner's Report – Mechanics

The quality of candidates' scripts showed many pleasing features this session. The quality of their work was high both in its content and presentation. Few scripts indicated that candidates had been entered inappropriately.

This suite of examinations is not designed to test candidates knowledge of pure mathematics, but at all levels marks were lost through errors not related to the understanding of mechanics. These included:

- Not using positive and negative quantities consistently in a problem
- Not using constants of integration
- Not employing the specified variables (a request for an equation in T and a implies no other letter should be included)
- Not giving an answer in the specified way (express x in terms of … requires an equation starting $x =$ …)
- Giving the wrong quantity, though its value is correct (such as the angle with a direction of motion; often its supplement is given)

It is to be expected that errors in arithmetic will also be made, for which reason printed answers are sometimes included in questions where an early answer is used again later in a solution. Candidates should be alert to the reality that changing earlier work to contrive a given answer is likely to lead to a loss of marks when previously correct methods are altered. Continuing their solution and using the given value avoids this risk and consumes no time.

4728/01: Mechanics 1

General Comments

The scripts contained much work of high quality by nearly all candidates, with solutions well presented. In some cases, the answers given did not relate to the questions asked, for example in Q1, or did not conform to the specific requirement for the form of the answer, as in Q7(ii).

Comments on Individual Questions

1) (i) There were very many correct answers to this part of the question, though there were some slips in arithmetic, and some candidates found the X component by 14 – -9 = 23. However, the commonest error was the evaluation of the magnitude of the forces **P** and **Q**, and the angle each made with the x-axis.

(ii) This part of the question was answered correctly in many scripts, including some in which candidates had begun part (i) by finding **P** and **Q** but then reverted to using components in part (ii).

2) (i) Many scripts included clear answers, though the point on the graph for the time when the particle is at its greatest distance from A was sometimes not recognised. Indeed in all parts of this question candidates found underlying problems in interpreting the graph for the distances required, and partly correct solutions were frequent.

(ii) The most successful method appeared to be the evaluation of the areas of the four regions between the graph line and the t-axis, though other approaches were common and led to correct answers, most frequently finding the area of the first triangular region (240 m) and adding to it the rectangle (2520 m). A significant number of candidates calculated 290 x 12/2 = 1790 m.

(iii) Candidates who had evaluated the individual areas in part (ii) were able to re-use the values and obtain correct answers. Scripts in which the triangle + rectangle method had been used in part (ii) were often marred by adding the area of a triangle with height 24 ms^{-1} and base 40 s. Candidates seemed to be evaluating the area beneath the graph line, rather than finding the area between the graph line and the horizontal axis.

3) (i) Though given explicit instructions in how to begin the question, some scripts included the mass of the particle, or omitted a necessary force. It was also common for there to be a sign error, or an assumption that the normal reaction would always equal the weight of the block.

(ii) Correct solutions based explicitly on $R = 0$ were seen, though others suggested that candidates were equating $T\sin72$ and a normal reaction of $50g$ N. In some cases the further request to find m was overlooked, though it was included here to make it clear to candidates that the essential information needed was the given value of T. A significant number of candidates unable to find T knew that $T = mg$, but did not use the given value of 515 N to find m.

(iii) Candidates completed this part of the question well, but a few stated $T\cos72 = X$, but made no further progress by using $T = 515$.

4) (i) This task was tackled correctly by nearly all candidates.

(iia) Conservation of momentum was used appropriately by nearly all candidates, though some had terms of the wrong sign for the "after" momentum.

(iib) Many scripts had fully correct solutions, though some candidates considered only one of the two possible cases. Some candidates did not recognise the term "coalesce" (used in the specification), while others did not realise that the speeds of 1.5 ms^{-1} still applied.

5) (i) Nearly all candidates gained full marks, but problems with the consistent and correct use of signs spoiled some solutions.

(ii) Very many correct solutions were seen.

(iii) Candidates unfamiliar with this type of question found its unstructured style a major problem. Fully correct answers by any of the possible methods were rare. Generally where any progress was made, its initial steps were to find the time for each particle to reach its highest point. Progress beyond this was unusual, generally for the reasons mentioned in part (i).

6) (i) Rarely was any error seen in answering this part.

(ii) The most frequently awarded mark was 3. Scripts often lack any reference to an integration constant, or else quoted "$t = 0$, $v = 0$, so $c = 0$"; though these led to the printed answer, they meant the loss of 2 marks.

(iii) Here also candidates lost marks as a result of ignoring the integration constant. It was also common for the values of t to be recast as 0 and 10, rather than retaining the correct values of 10 and 20.

7) (i) Nearly all candidates answered this correctly.

(ii) While most candidates were able to create appropriate equations for each mass, they sometimes included letters other than T and a, most commonly F. The most common error in setting up the equations was the omission of a component of weight when considering the block. There were few attempts based on the incorrect idea of using Newton's second Law "around a corner".

(iii) When the correct equations had been formed in part (ii), this step was completed correctly. Candidates unable to find 2.24 made progress in the later stages by using the given value.

(iv) (a) Very many scripts had the velocity directly and correctly calculated, but a minority of candidates first found the time taken by the particle to reach the ground.

(b) Though some candidates recalculated the acceleration of the block, many ignored the string becoming slack and continued to use $a = 2.24$ ms^{-2}. Frequently this was used in conjunction with $u = 0$ and $s = 2.8$.

4729: Mechanics 2

General Comments

The majority of candidates were well prepared for the examination. The use of diagrams was better than in previous examinations although some candidates appear to be marking their forces and velocities on the question papers leaving examiners to interpret the sign conventions used. A small number of candidates confused energy/momentum and forces/moments. Most candidates appeared to have had sufficient time to complete the paper.

Comments on Individual Questions

1) The first question was answered well although there was a significant number of errors in transferring the angle of 35° to the answer paper. Such candidates lost one mark from three. As was the case with the whole paper, there were very few sin/cos errors in taking components.

2) Most candidates calculated the complete time of flight and were successful. A few used the time to the top. A minority quoted the formula for the range on a horizontal plane. Some of these misquoted the formula and consequently lost most of the marks. Examiners recommend the solution of such problems by using first principles.

3) (i) This was the first point in the paper where a significant number of candidates lost marks. This was due to the false assumption that the acceleration was constant. Candidates need to be aware that in work done/energy problems this does not need to be the case and that constant acceleration should not be assumed, indeed part (ii) emphasises that the acceleration changes with velocity. Weaker candidates incorrectly used $\frac{1}{2}m(v - u)^2$ for the change in kinetic energy.

(ii) This part was answered more satisfactorily than part (i) although some introduced weight into the equation of motion.

4) (i) Most candidates followed the instructions and expressed x and y in terms of t and were successful in deriving the cartesian equation quoted. A significant number of candidates quoted the trajectory formula and became unclear about their strategies.

(ii) Most candidates used the given equation, substituted $y = -25$ and solved the equation successfully. A significant number of candidates prolonged the problem by finding the time of flight by adding together at least two time intervals. However, the majority of these candidates gained all three marks.

5) (i) Well answered.

(ii) Well answered.

(iii) Many work done/energy equations were confused and a significant number of candidates lost marks. For example, the potential energy was frequently quoted twice in the energy equation.

6) (i) Most candidates scored full marks. Those who didn't were either not aware of the need to resolve vertically, took incorrect components, or ignored the tension in *AP*.

(ii) The majority of candidates scored full marks, although many unnecessarily calculated v first.

(iii) Errors occurred when candidates did not know that $v = r\omega$ or, as was frequently the case, they quoted kinetic energy = $\frac{1}{2}mv^2$ but failed to square the velocity.

7) (i) Well answered.

(ii) Well answered.

(iii) Well answered by most candidates. Some stated $f \geq 1/3$ and lost one mark.

(iv) As in previous examinations, a large number of candidates failed to take the change in direction of motion into account when calculating the impulse.

(v) Finding the speeds of A and B required considerable care and marks were frequently lost due to inconsistencies in the directions of motion of A and B before and after the collision.

8) (i) Most candidates were successful in taking moments about one of four axes. Some were not clear whether the distance from O to the centre of mass of the hemisphere was 0.3 m or 0.5 m. Some candidates complicated the problem by calculating the volumes of the two solids.

(ii) The instruction was to take moments about O. A large number of candidates made errors through not involving distances and/or having more forces involved in the equation than just F and T.

(iii) Only a few candidates were successful in resolving parallel to the slope. However, many successfully resolved vertically and horizontally and eliminated the contact force. Candidates who scored full marks in this part often scored very highly in the whole paper.

4730: Mechanics 3

General Comments

Candidates were well prepared for examination and their work was generally of a high standard and was well presented in content and presentation. There is no evidence that candidates had too little time to complete the paper.

Comments on Individual Questions

1) Almost all candidates were able to score well in both parts of this question. Most candidates used the relation between the velocity and the displacement of the particle, which is the simpler approach, rather than explicit expressions for v and x in terms of t.

2) This question was the least well attempted of the first five. A very significant proportion of candidates interpreted 'which reduces the speed of the ball to 7ms^{-1}, as though it implies that there is no change in direction. Such candidates could, perforce, score one mark at most for the question.

   Among other candidates, those who took the hint of 'using an impulse-momentum triangle' were much more successful than those who resolved impulse-momenta in mutually perpendicular directions. In the former case the most common error was to find the internal angle of the triangle that is opposite to the side representing the final momentum, without subtracting it from 180° to obtain the required answer.

   Candidates who resolved impulse-momenta were divided roughly equally between those who used directions parallel to and perpendicular to the initial direction of motion, and those who used directions parallel to and perpendicular to the impulse. Candidates generally wrote these equations accurately; in particular it was pleasing to see that the equations were dimensionally balanced in almost every case.

   Unfortunately the candidates were usually unable to proceed beyond this stage. Among those who did, many eliminated the wrong angle from the simultaneous equations, offering the angle found as the solution. Some candidates did find the relevant value 39.8°, as in the case of most candidates using the triangle method, but failed to realise that this is not the angle required explicitly by the question.

3) (i) & (ii) Both of the first two parts of the question were very well attempted.

   (iii) A significant minority of candidates scored fewer than two marks in this part of the question, some making no attempt at all.

   Candidates who used $\mathrm{d}x/\mathrm{d}t = u\mathrm{e}^{-2x}$ were usually successful. However some candidates did not separate variables and 'integrated' directly to obtain $x = u\mathrm{e}^{-2x}t + \mathrm{C}$. Others did not include a constant of integration and therefore had no occasion to apply $x(0) = 0$.

   Many candidates used $\mathrm{d}v/\mathrm{d}t = -2v^2$, solving for $v(t)$ by integrating after separating variables. Most did this correctly, but very many omitted the constant of integration and the subsequent use of $v(0) = u$.

4) This question was very well attempted, and it was rare to see fewer than eight out of ten marks scored.

5) (i) This part of the question was very well attempted.

(ii) A significant number of candidates omitted this part of the question. Those who did attempt it generally used a correct method, and any errors made were usually due to errors of signs.

6) (i) This part of the question was very well attempted.

(ii) Many candidates adopted a correct strategy for dealing with this part. The most common error was to omit the initial elastic potential energy from consideration. A common error among candidates using the principle of conservation of energy to obtain an equation in an unknown distance, say X, was to have the incompatible 160gX for the change in GPE and $196X^2$ for the final EPE.

(iii) Most candidates considered the energy at the net and at *O*, and almost all were successful. Some candidates used two stages, finding the kinetic energy at the point where the rope becomes slack at the end of the first stage, for use in the second stage where it has to be shown to be equal to the further gain in GPE. Candidates who used two stages were generally less successful than those who used one stage.

(iv) Most candidates were able to state two suitable assumptions, although many included ‘the performers were modelled as particles’. This is not suitable because it is not an *additional* assumption.

7) (i) Most candidates obtained a correct expression for the radial acceleration, but errors were made in applying Newton’s second law. These included the omission of the mass from one or both of the relevant terms, and an error in the component of the weight of *P* such as the omission of g, or writing $\sin 60^o$ instead of $\cos 60^o$.

(ii) Because of the given answer for *Q*’s initial speed, most candidates realised the need for *P*’s speed on impact to be 2.1ms^{-1}. Many candidates were unable to obtain this value legitimately. The most common error was to omit the kinetic energy of *P* at $\theta = \pi/3$, on applying the principle of conservation of energy. Some candidates tried to use the irrelevant principle of conservation of momentum, when a value different from 2.1ms^{-1} was found for *P*’s speed on impact.

(iii) Most candidates applied Newton’s second law to *Q* and almost all did so in the direction perpendicular to *OQ*. However some candidates applied the law radially and thus made no progress.

Most candidates obtained $-(m)g\sin\theta = (m)r\ddot{\theta}$, but others made errors which included the omission of g, the omission of *r* and the inclusion of *m* on only one side of the equation. Candidates who introduced arc length or horizontal distance usually got into a muddle.

The only angle given in the question is the initial value of angle *POQ*. This value is $\pi/3$ so it is clearly necessary to demonstrate that the maximum angular displacement of *Q* is small in order to justify the use of $\sin\theta \approx \theta$. However this was almost universally omitted.

(iv) This was very well attempted, although a significant minority thought the required time interval is the period.

4731: Mechanics 4

General Comments

There were a good number of excellent scripts, with about a quarter of the candidates scoring 60 marks or more (out of 72); but there was also quite a significant proportion who were clearly not ready for an examination at this level, with about a quarter of the candidates scoring less than half marks. Most candidates seemed to have answered all that they could in the time allowed.

Comments on Individual Questions

1) This question, on constant angular acceleration, was very well answered, with about 85% of the candidates scoring full marks. The only errors were slightly mis-remembered formulae and careless arithmetic.

2) Almost all candidates knew how to find the centre of mass of a solid of revolution, and about three quarters of the candidates obtained the correct answer. When marks were lost it was usually because of careless slips, although some did attempt to use the formulae for finding the centre of mass of a lamina.

3) This question, on rotation and energy, was quite well answered, with about 40% of the candidates scoring full marks.

   (i) Almost all candidates added the two given moments of inertia; but some thought this gave the moment of inertia about *G* and followed it with an application of the parallel axes rule to find the moment of inertia about *A*.

   (ii) There were a few attempts to use constant acceleration formulae, but the great majority of candidates realised that they should consider energy. Most had all three terms (potential energy, kinetic energy and the work done against the couple) in their equation, but the loss of potential energy was often incorrect and the angle turned through was quite often different from $\frac{1}{2}\pi$; many candidates had sign errors in their equation.

   (iii) Some candidates assumed constant acceleration, but most did attempt to apply the equation of rotational motion. It was quite common for one of the terms contributing to the moment (either the weight or the frictional couple) to be omitted from the calculation.

4) This question required the application of the parallel axes rule to an elemental disc before integrating to obtain the moment of inertia. This has not been examined before, and about half the candidates were unable to carry this out, scoring 4 marks or less in the question. On the other hand there were many who proceeded with confidence, and about 30% of candidates scored full marks.

   (i) Generally, candidates either knew what to do, in which case they usually obtained the moment of inertia correctly; or they had no idea, in which case they made no sensible attempt or perhaps earned one mark for writing down the moment of inertia of an elemental disc about its diameter.

   (ii) This part was well answered, and the period of the compound pendulum was very often found correctly.

5) This relative velocity question was answered much better than similar questions in the

past. While about 20% of candidates had little or no idea and scored 2 marks or less, this is a lower proportion than usual. About one third of the candidates answered the whole question correctly.

(i) Most candidates made a good attempt to find the course for interception. Those who drew a velocity triangle were much more likely to be successful than those who considered components. The method for finding the time was also well understood.

(ii) Although this part was quite often omitted, a good proportion of the candidates did know how to find the course for closest approach.

6) This question, on rotation and the force acting at the axis, was well answered, with half the candidates scoring 12 marks or more (out of 15) and about 40% scoring full marks.

(i) Most candidates were able to find the moment of inertia, and obtain the given angular acceleration.

(ii) Most candidates found the angular speed correctly. The majority used conservation of energy; some integrated the angular acceleration with respect to θ to obtain $\frac{1}{2}\omega^2$.

(iii) Most candidates knew what to do to find F and R, although very many made errors of detail, such as using a or $\frac{2}{3}a$ for the radius instead of $\frac{1}{3}a$, and sign errors in the equations of motion.

(iv) This part was very well understood, although incorrect expressions for F and R frequently prevented the given result from being obtained.

7) This question, on the energy approach to equilibrium, was found to be the most difficult question, with part (iii) defeating the great majority. Only about 15% of candidates scored full marks, and about an equal number scored no marks at all.

(i) Most candidates made a good attempt to find the total potential energy, and a good proportion were successful. Some used rather inelegant methods, such as the cosine rule to find the length of the string AR.

(ii) Most candidates knew how to find the position of equilibrium.

(iii) The kinetic energy term was usually wrong, or missing altogether. Nevertheless, a good number of candidates differentiated their energy equation correctly.

(iv) Most candidates who attempted this part were able to use small angle approximations to obtain a simple harmonic motion equation, and to find the period correctly.

Chief Examiner's Report – Probability and Statistics

As usual there was much good work seen on all statistics units. It is good to see that note has been taken in many Centres of the importance of stating hypotheses and of giving conclusions in the context of the question (though see below). There are, however, three areas which continue to need particular emphasis.

- Verbal answers are still often poor – candidates' understanding seems to have been outstripped by their capacity to do calculations, which is, frankly, not the most important part of statistics. In particular, verbal answers this year laid bare massive misunderstandings of the concept of probability density function.
- Candidates vary very much in their ability to use formulae from the *List of* Formulae, MF1. The attention of Centres is drawn to the remarks on this, particularly under Probability and Statistics 1 (4732).
- Questions involving the use of binomial and Poisson distributions, with or without tables, continue to be the least well done sections of their respective specifications.

The attention of Centres is also drawn to the following intentions for future examinations:

- Statements of hypotheses should include definitions of the meaning of the symbols used, for instance:

  "$H_0 : \mu = 0$, $H_1 : \mu > 0$, where μ is the population mean difference in blood pressure measurements."

  In the immediate future the absence of such a definition will not be penalised unless it is explicitly requested, but it is intended that in due course such statements should be made as a matter of course. It is certainly good practice, and it may well also help candidates to focus on the key difference in the roles of population parameter and sample statistic in hypothesis tests, which is at present a widespread weakness.

- Statements of the conclusions of hypothesis tests should not be given in too assertive a manner. Thus not "aneroid readings <u>do not</u> overestimate blood pressure" but "<u>there is insufficient evidence that</u> aneroid readings overestimate blood pressure". Over-assertive statements of conclusions may not receive full credit in future examinations.

4732: Probability and Statistics 1

General Comments

Most candidates showed a good understanding of much of the mathematics in this paper and there were some very good scripts. There was a wide range of total marks. Algebra in question 8(ii) was sometimes weak. There were several questions that required an interpretation to be given in words, and these were answered fairly well on the whole. More able candidates gave some very good written explanations. However, many candidates lost marks unnecessarily because their answers did not refer to the context.

This year again it was pleasing to note that very few candidates ignored the instruction on page 1 and rounded their answers to fewer than three significant figures, thereby losing marks. However, in a few cases marks were lost through premature rounding of intermediate answers.

The only question which made a significant call upon candidates' knowledge of Pure Marthematics was question 9(ii)(b), which was not well answered.

A few candidates appeared to run out of time.

Most candidates failed to fill in the question numbers on the front page of their answer booklet.

Very few candidates scored full marks. This was due mainly to the difficulties found by candidates in questions 7 and 9.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae

The *List of Formulae*, MF1, was useful in Qs 2, 6(i)(a), 7 (for binomial tables and formula) and 9(ii)(b) (for a Pure Maths formula). However, a few candidates appeared to be unaware of the existence of MF1. Some candidates tried to use the given formulae, but clearly did not understand how to do so properly (e.g. Σx^2p was misinterpreted as Σxp^2). Some candidates found Σxp or Σx^2p correctly but then divided by 4 or 6 (but not usually in both of these formulae. Why not?). For the variance, many candidates omitted to subtract μ^2 from Σx^2p, or subtracted just μ. A few candidates used the less convenient version, $\Sigma(x - \mu)^2p$, from MF1, leading to arithmetical errors in most cases. A few candidates quoted the formula for Spearman's rank correlation coefficient wrongly. Others interpreted Σd^2 to mean $(\Sigma d)^2$. In Q6(i)(a) some candidates quoted their own formulae for r_s, rather than using the one in MF1. Usually these were incorrect. Some candidates' use of the binomial tables showed misunderstanding. Others used the binomial formula rather than the tables where the latter was clearly more appropriate (in Q7(ii)). Perhaps some centres advise students to use the formula for all binomial calculations, since it is always applicable, whereas the tables can only be used for those values that are included therein. This is bad advice.

It is worth noting yet again, that candidates would benefit from direct teaching on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1. They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

Comments on Individual Questions

1) This question was fairly well answered – better, on the whole, than in the past. A few candidates made the errors mentioned above and a surprising number of relatively weak candidates seemed to be unaware of probability distribution theory and found, for example, $\Sigma x/4$.

2) This question was also well answered, despite its being presented in a different form from usual. A few candidates omitted the "1–" or placed this in the numerator. A large minority quoted the formula correctly, possibly also substituted values correctly, but then made one of these two errors.

3) (i) Most candidates answered this part correctly. A few used a permutation.

(ii) Most candidates also answered this part correctly. The most common error was to add the two combinations. A few candidates calculated $\frac{^{15}C_7}{3!\times 4!}$ or similar. Some found the correct answer, but divided it by $^{15}C_7$, wrongly thinking that they needed to find a probability. In view of the wording of the question (the phrase "without regard to order" not being repeated) permutations were accepted in this part. However, most of the small number of candidates who used permutations thought that $^9P_4 = {}^{9!}/_{4!}$.

4) This question tests understanding of the idea of conditional probability. Candidates, however, tended to fall into one of two categories. Most did not recognise the significance of the phrase "given that", and treated both parts (a) and (c) as "AND" probabilities. At the other end of the scale were those who used the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$. This led to long methods in parts (a) and (c), sometimes arriving at a correct answer, but often not. In fact this formula is not required for this module; it appears only in the specification for module S4. Whenever conditional probability is tested in S1, as in this question, it will only involve an understanding of the idea of conditionality, not any formal treatment.

(i)(a) Very few candidates realised that the answer could be written down immediately. Most found P(BB).

(b) This part was answered correctly by most candidates.

(c) Almost all candidates found P(BBB or BWB).

(ii) Many candidates gave the answer "Yes" because discs are removed until the first success. Many stated "No because there are not two outcomes". Some gave a correct answer, but without referring to the context, for example "No because the probability is not constant" or "No because there are a limited number of trials". Some gave an inadequate answer, "No because the discs are not replaced". None of these candidates gained the mark. Very few stated "No" either because the number of discs is limited or because the probability of removing a blue disc is not constant.

5) (i) This part was well answered.

(ii)(a) Many candidates used the 1991 total, but read from the 2001 curve. A few did the opposite. Many candidates showed no working and therefore lost marks.

(b) This part was well answered. A few candidates referred to the larger IQR for 2001, suggesting that this implied that the 2001 mothers were older. A very small minority gave a sociological answer.

6) (i)(a) This part was well answered, with a very small minority using an incorrect formula or simply $S_{xx} = \Sigma xy$ etc.

(b) Most candidates gave the correct value for r_s, but many explanations were incorrect or inadequate, for example there is strong positive correlation. Some stated that $r_s = 0.675$ (as r) or some other value close to 1. A few stated that $r_s = 0$.

(ii)(a) Many candidates understood the point, but gave poor explanations, for example it will increase because the points removed are "low", or because there are now fewer points. Some candidates attempted explanations that referred to the values in the formula. None of these was valid.

(b) Again, some candidates understood that r_s would be unchanged, but could not give a convincing explanation.

(iii) Most answered this correctly. A few gave 14.9 or a value clearly too low, such as 13.7.

(iv) Some candidates gave, parrot fashion, the standard response, namely that the regression line is better because it takes account of all the values. Many others showed an ill-founded faith in statistical formulae, declaring a belief that any calculation based on an actual formula is bound to be more reliable than reading a graph by eye (involving "human error"). All answers of these kinds missed the point, which is the obvious curved nature of the graph.

7) (i) The two conditions required are independence and constant probability, both of which must be stated with reference to the context, i.e. mentioning vouchers. Some candidates attempted to do this, but their answers were confused, for example "The vouchers are independent". Other candidates gave these two conditions in general terms and gained no marks. Many others gave general conditions for a binomial distribution or simply restated facts given in the question, for example "Two possible outcomes", "Fixed number of trials" and "She buys one packet each week".

(ii) Some candidates used the table, but looked up the value for 5 rather than 6. Others subtracted the values for 6 and 5 or for 7 and 6. Still others added the values for 0, 1, 2, 3, 4, 5 and 6. Many candidates used the formula instead of the table. Of these, some just found $P(X = 6)$. Others attempted to add all the terms, but omitted one. Even those who included all the correct terms sometimes made arithmetical errors. For those who knew how to use the tables, there were two very simple marks here. For those who insisted on using the formula, there was a great deal of time-wasting arithmetic to be done.

(iii) Only a minority of candidates recognised that this simply required subtracting their answer to part (ii) from 1. Most started from scratch, with many finding just $P(X = 7)$. Candidates should note that a question with a tariff of only one mark is likely to involve little or no calculation.

(iv) Only a minority of candidates recognised the need for a binomial with 6 successes in 11 trials, followed by a single success. Many just considered 12 trials and found $P(X = 7)$ or $P(X = 6)$ or $P(X \geq 7)$.

8) (i) This question was well answered by a good number of candidates. Others halved 0.04 instead of finding the square root. Some just found $(1 - 0.04)^2$.

(ii) The responses to this question were very mixed. Many started with $pq = 0.42$, although even some weaker candidates arrived at $pq = 0.21$. Some of these "spotted" that $0.3 \times 0.7 = 0.21$, and just gave these two values as the answers for the possible values of p (perhaps using faulty reasoning, but it was impossible to tell). A few started with $pq = 0.42$, and gave answers of 0.6 and 0.7. However, many candidates did as expected. They combined $pq = 0.21$ with $q = 1 - p$ and obtained the correct equation, $p - p^2 = 0.21$. Some could not proceed from here, but those who were able to rearrange this equation into the necessary form usually achieved the correct answers. A few candidates started with $2pq = 0.42$, put this together with $(p + q)^2 = 1$, and arrived at a quartic equation. Some of these succeeded in reaching the correct answers.

9) (i)(a)
(b) These questions were well answered by most candidates.

(i)(c) This question gave rise to all the usual errors such as $(^4/_5)^3$, $1 - (^4/_5)^4$ and $(^4/_5)^4 \times {}^1/_5$. Those who used the long method often omitted one term or added a bogus extra one ($P(X = 0) = (^4/_5)^{-1} \times {}^1/_5$), presumably through mindlessly quoting the formula for geometric probabilities, rather than considering its meaning.

(ii)(a) Many candidates did not understand the question. Amongst those who at least appreciated what was required, explanations were generally quite good.

(ii)(b) Many candidates started with the given answer of $\dfrac{1}{1+q}$ and attempted to match this up in some way with the formula $\dfrac{a}{1-r}$. Only a minority of candidates understood that they should start with the series given to them in part (ii)(a) and proceed to sum it. Of these, many saw that $r = q^2$, but could not get beyond $\dfrac{p}{1-q^2}$. Some went on to find $\dfrac{1-q}{1-q^2}$ or $\dfrac{p}{p(2-p)}$ and then gave the final answer (which was given in the question) without showing the last step. One ingenious candidate gained full marks by arguing thus: $p + pq^2 + pq^4 + \ldots$ $= (1 - q) + (1 - q)q^2 + (1 - q)q^4 + \ldots$ $= 1 - q + q^2 - q^3 + q^4 - q^5 + \ldots$ which is a GP with first term 1, and common ratio $-q$, and hence yields the required expression. A few others recognised this series as the binomial expansion of $(1 + q)^{-1}$ but simply quoted this, rather than showing its derivation from the general series for $(1 + x)^n$. These gained two out of the four marks. Strangely, a large minority of candidates quoted the PGF for the binomial distribution, $\dfrac{pt}{1-(1-p)t}$, but none made any progress from here.

4733: Probability and Statistics 2

General Comments

This paper had a higher than usual proportion (11 marks out of 72) of questions requiring verbal answers. These were generally poorly answered, revealing many candidates who could quote formulae and do a wide range of calculations but seemed to have very little idea of what they meant. Many answers showed that these questions had not been read with sufficient care. There seemed to be a good deal of "teaching to the test"; those questions that were dissimilar to those asked in the past were poorly done whereas a question such as Q8, which was conceptually complicated but which had been asked in similar form in the past, was answered very well.

Candidates seem to be uncomfortable with the concept of a distribution. When a question is asked that requires modelling assumptions, the first reaction of many, instead of thinking of the shape or type of the distribution, is to write down numerical conditions on parameters. It may be worth spelling out that such numerical conditions usually refer only to *approximations to other distributions*.

Examiners noted that some common topics, such as hypothesis testing using discrete distributions, were even less well done than in the past, despite regular mention in Reports that this was a weakness. Some candidates seem to be unable to do hypothesis tests using any distribution other than the normal.

More candidates are using electronic calculators that do a wide range of statistical calculations in a single step. Most such candidates clearly take trouble to write down sufficient details of their calculations for examiners to see what they are doing, but a few do not; they are reminded that it is hard to give any marks at all for a wrong answer with no supporting working.

Few candidates seemed to suffer from time pressure. However, there seemed to be more very weak candidates than in the past.

The attention of Centres is particularly drawn to the comments on Qs 6(i) and 7(ii).

Comments on Individual Questions

1) (i) For many this was a straightforward start, although some forgot the factor $n/(n-1)$ for the variance. Those candidates who used a single formula to get the unbiased variance estimate often could not remember it correctly; they would be better advised to remember $\Sigma x^2/n - \bar{x}^2$ and then multiply it by $n/(n-1)$.

   (ii) Here the question did not state the distribution of X. Better candidates easily saw that the Central Limit Theorem could be applied, but many did not seem to think of whether they needed the shape of the distribution at all. Some seemed to assume that it had to be normal because they were given the mean and variance.

2) Apart from some numerical errors, most candidates saw that this was B(130, 1/40), but some failed to read the question properly, using that distribution for their calculations instead of "a suitable approximation" as instructed. As usual, some tried to use a normal approximation, which is not valid here as $np < 5$. Among those who got as far as Po(3.25), some attempted to use tables, either for Po(3.3) or Po(3.5). However, a majority of candidates got the correct answer.

3) Most correctly stated that the relevant distribution was binomial. However, part (ii)

produced few good answers, with most regurgitating the conditions for a binomial distribution to be valid. This was not what the question asked; candidates had to consider the properties of a random sample. The specification mentions appreciating “the benefits of randomness in choosing samples”, and the points here are that in a random sample each member of the population is equally likely to be chosen, and the choices are independent of one another. As a result, the conditions required for a binomial distribution are met. Thus the common answer “people’s opinions are independent” was off-target; the important thing was that people were *selected* independently.

4) (i) The first part of this question seemed to leave most candidates at a loss. Most said things like “np has to be large” or “X has to be selected independently” or even the absurd “X has to be large”. Those who said “the distribution has to have a bell-shaped curve” were begging the question, though they received some credit. The best answers were qualitative, for instance “Equally likely to be above or below the mean and most likely nearer the mean”

(ii) This was fairly well answered, though the factor $\sqrt{20}$ in the denominator was often missing and 0.0468 was often seen instead of 0.9532.

5) (i) This was usually correct.

(ii) Many correct answers were seen here as well, although the usual error in using tables of discrete distributions, $P(R > 23) = 1 - P(R \le 22)$, was also common. Some weaker candidates attempted to “scale” the calculation by finding $P(R > 23/6 | \lambda = 2.5)$.

(iii) Here many candidates who were looking for a probability just below 0.1 seemed to find 0.0093, while the vast majority failed to divide their parameter value by 6. However, most showed sufficient working to be given at least partial credit.

6) (i) This was an entirely routine example of a hypothesis test using a binomial distribution, and it was remarkably poorly done. It was rare to see candidates at the level of, say, grade C getting it even nearly correct. First, the hypotheses were often wrongly stated, using letters such as μ or e instead of p. Many weaker candidates used the sample value of p, namely 1/20, or a mean of 3.8, either in a normal distribution or as some sort of critical value, and even among those who used the correct distribution B(20, 0.19), far too many calculated merely P(1) instead of P($\le$ 1). This topic has always been a weak area on the specification, but this year it appeared that even fewer candidates than usual were properly familiar with it.

(ii) The rider was often well answered. It was good to read a range of relevant sensible suggestions.

7) (i) It was surprising to see how many candidates continued their horizontal straight lines right across their diagram, and well past the limits of their parabola, oblivious of the fact that the probability density function changes from ½ to 0 outside the range [–1, 1]. Otherwise the diagrams were often well drawn, though some drew the straight line vertical or with gradient ½, and some drew the parabola going through (0, 3/2). When doing such a question it is wise to bear in mind the fact that the areas under the curves must be equal.

(ii) Quite apart from those who failed to address the question and wrote merely about the shape of the graphs instead of the random variables, it was clear that many were vague as to what the variables in graphs of probability density functions are. It would appear that many candidates have at the back of their minds an idea that x is some arbitrary parameter and the value of the random variable is measured on the y-axis, so that answers such as "S is constant but T varies", or "S is equally likely to happen for any value of x but T is more likely to happen for values of x close to ±1" were common. A similar question was asked in 2005 and it is disappointing that the quality of answers has not improved.

(iii) A number of candidates automatically calculated the mean and variance of the distribution, often going on to try to find t using a normal distribution with the same mean and variance. A few tried to integrate $f(x)$ between -1 and 1. However, those who knew what to do generally got at least 4 of the 5 marks; common mistakes included equating the integral between -1 and t to 0.2 instead of 0.8, failing to deal with the signs correctly, or pressing the square root button instead of the cube root button at the end.

8) (i) This, by contrast, was the best answered question on the paper, and it was not at all uncommon for answers to obtain full marks. As the concepts involved are far from straightforward, it is clear that this material has been taught and learnt well. The most common errors were in omitting the $\sqrt{n}$ factor, or in treating 12.25 as σ rather than σ^2.

(ii)(a) Many gave the critical value ("$c = 63.81$") rather than the critical region ("$c > 63.81$"), though on this occasion they were not penalised.

(b) Weaker candidates tended to use the wrong tail of the distribution, or to attempt to find $P(\bar{Y} < 63 \mid \mu = 65)$.

(iii) Most gave a satisfactory answer to the last part, though it is worth noting that comparison of the sizes of Type II errors for a given wrong μ is sensible only if, as here, the probability of a Type I error is almost the same for both tests.

9) (a) The specification states that the conditions for a normal approximation to the binomial to be valid are $np > 5$ and $nq > 5$. Some used npq and some, having obtained $n > 6\frac{2}{3}$ and $n > 20$, then either did not write down a final answer or gave $6\frac{2}{3} < n < 20$.

(b) This was a good discriminator at the end of the paper. Most candidates were able to get somewhere with it. The usual mistakes were omission of the continuity correction in the first part, incorrect signs of z, and, in part (ii), use of $npq = \sigma$ instead of σ^2.
The simultaneous equations to be solved are $70.5 - \mu = 1.75\sigma$ and $46.5 - \mu = -2.25\sigma$
It is amazing that so many candidates attempt to solve a pair of equations like these by substitution, when elimination is so very much easier and when equations with exactly this structure appear in almost every paper. Those who attempted to use np and npq at an early stage, in place of μ and σ, tended to make life harder for themselves. A good deal of follow-through marking was allowed in this question, so that, for example, those who omitted the continuity correction could still get 8 marks out of the possible 10.

4734: Probability & Statistics 3

General Comments

The paper was well-received by a majority of candidates who, apart from in a few areas, were able to perform admirably.

In the hypothesis tests some used the critical value (or region) of the test statistic rather than calculating the value of t or z or χ^2 from the sample. Whilst being correct, it does take longer and is more prone to error.

Also in the tests, marks were often lost in the statement of hypotheses. For parametric tests these should be given in terms of population parameters, for example μ or p, which (ideally) should be defined. The conclusion of a test should always include a specific comparison of the test-statistic with the relevant critical value (for example 1.558 > 1.363) or a clear statement of the critical region and why the test statistic is, or is not, in it. The examiners require a conclusion in context, which most candidates are now giving, but when H_0 is not rejected, some candidates state that there is evidence that H_0 is true. This is not strictly correct, the test can only indicate that there is insufficient evidence for H_1. The conclusion should be in words and avoid the use of the parameters in the hypothesis.

Graphical calculators can perform many of the tasks required on the paper but candidates should ensure sufficient detail is given of what has been found. In Q4, it was expected that candidates should have indicated how the expected values were obtained.

In some cases the accuracy of calculations was not acceptable. Final answers should mostly be given to 3sf, and this requires intermediate values to, at least, 4sf.

Comments on Individual Questions

1) This proved to be an easy question whose pitfalls were mainly avoided.

2) (i) Some candidates did not appear to notice the bars over the Xs and others did not give the parameters.

   (ii) Although many candidates could obtain the correct answer, 30% of the candidates failed either by using an incorrect variance or calculating z wrongly or not obtaining the tail probability.

3) Most candidates knew that a t-distribution was required and could obtain the critical value correctly. Only a few stated that the sample needed to be random for the validity of the test.

4) (i) Almost all candidates scored the possible 2 marks.

   (ii) This was very well known, including the application of Yates' correction. Many candidates lost a mark through not indicating how their E-values were obtained.

5) (i) Since $\mu \approx 0.80$ had to be shown, it was necessary to give it to at least 3dp before rounding.

   (ii) This was usually well done if the correct mean had been obtained.

   (iii) Some candidates misinterpreted 'total' and others added the two probabilities, rather than multiplying them.

6) (i) Most candidates could correctly obtain the required confidence interval.

(ii) The interval is not exact because the distribution of the sample proportion is only approximately normal. Also, the variance of the sample proportion uses the sample parameters and this leads to inaccuracy. However, the fact that a continuity correction had not been used by the candidate made the interval more inaccurate.

(iii) Many candidates were aware that the best estimate of the common proportion was required. Some, however, used a variance' of $p_\alpha q_\alpha/200 + p_\beta q_\beta/150$ and this was penalised.

7 (i) This is the most difficult part of the syllabus, and only a minority of candidates could obtain the correct result. Many were able to start from $P(Y < y)$ but then obtained the y in terms of x rather than the Y in terms of X.

(ii) Most candidates knew the relation between $g(y)$ and $G(y)$ and could score a mark.

(iii) Candidates usually knew which integral was required but many could not obtain it accurately.

8) (i) Most of this part used S2 material and many candidates could score high marks. It was not always realised that the table was symmetric, which would have made the calculations simpler. Candidates often forgot to check that the sum of the frequencies was 50.

(ii) This straightforward goodness-of-fit test was very well answered.

(iii) Apart from a few with an incorrect variance, a majority of candidates achieved full marks.

(iv) The point of this part was that, whatever the distribution of Y, the Central Limit Theorem applies for a sample size of 50. Only a minority mentioned this.

4735: Probability & Statistics 4

General Comments

There were many excellent scripts on this paper which perhaps contained more straightforward questions than usual. Q3 was very well received and only parts of Qs 6 and 7 caused any difficulty.

Comments on Individual Questions

1) (i) This proved to be an easy start and was tackled successfully both by appealing to a Venn Diagram and also by using appropriate formulae.

(ii) Several candidates did not *deduce*, as required by the question, and others did not supply enough detail in showing the given result.

2) Most candidates used the correct Wilcoxon test, which is described in detail in the formula booklet. The table of critical values requires a bottom-up ranking so those (few) who ranked top-down obtained the wrong value of W. The hypotheses should be given in terms of the relevant medians, m, but if in words then *population median* should be stated.

3) (i) There was little in this part that caused problems. As in Q1(ii) some candidates lost marks through not giving sufficient detail in obtaining the given answer.

(ii) Some candidates just wrote down the answers. One mark was given for a method, and candidates who did not indicate how at least one of the values were obtained were penalised.

4) (i) Candidates were very familiar with the sign test and most could carry it out accurately. The test was for $m > 2.70$, so really $P(X \geq 13)$, where X was the number of + signs, was appropriate. Since B(20, 0.5) is symmetrical $P(X \leq 7)$ was acceptable.

(ii) The other possible test (Wilcoxon signed rank) was stated by a majority of candidates who also could give the requested advantage and disadvantage

5) (i) Only a few did not see that the result followed from the integral of the p.d.f.

(ii) This was not the first time this type of question had been asked, and there were some excellent responses. The biggest problem was converting the dx to du.

(iii) This only depended on the use of the correct moment-generating function, and since this appeared in part (ii) most could obtain the correct answers. Results were obtained either by differentiation of the m.g.f. or by expanding it in powers of t. There was the occasional error in the latter method when $E(X^2)$ was stated to be half, rather than double, the coefficient of t^2.

6) (i) (ii) & (iii) The first three parts of the question was illustrating how the mean and variance of a binomial distribution could be found easily using the p.g.f. of the Bernoulli distribution. It was gratifying to find so many good solutions to this, displaying familiarity with the generating functions and the distribution.

(iv) This proved to be a discriminator. However, some smart candidates identified $e^{-(1-t)}$ as the p.g.f. of a Po(1) variable and were able to obtain the required probability by enumerating the possible values of Y and Z. Those who used the p.g.f. often expanded $e^{-(1-t)}$ in powers of $(1 - t)$ which did not lead to the answer.

7) (i) Many confused $\overline{X}$ with μ and wrote $\overline{X} = \int_0^\theta \frac{x}{\theta} dx$, which, although correct, requires some explanation.

(ii) Those careful with their algebra and calculus could score high marks on this.

(iii) Most candidates could find $\text{Var}(T_2)$ but few could correctly obtain $\text{Var}(T_1)$, often misreading it as $\text{Var}(2X)$. However, the idea that the better unbiased estimator was the one with smaller variance was known by most candidates.

4736: Decision Mathematics 1

General Comments

There were many very good scripts and only a few really poor attempts. There were very few candidates who could not attempt every question although some candidates threw marks away by not following the instructions in the questions.

Comments on Individual Questions

1) (i) While most candidates knew what a cycle is, some drew a diagram rather than ‘writing down’ their example as instructed in the question.

(ii) Many candidates claimed that the given trail is not a path because it does not visit node *W*, while others claimed that the problem was that it did not start and end at the same node. The actual issue was that the trail passed through node *Y* twice (it contained a cycle within it).

(iii) Most candidates found the correct value of 5, some by working out $n – 1$ and some by drawing an example of such a tree.

(iv) Generally well answered, although some candidates thought that both graphs must be of the same type. Most, however, knew the definition of Eulerian and semi-Eulerian in terms of the number of odd nodes and were able to count the numbers of odd nodes correctly.

(v) This part needed some careful reading, and candidates who gave the number of times the etching tool needed to be repositioned for each graph were given 1 mark. Some candidates just gave the total number of times the etching tool needed to be positioned, instead of giving the number for each graph, and some candidates included the lifting of the tool at the end in their counts.

(vi) Many correct answers. The most common incorrect answers were from candidates who had included the arc *QU* in their count or candidates who had assumed that they needed to duplicate existing arcs.

2) (i) Several candidates began their answer by writing down all the information given in the question but then did not indicate which was the constraint from the total area. A number of candidates claimed that $40g+d+f = 120$.

(ii) This part required candidates to find the relevant words from the text, not to interpret the constraint in context.

(iii) Another simple request that several candidates made more complicated than it needed to be.

(iv) Most, but not all, candidates were able to write down the constraint $g \geq 40$. Several were able to identify 10 as the minimum value for *d* and a few found 20 as the minimum for *f*.

(v) Several candidates merged their answers to parts (v) and (vi), either going straight into part (vi) or answering part (v) but then omitting the objective from part (vi)

(vi) Very few candidates were able to set up the LP problem correctly. Many omitted at least one constraint and several were not able to convert inequalities into equations by adding slack variables.

3) (i) Candidates were asked in the question to 'write down the list that results at the end of each pass', several did not make it at all clear where one pass ended and the next began. Some candidates crossed out their numbers in working from one pass to the next making it almost impossible to give them any credit. The majority of candidates did seem to be using shuttle sort, rather than bubble sort, although some were applying a hybrid mix of the two. The second pass involved a comparison but no swaps and a number of candidates omitted to record this pass.

Candidates were asked to 'record the number of comparisons and the number of swaps that are made in each pass', this required the separate values (given numerically), not just tallies followed by a total.

Some candidates applied shuttle sort correctly except that in the second pass they claimed to have done two comparisons. When a comparison does not result in a swap there is no need to shuttle back down, so only one comparison was needed.

(ii) Most candidates were able to follow the instructions up to Step 6, but several then either assumed that the final list would be the sorted list or went back to Step 1 to execute a second pass that resulted in the sorted list. The candidates who did exactly what the question asked usually achieved full marks.

4) (i) Nearly all candidates recognised what they were supposed to be doing here, some left out one or more of the slack variables and a few gave the objective row as $P + 3x – 5y = 0$, or even $P – 3x – 5y = 0$, rather than $P – 3x + 5y = 0$.

(ii) Most candidates were able to find the correct pivot choice for their tableau and many were able to explain their pivot choice well. Some candidates just referred to 'choose x because negative' rather than 'choose the x column because it has a negative value in the objective row', or equivalent. Most candidates then showed the calculations of the three ratios and explained that $10 \div 1$ gave the least non-negative ratio, some candidates lost a mark here by not being specific enough.

(iii) Most candidates showed that they knew how to perform the pivoting operations, even if they had made earlier errors, and the majority of candidates were able to read off the values of x, y and P from the resulting tableau. Some candidates tried to solve simultaneous equations to find x and y and some continued to an unnecessary second iteration.

(iv) Most candidates were able to substitute $x = 11$, $y = 0.2$ into the objective to find the improved value of P. Rather fewer candidates verified that this was a feasible solution by showing that these values fitted all the constraints.

5) (i) Most candidates were able to find the shortest path and its length but several made errors in applying Dijkstra's algorithm. Many candidates recorded temporary labels that were larger than the current value (for example, recording 90 at I when the current value was 75) and some omitted necessary temporary labels (for example, only recording 70 at F when the temporary label 90 should have been recorded first, working from J, and then updated to 70). Several candidates crossed out values, making it unclear whether they had deleted them or updated them and some candidates only recorded values as permanent labels

rather than recording them as temporary labels first. As well as issues with the temporary labels, several candidates did not record the order of assigning permanent labels correctly, even when their permanent labels were correct.

(ii) Many candidates achieved full marks on this part, whilst others tried to construct a route without using an algorithm at all. The majority of candidates recognised this as being a route inspection problem and were able to identify the odd nodes. Most candidates gave weights to all the pairings of odd nodes and hence found the appropriate arcs to duplicate. A few candidates were using the correct method but lost marks through missing out detail in their working.

(iii) The majority of candidates interpreted 'Janice wants to visit every cleaning station using the shortest route possible' to mean that they had to construct a minimum spanning tree. The fact that Q6 asked for a minimum spanning tree should have alerted candidates to the fact that this was unlikely. What the question had asked candidates to do was to construct a simplified network with no repeated arcs and no arc that joins a vertex to itself and then to identify which standard network problem she needed to solve – this being the travelling salesperson problem (or, as one candidate wrote 'the travelling salesperson problem, also known as the travelling cleaning supervisor problem').

6) (i) Many candidates applied nearest neighbour instead of Prim (despite the fact that nearest neighbour was referred to in part (iii)), and some seemed to have worked out what they thought was the minimum spanning tree and then tried to circle elements in the matrix to match this.

(ii) Most candidates either correctly calculated the weight of the minimum spanning tree on the reduced network or they calculated the sum of the weights of the two shortest arcs from *B*. Some candidates calculated both of these values and added them to give a total of 29.

(iii) The majority of candidates applied nearest neighbour to reach all the vertices but did not then return to the start to form a cycle. A few gave the correct tour but then doubled its weight for their upper bound.

4737: Decision Mathematics 2

General Comments

The number of candidates for this unit continues to grow. There were many good scripts but a small number of candidates presented messy work that was difficult to understand. In particular, some of the graphs drawn had inappropriate scales making them difficult to use properly. The use of an insert made the dynamic programming and labelling procedure questions accessible to most candidates.

Comments on Individual Questions

1) (i) Most candidates were able to copy the table accurately, although a few omitted the row and column headings and some did not put equal values in the dummy row.

(ii) The application of the Hungarian algorithm was usually done well. A few candidates reduced the rows and then tried to reduce the columns ignoring the zero entries, but most were able to achieve a reduced cost matrix. Most candidates then attempted an augmentation, although some subtracted 100 from the uncovered entries but only added 1 to the entries that were covered twice and a few candidates crossed out the zero entries using four horizontal or four vertical lines even though the matching was not yet complete.

(iii) Most candidates were able to find the minimum cost matching and to cost it correctly.

2) (i) Only a few candidates were not able to find the expressions for the expected pay-offs; a few could not simplify their expressions correctly.

(ii) Some candidates drew tiny little graphs, or enormous graphs within which the required part was tiny. The horizontal axis should show the probability p from 0 to 1, and no further than this, and the vertical axis should show the expected pay-off, which in this case needed to extend from -2 to +5. The lines, representing the expected pay-off for each choice of strategy by Bea, should be ruled and should extend across the width of the graph from $p = 0$ to $p = 1$.

Having drawn the graph, the optimal value for p is at the highest point on the lower boundary, which in this case was when p took the value 0.5.

(iii) Most candidates realised that they could put their chosen value for p into the expected pay-off expressions to find the minimum expected pay-off. The explanations of why Amy might gain more points than this on average were often confused but generally referred to the fact that if Bea plays strategy Z, some of the time, then Amy would win more than when Bea plays X or Y. Amy is playing her optimal strategy but if Bea does not also play her optimal strategy then Amy may do better than she expected.

(iv) The majority of candidates correctly identified Bea's minimum expected loss as being the same as Amy's minimum expected gain. Some, however, assumed that the question wanted them to find the play-safe strategies and proceeded to reject strategy Y, whereas in fact it was playing Z that could have increased Bea's expected loss. Neither player should stick to any one strategy all the time as this would, in due course, be exploited by her opponent.

3) (i) Most candidates attempted to draw an activity network using activity on arc.

Several had precedence errors, in particular G was often shown following F, and many used unnecessary extra dummy activities. Some candidates seemed to treat dummy activities as being a way of dealing with the start and end of every activity. The activities, including the dummy activities, need to be shown with a direction and the network must have a single start and a single end point.

The two dummy activities that were required here were both needed to resolve precedence issues, the first because D follows both B and C, and the second because both F and G follow from D. Many candidates said that the second dummy was needed because G follows D and E, and some gave either broad references to 'following from or joining into two activities' or else listed every precedence, whether it was relevant or not. Dummy activities may be needed either to resolve precedence issues or to avoid having two activities with a common early event and a common late event.

(ii) Candidates usually attempted to carry out a forward pass and a backward pass. Where possible, their networks were followed through. The minimum completion time should have been 14 days and the critical activities were A, C, D and F.

(iii) Several candidates drew suitable graphs, it was not necessary to indicate the individual activities but this was not penalised. Some candidates left holes in their graphs or had blocks that hung out over empty space, and a few drew cascade charts or assumed one worker for each activity.

(iv) Although many candidates said that with the current schedule there was one day on which five workers were needed, rather fewer of them explained why this could not be resolved by resource levelling without increasing the project duration. Most candidates identified that activity F needed to be started one day later, although a few said that the critical activities could not be moved.

4) (i) The use of the insert made this a high scoring question for most candidates.

(ii) The minimax value was usually stated correctly, but some candidates were not able to track back through the table to read off the minimax route. The action values at each stage correspond to the state values at the previous stage.

(iii) Most candidates were able to complete the network correctly, although sometimes it was not obvious which arcs some of the weights were attached to. Some candidates were not able to extract the information from the table to find the arc weights for stage 2.

5) (i) Several candidates were able to state the route of the current flow as SEIT. A few gave routes through which an additional 6 litres per second could flow.

(ii) Nearly all the candidates realised that the capacity of pipe AG is 6 litres per second and several knew that the direction in which the fluid could flow was from A to G. Some candidates gave directions such as 'left to right' or 'from S to T', which whilst being true were not necessarily sufficient.

(iii) Most candidates were able to identify the cut arcs as being AG, BF, CF, DF and EI. The arcs AG, BF and CF were usually dealt with correctly to give 6 + 2 + 4. The direction in which fluid could flow in arc DF meant that the maximum possible flow across the cut from the source to the sink was 0, and the arc EI currently has 6 litres per second flowing from the source to the sink but can take another 2, giving a capacity of 6 + 2 + 4 + 0 + 8 = 20 litres per second for this cut (without regard to the rest of the network).

(iv) Candidates were usually able to list two, or sometimes three, flow augmenting routes. Some candidates did not give flow augmenting routes and described the flow in pieces, this was not appropriate for the use of the labelling procedure. Most candidates were able to update the labels appropriately, with only a few sending more along a pipe than it could accommodate. The explanations of how candidates knew that this was the maximum flow either involved stating that the current flow was 13 litres per second and identifying a cut of capacity 13 or stating that the arcs into T (i.e. GT and IT) were both saturated and so no more can flow into T. Some candidates rather confusingly talked about a 'zero cut' when they appeared to be discussing a cut across which no more can flow from the source to the sink.

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