GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Report on the Units

January 2009

3890-2/7890-2/MS/R/09J

This report on the Examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the syllabus content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the Examination.

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REPORTS FOR THE UNITS

GCE Mathematics and Further Mathematics Certification

Optimising Grades for GCE Mathematics Qualifications

Centres are reminded that when candidates certificate for a GCE qualification in Mathematics they are strongly advised to recertificate for any GCE Mathematics qualification for which they have previously certificated.
For example

- a candidate certificating for A level Mathematics is advised to recertificate for AS Mathematics if this has been certificated in a previous session.
- a candidate certificating for A level Further Mathematics is advised to recertificate (or certificate) for AS Mathematics, A level Mathematics and AS Further Mathematics.

The reason for this is to ensure that all units are made available to optimise the grade for each qualification.
Certification entries are free of charge.

The table below summarises this.

Qualification	
7890	Candidates are **strongly advised** to apply for recertification for 3890 in the same series as certificating for 7890 if this has been certificated in a previous session.
3892	Candidates are **strongly advised** to apply for recertification (or certification) for 3890 (and 7890 if enough units have been sat) in the same session as certificating for 3892. If a candidate has certificated or is certificating for AS Mathematics or A-level Mathematics with a different specification or Awarding Body then a Manual Certification form* **must** be completed and returned to OCR.
7892	Candidates are **strongly advised** to apply for recertification (or certification) for 3890, 7890 and 3892 in the same series as certificating for 7892. If a candidate has certificated or is certificating for A-level Mathematics with a different specification or Awarding Body then a Manual Certification form* **must** be completed and returned to OCR.

Manual Certification for Further Mathematics

It is permissible for candidates to enter for GCE Further Mathematics with the OCR specification if they have previously entered (or are simultaneously entering) for GCE Mathematics with another specification or Awarding Body. In this case OCR has to check that there is no overlap between the content of the units being used for the GCE Mathematics qualification and the GCE Further Mathematics qualification.
A Manual Certification Form must be completed for each candidate.

***A copy of the Manual Certification form is available on the GCE Mathematics pages on the OCR website. It may be photocopied as required, and should be returned to:
The Qualification Manager for Mathematics, OCR, 1 Hills Road, Cambridge, CB1 2EU; Fax: 01223 553242.
An electronic copy of the form may be requested by emailing fmathsmancert@ocr.org.uk
When completed, the form can be returned to the same email address.

Chief Examiner's Report – Pure Mathematics

The vast majority of candidates fully appreciate that it is in their interests to provide solutions that enable examiners fully to assess their understanding of the mathematics being tested. One aspect where this is particularly true concerns the use of calculators. In general, candidates make good use of modern calculators but are also aware of circumstances when over-dependence on calculators will not be appropriate.

The following highlights some of the relevant points concerning the use of calculators in the Core Mathematics units.

4722 Q2 Many candidates did not have their calculator set to the correct mode.

Q4 Candidates were fully aware that the wording of the question meant that an analytical approach was required.

Q5 Some final answers were inaccurate because answers at intermediate stages had been rounded and then used for subsequent calculations.

4723 Q2 Candidates provided sufficient detail to indicate that Simpson's rule was being attempted. No great detail was required but an attempt consisting of an answer only would have scored zero. For example, it was perfectly acceptable to write

$$\text{Approx value} = \tfrac{1}{3} \times 2(\ln 4 + 4\ln 6 + 2\ln 8 + 4\ln 10 + \ln 12) = 16.27\ .$$

Q6 Candidates carried out the iteration in part (iii) very well, using their calculator to generate the terms efficiently and recording the values in their solutions.

4724 Q4 There was evidence that a few candidates, having attempted the exact value by an appropriate method, then used their calculator to check that the answer provided by the integration facility of the calculator agreed with their exact value.

Q8 In part (ii), a neat solution using index properties was much more convincing than an attempt in which a calculator was involved.

4721 Core Mathematics 1

General Comments

In general, candidates coped well with this paper. Most worked through the questions in order and were able to attempt every question, although questions 8 and 10(iii) proved challenging for all but the most able. The majority of scripts showed an appropriate amount of working, although candidates should be encouraged to write a few words to explain their reasoning in questions like 7(iv) which required the verification of a given statement.

Unfortunately, examiners reported that many candidates' work again showed evidence of poor arithmetic skills, with candidates of all abilities unable to calculate 48/8 or 20^2 correctly. Thebe were also many cases where an error resulted from carelessness in dealing with negative numbers.

It was pleasing to note an improvement in the proportion of candidates able to recognise and solve a disguised quadratic equation. However, manipulation of indices and surds remains an area of the specification where candidates lack understanding.

There appeared to be adequate time to complete the paper and few candidates scored fewer than 20 marks in total.

Comments on Individual Questions

1) This opening question proved straightforward for many candidates and they gained all 3 marks. However, an equally large number scored only 1 mark, usually by correctly simplifying $\sqrt{45}$ (although there were many cases of $9\sqrt{5}$ or $5\sqrt{9}$ seen). Some candidates understood how to rationalise the denominator of the fraction but others multiplied the entire expression by $\sqrt{5}$ or wrote $20 = \sqrt{4}\sqrt{5}\sqrt{5}$ and cancelled, obtaining $2\sqrt{5}$.

2) (i) Most candidates knew that a power of $\frac{1}{3}$ was needed although a significant proportion left their answer as $x^{\frac{6}{3}}$ which was surprising. By far the most common wrong answer was x^3 although $x^{\frac{1}{18}}$ was also seen.

(ii) This question was extremely poorly done by candidates of all abilities and, as in previous papers, reflected the inability of many to simplify expressions involving indices and fractions. The most prevalent error was to expand $(10y)^3$ as $10y^3$, although there were also many examples of $30y^3$ or $1000y$. Of those who correctly wrote 1000, there was a worryingly large number who then simplified the constant to 501.5. Some candidates split the original expression into $\frac{3y^4}{2y^5} \times \frac{(10y)^3}{2y^5}$.

3) The responses to this question were varied. Many candidates recognised that it involved a disguised quadratic, made an appropriate substitution and found the roots of the resulting quadratic correctly, earning the first 3 marks. However, candidates were less likely to complete the question correctly. Some tried to cube root, rather than cube, their values, while for others cubing $\frac{2}{3}$ correctly proved impossible, with $\frac{8}{3}$ often seen. Weaker candidates failed to make any progress at all. Some cubed each term separately, others attempted to multiply the original equation by 3. These incorrect methods gained no marks at all.

4) (i) The quality of graph sketching proved to be very centre dependent. It is pleasing to report that far fewer candidates are using graph paper to sketch graphs and only the weakest candidates are working out coordinates and plotting points to establish the shape of the graph. However, graph sketching is still an area needing improvement. Candidates must be encouraged to draw axes with a ruler and, in this particular question, candidates whose freehand axes were non-perpendicular or curving made it difficult for them to show the asymptotic nature of the curve clearly. It was common to see the graph of $y = \frac{1}{x}$ rather than $y = \frac{1}{x^2}$ and, less predictably, there were also numerous candidates who sketched the graph of $y = -x^2$.

(ii) In general, candidates' understanding of graph transformations has improved with many answers gaining both marks here. The most commonly seen wrong answer was $y = \frac{1}{x^2+3}$, with far fewer cases of $y = \frac{1}{x^2} + 3$ and only a handful of alternatives. A small number of candidates lost a mark because they failed to write '$y =$' in their answer.

(iii) This question was successfully tackled by almost all candidates although a great variety of combinations of 1, 4 and $\frac{1}{4}$ were seen among the incorrect answers.

5) (i) As in previous papers, this straightforward question on differentiation was a useful source of marks for even the least able candidates. This first part was well answered by almost all, with only a few answers of $-50x^{-4}$ seen and even fewer completely wrong expressions.

(ii) It is pleasing to note that almost all candidates immediately rewrote $\sqrt[4]{x}$ as $x^{\frac{1}{4}}$ and then differentiated correctly. The answers $\frac{1}{4}x^{-\frac{1}{4}}$ or $\frac{1}{4}x^{-\frac{5}{4}}$ were seen but these were rare. Some candidates finished by changing their derived expression back into surd form which was not necessary but showed a pleasing confidence in dealing with the notation.

(iii) The vast majority of candidates knew how to expand the cubic expression although there were plenty of careless errors made. Some candidates reversed the signs of all the terms to make the expression easier to differentiate, overlooking the fact that the expression was not set equal to zero. However, most candidates were able to score full marks in this part.

6) (i) Once again, relatively few candidates could complete the square correctly. All but the very weakest candidates established the correct values of p and q but the expression $5(x+2)^2 - 12$ was seen much more commonly than the correct answer. Some candidates multiplied out their final expression and then corrected their value of r. Such checking is to be strongly encouraged as completing the square for a quadratic involving an x^2 term with a coefficient other than 1 continues to prove challenging for most.

(ii) Only a small minority of candidates gave the correct equation for the line of symmetry. The most common incorrect answers were $y = -2$ and $y = -5x^2 - 20x + 8$. Despite the fact that the answer was worth a single mark, some candidates differentiated, set the resulting expression equal to zero and found the coordinates of the minimum point. This was acceptable if they then gave the equation of the required line but few candidates seemed able to make the link between the vertex of a quadratic graph and the line of symmetry.

(iii) Almost all candidates knew the formula for the discriminant and wrote $20^2 - 4 \times 5 \times -8$ correctly. However, a disappointingly large number could not evaluate this expression without error. The incorrect value 240 was more prevalent than the correct value and errors such as $20^2 = 200$ were also frequent. A small number of candidates, obviously unfamiliar with the term 'discriminant', differentiated the given expression.

(iv) The vast majority of answers were correct and a valid reason was often given. It was surprising to see candidates work out the discriminant all over again despite having found this value in part (iii) – an indication that candidates too often fail to detect the connection between different parts of a question.

7) (i) This question was very well answered. Most candidates substituted $x = 10$ into the given equation and rearranged to find k. A smaller number chose a much more complicated route by rearranging the original equation into $y = mx + c$ form and then substituting (2, 1) and (10, k) into the equation $\frac{y_2 - y_1}{x_2 - x_1} = m$. This method was less successful. Regardless of method, there were many careless sign errors made and it was fortunate that, having realised in part (iv) that the length of the line AB should equal 10, some candidates were able to examine their earlier working and correct it.

(ii) All but the very weakest candidates knew how to find the length of a line and did the calculation correctly. Again, of those who had an incorrect value, many were able to revisit this calculation after part (iv) and make a correction. However, only those with correct working shown leading to the value of 10 could score full marks in this part.

(iii) Almost all candidates could correctly state the coordinates of the centre and the radius, although (-6, 2) was occasionally seen, as was $r = 25$ or $r = \sqrt{65}$.

(iv) It was clear that candidates were unfamiliar with this type of request. Even candidates scoring almost full marks overall often lost one mark here. The majority of candidates wrote one statement only, usually '$AB = 2 \times r$' or even '$10 = 2 \times r$' with no mention of AB. Midpoint of AB = (6, -2) was also frequently seen with or without supporting working as was the statement that both A and B fitted the circle equation given in part (iii). But few candidates realised that their one fact, even if properly checked, was insufficient for a complete verification. A few candidates showed that the line $3x + 4y - 10 = 0$ passed through the centre of the circle and that AB was of length 10 without realising that this did not prove that AB was a diameter.

8) (i) This question proved to be the most difficult on the paper, with only the best candidates scoring more than half the available marks. Many candidates could not deal with the quadratic equation as given and attempted to rearrange it before starting, often failing to do this correctly. Others used the quadratic formula with $a = 5$ and $c = -1$, while those who substituted into the formula correctly then failed to deal with the negative values in the discriminant and obtained 44 instead of 84. The minority who had reached $\frac{8 \pm \sqrt{84}}{-2}$ correctly often could not simplify this correctly, most commonly ending up with $-4 \pm 2\sqrt{21}$. Those candidates who chose to complete the square after reversing the signs in the original equation had the simplest route to the correct roots.

(ii) As in other questions, candidates failed to see the link between the parts of this question, with many solving again here, often by a different method. Regardless of the roots found, few candidates could construct the correct inequalities, most giving the region between their roots.

(iii) Some candidates wrote a page or more of working but failed to make any attempt at a sketch so could not earn any marks at all in this part. Of the curves drawn, most were either positive or negative cubics with at least some of the intercepts correct. However, few were fully correct, usually because they had the wrong roots or correct roots in incorrect positions. Many candidates knew that the y-intercept was (0, 20), but drew their curve with a negative y-intercept.

9) This question was very well answered by most, with a large number of perfect solutions from candidates of all abilities. However, a significant minority found p by solving $y = 0$ rather than $\frac{dy}{dx} = 0$. They often continued by differentiating to determine the nature of the stationary point. There was some confusion about how to classify the stationary point, with some candidates stating 'min because $x > 0$' and others solving $\frac{d^2y}{dx^2} = 0$. Those who used the more laborious methods of investigating the sign of either the y coordinates or $\frac{dy}{dx}$ either side of $x = 4$ often gained full marks, provided there were no errors in their calculations.

10) (i) Although this question was generally well done, there were a surprising number of candidates who did not realise that they needed to differentiate to find the gradient and simply divided the y-coordinate by the x-coordinate. A disappointingly large number of those who did differentiate wrote $\frac{dy}{dx} = 2x$.

(ii) This question was also tackled well by most, although there were plenty of arithmetic and sign errors seen. Some candidates failed to use the negative reciprocal of the gradient in part (i) but the majority scored at least 3 marks here.

(iii) This last question proved demanding with only the most able candidates scoring well. It proved very interesting to see the different approaches used. There was an evident lack of appreciation that not only did the curve and the line have to meet but also that their gradients had to be the same at the points of intersection. Weaker candidates gave up after setting the equations of the line and curves equal while others did pages of working and tried multiple approaches without making any significant progress. A good number of candidates were aware that the determinant should be used but could not extract the x coefficient from the equation $x^2 + x - kx + 4 = 0$. Those who substituted $k = 2x + 1$ into the equation of the line had an easier route to a solution. A large number of candidates simply assumed that the line passed through the point (2, 6) and used this to find k, without considering whether the resulting line was tangential to the curve.

4722 Core Mathematics 2

General Comments

This paper was accessible to the majority of candidates, and overall the standard was very good. Candidates seemed well prepared for the paper and familiar with the topics being tested. There were a number of straightforward questions where weaker candidates who had mastered routine concepts could gain marks, and there were also aspects to challenge the most able.

Most scripts showed clarity of presentation, with candidates making their methods clear. This is particularly important on questions where they are asked to show a given answer. On scripts where presentation is poor it can be difficult for examiners to decipher what has been written, and some candidates lost marks through misreading their previous work. Candidates also need to be able to use the correct mathematical conventions to convey their meaning, especially the appropriate use of brackets. When angles were given exactly in radians it was sometimes unclear whether π was intended to be in the numerator or the denominator, and in other fractions the extent of the fraction line was not always made clear.

Candidates must ensure that they read the question carefully, particularly noting when exact answers are requested. They also need to ensure that they are familiar with the appropriate terminology for this module. A number of marks were lost through a lack of understanding of words such as root, factor and coefficient.

Some candidates struggle to make efficient use of their calculator, especially in ensuring that it is in the correct mode for questions involving trigonometry. They also need to make effective use of the memory facility as using rounded values throughout a question will often result in a loss of accuracy in the final answer. It was also noticeable that in both Questions 5 and 6(ii) a number of candidates were unable to correctly round their calculator display. When using a calculator it is still important that full details are shown – examiners cannot award method marks if little or no method has been shown

Comments on Individual Questions

1 (i) The majority of candidates made a very good attempt at this question, with very few attempting to differentiate. Some candidates correctly integrated the first two terms but then failed to deal with the third term. Others failed to gain the final mark either by omitting the $+c$, or by leaving a dx in the answer.

(ii) This part was also very well answered with the majority of candidates appreciating the need to rewrite the integrand using index notation prior to integration and most did this correctly. The integration was also usually correct, though some failed to simplify their final answer. A few candidates could not divide 12 by 1.5, leading to an incorrect coefficient of 18. It was more common for the constant of integration to be omitted in this part, even by candidates who had included it in part (ii).

2 (i) Most candidates performed the correct procedure but too often the final solution was given in a non-exact form.

(ii) Many candidates made a good attempt at this question, though using a decimal answer from part (i) often led to an inaccurate final answer. Some candidates used a mixture of degrees and radians throughout the question, but failed to ensure that their calculator was in the appropriate mode when attempting the length of the chord. Most candidates correctly quoted the formula for the length of the arc, but this was occasionally spoilt by using an angle in degrees not radians. It was also disappointing to see some candidates working from fractions of the circumference to attempt the arc length rather than being familiar with the necessary formula. A number of candidates lost marks by failing to read the question carefully; it was surprisingly common to see attempts at the perimeter of the sector and the area of the segment.

3 (i) This was very well answered, with many candidates gaining both of the marks available. Some did not give exact answers, and others attempted an iterative method.

(ii) This was a straightforward question for the candidates, most of whom gained full marks. A few attempted to find an expression for the n^{th} term using $a + (n - 1)d$ rather than the given u_n, but then made a sign error on their value for d.

(iii) Most candidates appreciated that a summation was required, though a few simply found the sum of the first 20 natural numbers and others attempted the sum of a GP. However, the majority could quote and attempt to use a correct formula, though a significant number used at least one incorrect value. The most common mistake was to have a difference of ⅔ rather than - ⅔, despite the evidence in part (i). The most successful candidates found the value of the 20^{th} term from the given definition and then used the appropriate formula for the sum of an AP.

4 Candidates adopted a number of different methods to deal with this question, with varying degrees of success. The most successful approach was to find the area of the rectangle, though this was often done using integration, and then subtract the area under the curve. A number of candidates stopped having found only the area under the curve. Some candidates attempted to combine the two functions prior to integration, but this was often done in the incorrect order or came as a result of first equating the two functions. Of those who subtracted the function from 19, a lack of care with brackets sometimes resulted in $22 - x^4$ rather than the correct integrand. Irrelevant of the method used, there were very few mistakes made with the actual integration, and the majority also attempted the correct use of limits though some tried to use 3 and 19 instead. A few candidates chose to rearrange the function to one in terms of y and then attempt integration, but this was rarely successful.

5 (i) This question was generally well answered with most candidates able to make confident use of the sine rule. However, some candidates failed to ensure that their calculators were in the correct mode and others made rounding errors.

(ii) Most candidates appreciated the need to use the cosine rule, though a surprising number first calculated TB and then used this length rather than simply considering triangle ATC. Premature rounding throughout the question often led to an inaccurate final answer. A few candidates did not read the question carefully and failed to place C in the correct position.

(iii) Very few candidates appreciated what was required in this question and either simply compared the lengths of TA, TB and TC or assumed that the shortest distance would occur at the midpoint of AC. The more able candidates found the perpendicular distance from T but only the most astute checked that this point of closest approach occurred on AC. There was no intention to mislead the candidates so either answer was given full credit.

6 (i) Very few candidates failed to gain both of the marks available in this part of the question.

(ii) This part was also done very well, though a few candidates used n as 20 rather than 30.

(iii) This final part of the question proved to be much more challenging for candidates, and very few gained all four of the marks available. Most could state a correct expression for u_p, though there were errors on the index and a few candidates used the sum formula instead. The most common error was for u_p to become 18^{p-1}. Whilst most candidates attempted to use logarithms, only the most able appreciated the need to rearrange the equation beforehand, and $\log(20 \times 0.9^{p-1})$ was rarely dealt with correctly. Having done everything else correctly, a number of candidates lost the final mark by leaving their answer as a decimal or an inequality. Very few appreciated the need for p to be an integer and, of those who did, most rounded down to the nearest integer rather than giving careful consideration to the situation. Other candidates failed to gain full credit due to not changing the direction of their inequality sign at the appropriate point.

7 (i) Most candidates gained the first mark for attempting a binomial expansion, and many also gained the second mark for then equating this to 24. A few did not understand the meaning of 'coefficient' and x^2 was often left in the expression, usually to then be replaced with 24. The most common error was a failure to use brackets resulting in $6k^2a = 24$. There were then attempts to produce a convincing proof, but any errors in working were penalised. Some candidates assigned integer values to a and k and attempted a numerical proof. As with previous questions on the binomial expansion, the most successful candidates made effective use of brackets.

(ii) Most candidates stated the correct coefficient, but many then struggled to make further progress in solving the two equations. There were several long-winded methods involving squaring and cubing that gave ample opportunities for slips, and other incorrect methods such as subtraction. However, a pleasing number of elegant and concise solutions were also seen and this was a relatively straightforward question for many.

(iii) Many candidates substituted their values into a correct expression, though omission of brackets led to a number of wrong answers.

8 (a)(i) This question was very well answered, though a few candidates left logs in their final answer involving p and q.

(ii) Most candidates gained some of the marks available, but fully correct solutions were less common. The mark for $-q$ was usually gained, but many candidates then struggled to apply both the addition and the power laws to the remaining term in the correct sequence.

(b)(i) This part was poorly done. Most candidates seemed familiar with the subtraction law but spoiled their solution with a number of other errors. A common first step was $\log x^2 - \log 10$. Some candidates stated the correct expression but then continued with an attempt at cancelling, resulting in $\log(x - 10)$. This wasn't penalised in part (i), but meant that candidates struggled to make much progress in part (ii).

(ii) This proved to be a challenging question for all but the most able and very few completely correct solutions were seen. Many candidates gained only one mark for recognising that 2log3 could be expressed as log9, and a number failed even to get this mark. The link between the two parts of the question was not always appreciated and many candidates started afresh in part (ii). However, the more able candidates used their correct expression from (i) and hence easily solved the given equation. Of these, very few appreciated that once the solutions 10 and –1 were found, the latter had to be discarded as invalid.

9 (i) This question was generally very well answered. Most candidates demonstrated that f(1) = 0, and others showed a remainder of 0 after long division. A few candidates did not address this part of the question at all. A variety of successful methods were then employed to find the quadratic factor, including division, inspection and coefficient matching. Having obtained the correct quadratic factor, a number of candidates then struggled to find the roots. Linear factors of $(x + 3)(x - 3)$ and $x(x - 3)$ were common, and a surprising number resorted to the quadratic formula, often resulting in an unsimplified surd. It was obvious that a number of candidates were unfamiliar with the distinction between a root and a factor as it was common to see a correct 3 term factorisation, but with no attempt at the roots.

(ii) This was also well answered with most candidates making the link between the two parts of the question, though a few made a new attempt at solving the cubic. Whilst some candidates found the tangent of their roots, most equated tan x to their roots and attempted a solution. Some stopped after finding the principal solutions to their three equations, whereas others dismissed $-\frac{1}{3}\pi$ as being out of range and only ended up with four of the required solutions. Most candidates who worked in degrees subsequently changed their answers to radians, though a few lost marks by either failing to do so or by giving decimals rather than appreciating that exact solutions were required. Most candidates scored well on this question, but only the most able obtained full marks.

4723 Core Mathematics 3

General Comments

There were plenty of marks among the first six questions accessible to all candidates and it was pleasing that there were relatively few candidates scoring low marks on this paper. The final three questions presented more challenges. There were many excellent responses to these questions but the depth of mathematical understanding and levels of algebraic and trigonometric skill required meant that some candidates struggled to record more than just a few marks from this part of the paper. The comprehensive attempts made by many candidates suggested that there were no particular time pressures unless candidates had adopted unnecessarily lengthy processes; there were some protracted attempts at questions 2, 4, 7 and 8 which might have led to the candidates involved struggling to offer complete solutions to all the questions.

The final two questions included some given answers to be confirmed. It is important that candidates approach such questions carefully and thoroughly. In particular, if a slip has occurred and been discovered, that error must be corrected throughout the solution and not just in the final two lines of working. Full marks are not awarded to a solution containing errors even if the given result appears finally to be confirmed.

It is expected at this level that candidates will be able to deal effectively with equations which are straightforward if a little unconventional. For example, on this paper, many candidates struggled to find a neat method of solving one or more of $2x\ln x - x = 0$, $3^k = 27$ and $32\cos^6\theta - 48\cos^4\theta = 0$.

There were several instances of candidates offering more than one solution to a particular question without indicating which solution they wished to be assessed. In such circumstances, it is the final solution which examiners will mark, even if assessment of an earlier attempt would have led to a higher mark.

Comments on Individual Questions

1) These two straightforward requests enabled most candidates to make a successful start to the paper. There were some errors with part (i), $16e^{-2x}$ being the commonest. Part (ii) was usually answered correctly although those candidates who felt the need to carry out a related differentiation first sometimes confused themselves and concluded with $4x+5$ raised to the wrong power. One mark was available for the inclusion of the arbitrary constant at least once; many candidates failed to earn this mark.

2) Most candidates carried out the calculation in part (i) efficiently and accurately. Some candidates associated 4 and 2 with the wrong y-values and a number used values of $\frac{1}{x}$ instead of $\ln x$. It was not uncommon for candidates to be unaware of the general structure of the expression being evaluated; absence of necessary brackets led them to evaluate $\frac{2}{3}(\ln 4 + \ln 12) + 4(\ln 6 + \ln 10) + 2\ln 6$. Part (ii) was not answered so well, with many candidates not recognising that the answer is simply 10 times the answer from part (i). The wrong answer $(16.27)^{10}$ appeared very frequently.

3) This was a routine request for many candidates and they obtained the correct answers without any difficulty. However there were problems for many other candidates. It was surprising how many were unsure about the identity $\tan^2\theta \equiv \sec^2\theta - 1$; they either seemed to have no knowledge of the existence of such an identity or, if they had some notion, they made sign errors in its use. Some candidates started by replacing $\tan^2\theta$ by $\dfrac{\sin^2\theta}{\cos^2\theta}$ and sometimes succeeded in reaching the correct expression in $\sec\theta$. An error in this basic work from part (i) led to problems with the equation in part (ii) although some credit was still available for an appropriate attempt. By no means all candidates knew how to deal with $\sec\theta = 2$; attempts such as $\tan\theta = \frac{1}{2}$ and $\theta = \dfrac{1}{\cos 2}$ were sometimes noted.

4) For each curve, it tended to be the case that the differentiation was carried out well but that finding the location of the stationary point presented more problems. In part (i), most candidates obtained the correct $40x(4x^2+1)^4$ for the first derivative. A minority of candidates then concluded immediately with the correct $x = 0$, readily recognising that $4x^2+1$ cannot be zero. Many others embarked on an attempt to solve $4x^2+1=0$, often ending with $x = -\frac{1}{2}$.

   In part (ii), the vast majority of candidates wisely attempted to use the quotient rule. (Using the product rule is distinctly unhelpful in this case.) There were some errors, often caused by confusion between *u* and *v*. Candidates were expected to present the derivative correctly; it was common for the denominator to be shown as $\ln x^2$ rather than as $(\ln x)^2$ and candidates doing so did not earn the mark for the derivative. Many candidates struggled to solve $2x\ln x - x = 0$. A few candidates commendably explained why $x = 0$ is not a possible answer as well as providing the correct $e^{\frac{1}{2}}$.

5) This question was answered extremely well and it was very common for candidates to score full marks. In part (i) some candidates showed their awareness of the properties of exponential growth by completing the table before finding the value of *k* but it was more usual for *k* to be found first. The vast majority recognised the need for differentiation in part (ii) and used it accurately in finding the rate. A few candidates were casual with their value of *k*, using an over-approximated value of 0.03 instead of the more accurate 0.033. Doing so led to significant errors in the answers to both parts of the question.

6) This question was another good source of marks for many candidates. The process for finding the inverse function was well known and only careless slips marred some attempts. In part (ii), the first mark was easily earned with reference to reflection in the line $y = x$ but few candidates earned the second mark. Nothing lengthy or sophisticated was expected, merely the observation that, at *P*, the line $y = x$ and the curve $y = \sqrt[3]{\frac{1}{2}x+2}$ meet and therefore $x = \sqrt[3]{\frac{1}{2}x+2}$ at that point.

   The iteration in part (iii) was done very well by most candidates. All but a few provided the necessary evidence although the practice of some candidates in giving the iterates themselves just to 2 decimal places is not good practice; candidates are advised to give greater accuracy in these values before giving the final answer to the requested accuracy. In a few cases, the solution to part (iii) consisted only of the answer 1.39; such attempts earned no marks because there was no evidence of the method which had been adopted.

7) The vast majority of candidates recognised that a translation and stretch were involved in part (i) but the terminology used and the details given were often unacceptable. The term 'translation' was expected and many candidates managed to give the correct details too, although the double negative of 'translation in negative y-direction of $-a$ units' appeared occasionally. Candidates were less sure of the stretch; it was sometimes described as in the y-direction and often as in the x-direction with scale factor k. A number of candidates seemed to confuse the word 'translation' with 'transformation'; it is important that these descriptions are given with precision of language.

The curve in part (ii) was usually drawn acceptably although some had the reflected part curving the wrong way or seeming to show a maximum point in the second quadrant. Candidates did not always make it clear what their answer was; candidates who drew only the curve $y = \left|e^{kx} - a\right|$ left no doubt but those who super-imposed the requested curve on a copy of the diagram given in the question often raised doubts as to what exactly was their intended answer.

Part (iii) was done poorly and there were not many candidates who managed to find the values of k and a efficiently. The usual approaches involved attempts at simultaneous equations or the squaring of equations and these led to muddled solutions which, at best, provided the correct answers as well as several incorrect ones. There was little evidence that the graph from part (ii) had been used to inform a method. Since the point (0, 13) lies on the reflected part of the curve, it must be true that $-(e^0 - a) = 13$ and this gives the value of a immediately. Further, the point ($\ln 3$, 13) lies on the original curve $y = e^{kx} - a$ and this gives the value of k. Even for those candidates with a viable method, there were problems dealing with the term e^{kx}; with substitution of $x = \ln 3$, this often appeared as $e^{\ln 3k}$, leading to $3k$ rather than 3^k.

8) Although there were a few candidates who attempted to integrate $\pi(\frac{6}{\sqrt{x}} - 3)^2$, most candidates were aware of the correct formula for finding a volume where rotation is about the y-axis. But, in many cases, the level of algebraic skill needed to express x^2 in terms of y was not apparent. Even for those candidates with a correct expression for x^2, many presented it in a form such as $\int (\frac{36}{(y+3)^2})^2 \, dy$ or $\int \frac{1296}{(y^2+6y+9)^2} \, dy$ which was not helpful for the subsequent attempt at integration. The number of candidates who reached the correct integral of $-432\pi(y+3)^{-3}$ and applied the limits clearly to confirm the given result was disappointingly small.

In part (ii), most candidates were aware of the need to find the product of $\frac{dV}{dp}$ and $\frac{dp}{dt}$ but the attempts at finding the former were seldom correct, despite the fact that, of course, the derivative is the same expression as the integrand in part (i), albeit with p involved rather than y. Some attempts to find $\frac{dV}{dp}$ looked more like integration and, in many cases, the principles of differentiating such an expression were just ignored.

9) The first two marks of part (i) were earned by many candidates but a fully convincing solution needed careful management of identities, signs and brackets and this was not always evident.

Part (ii) proved challenging and few candidates showed the necessary mastery of multiple angle expressions. A clear statement indicating the method to be used, based on either $\cos 6\theta \equiv 2\cos^2 3\theta - 1$ or $\cos 6\theta \equiv 4\cos^3 2\theta - 3\cos 2\theta$, was the expected opening. Some attempts started by squaring $4\cos^3\theta - 3\cos\theta$; candidates, with an eye on the given answer, then doubled the result and subtracted 1. Such unconvincing attempts did not receive credit.

Most candidates made some progress with part (iii) but candidates had to provide clear evidence for their conclusions to earn all the marks. Too often, attempts at solving $32\cos^6\theta - 48\cos^4\theta = 0$ involved over-enthusiastic cancelling with the loss of one of the possible values of $\cos\theta$. To earn the final two marks, candidates had to refer to the fact that $\cos^2\theta = \frac{3}{2}$ has no solutions and that $\cos\theta = 0$ leads to values such as $\pm 90°, \pm 270°, \pm 450°$, thereby confirming the odd multiples of $90°$. Some candidates left no doubt of their understanding by providing a sketch of $y = \cos\theta$ with the intercepts on the θ-axis indicated.

4724 Core Mathematics 4

General Comments

Again this January, there was a very wide range of responses; many of these were excellent but it was surprising how many candidates were entered who really did not understand most of the topics. It was also very disappointing to note that candidates made so many very simple arithmetic and algebraic errors at this stage of their mathematical careers. There seemed to be no problem with the length of the paper.

Comments on Individual Questions

1) This question gave candidates a good start and it was interesting to see the variety of their solutions. The obvious method of factorising numerator and denominator was the most common, the only problems arising from connecting $(x-4)$ and $(4-x)$ and the occasional factorising of $6x^2-24x$ as $6(x-2)(x+2)$ or $6(x-4)(x+4)$. A less obvious idea was to use partial fractions and this generally proved successful. A third small group of candidates decided to use long division; provided numerator and denominator were arranged consistently, this method produced the correct answer rapidly. Three versions of the correct answer were seen: $-\frac{5}{6x}$, $\frac{-5}{6x}$ and $\frac{5}{-6x}$. All were accepted as it was frequently impossible to distinguish one from another.

2) Most candidates used the correct method of integration by parts and there were relatively few errors in its application. However, modulus signs and '$+c$' were frequently omitted.

3) (i) The binomial expansion was well known; '$2x$' caused a few problems in that its square was often $2x^2$ and, for some candidates, untidy work meant that they could not read their fractional expressions accurately.

(ii) In general, this part depended on how the denominator of $(1+x)^3$ was treated. Those who converted it into $(1+x)^{-3}$ and then used multiplication were generally successful. However, those who retained $(1+x)^3$ usually wrote $\frac{1+x-\frac{1}{2}x^2+\frac{1}{2}x^3}{1+3x+3x^2+x^3}$ and either stopped or performed quite ridiculous 'cancellations' or even subtraction.

(iii) Although validity questions have often been asked, this proved to be harder than usual for many candidates and only about half produced the correct answer.

4) A few candidates integrated immediately with a result of $\frac{(1+\sin x)^3}{3}$, or something similar, but the majority was well aware of the correct approach. The error $(1+\sin x)^2 \equiv 1+\sin^2 x$ was seen too frequently but the attempts to deal with the integral of $\sin^2 x$ were very encouraging. Substitution of the limits, although initially correct in most cases, often contained sign errors when terms were collected.

5) (i) This part was handled well, particularly by those who converted $u = \sqrt{x}$ to $x = u^2$ and consequently used '$\mathrm{d}x = 2u\,\mathrm{d}u$'. There was some carelessness in style but candidates should always remember that, when the answer is given, working will be closely scrutinised for any error and it is expected that the answer will be given as shown in the question; so, in this example, $\int \frac{2}{u(1+u)}$ was not a satisfactory ending.

(ii) A few candidates decided that the result of the integration was $2\ln u(1+u)$ but the majority realised that they had to do something with $\frac{2}{u(1+u)}$ before they could attempt integration. Although resolution into partial fractions was the norm, there were many cases of $\frac{2}{u} + \frac{2}{u^2}$.
The use of the limits – either changing them or expressing the integral in terms of x and using the original limits – was generally handled well. There were a few candidates not appreciating the meaning of the root sign in $\sqrt{x}$ and ± 3 and ± 1 were seen; as natural logarithm was involved, the negative aspect was quickly dropped. Any suitable form of the answer was acceptable.

6) A question involving parametric equations in this unit is bound to involve the evaluation of $\frac{\mathrm{d}y}{\mathrm{d}x}$ so this was generally found at an early stage, irrespective of where it was needed – and some found a use for it in part (ii)!

(i) This part was generally well done. A few changed the order of parts (i) and (ii) and then used the cartesian form to find where the curve met the x-axis. A few used $x = 0$ as the equation of the x-axis.

(ii) This part was also well done. The comment concerning poor algebra was as relevant here as it was in question 4 – the squaring of a simple expression such as $(y+3)$ or $(1+\sin x)$ ought never to be wrong at this level of attainment.

(iii) A few found the equation of the normal and another group of candidates retained t or x or y in the value of the derivative but most understood the direction in which they should go. The only slight awkwardness involved those using the cartesian equation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$; some changed $x = y^2 - 5$ into $y = (x+5)^{\frac{1}{2}}$ and did not realise that the point where $t = 2$ lies on the lower half of the parabola and so the equation $y = -(x+5)^{\frac{1}{2}}$ should have been used. Although obviously incorrect, this error was viewed with more latitude than usual and only a small penalty was imposed.

7) Quite a few errors occurred here, generally from carelessness in arithmetic or in copying the details of the question.

(i) Most realised that the equation of the line through (9, 7, 5) and (7, 8, 2) involved a direction vector but $\mathbf{r} = \begin{pmatrix} 9 \\ 7 \\ 5 \end{pmatrix} + s\begin{pmatrix} 7 \\ 8 \\ 2 \end{pmatrix}$ was sometimes seen. Candidates making this error were still able to produce the necessary equations and solve them but, of course, showing consistency and finding the point of intersection were not possible. Conventional solutions in a topic such as this have improved but there were candidates who did not label their equations or state clearly which they were using for the solving process and which was being used for the consistency.

(ii) Almost all who started part (i) correctly were able to deal with this part accurately, the only common error being that sometimes it was the obtuse, rather than acute, angle that was given. Any candidate demonstrating the methods for finding the scalar product of any two vectors and the magnitude of any vector was able to obtain half the available marks in this part.

8) (i) This was well done. Very few started with $\frac{\mathrm{d}y}{\mathrm{d}x} =$ and, of those who did, hardly any made any subsequent use of it.

(ii) There was a lot of unclear thinking in this part. It was expected to be a straightforward request, with candidates showing that the point lies on the curve and that the gradient at the point is zero. Quite a number of candidates assumed that $\frac{\mathrm{d}y}{\mathrm{d}x}$ was given to be 0 and started to use the fact that the numerator of $\frac{\mathrm{d}y}{\mathrm{d}x}$ was 0; they then tried to solve $x^3 + y^3 = 6xy$ simultaneously with $6y - 3x^2 = 0$ and, in general, got into an algebraic mess. Those thinking clearly used one of two methods to prove the results – the laws of indices or their calculator. It was significant that the better candidates used indices.

(iii) Many candidates trying to find the value of a produced the equation $2a^3 = 6a^2$, divided by $2a^2$ and produced $a = 3$ without any regard for the possibility of a being equal to 0. It was expected that attention would have been drawn to the fact that, as given in the question, $a > 0$ to justify the existence of only one root. Almost everyone produced the gradient of -1 with a significant number demonstrating that it would not have mattered what the value of a was.

9) (i) Most received two marks for this, the main exceptions being those who used $\frac{\mathrm{d}t}{\mathrm{d}\theta}$ instead of $\frac{\mathrm{d}\theta}{\mathrm{d}t}$ and those who omitted the constant of proportionality.

(ii) The main error, and it was very common, was to integrate $\frac{1}{160-\theta}$ to produce $\ln(160-\theta)$ instead of $-\ln(160-\theta)$. Although candidates doing this were able to demonstrate correct methods thereafter, accuracy marks were lost. The position of the *k* after the separation of the variables was the cause of other mistakes; it need not have been, of course, but integrating $\frac{1}{k(160-\theta)}$ seemed much more difficult than integrating $\frac{1}{160-\theta}$. Those who had omitted the constant of proportionality in part (i) were now at a disadvantage as there was superfluous information.

4725 Further Pure Mathematics 1

General Comments

Most of the candidates showed that they had a sound knowledge of a good proportion of the syllabus, with questions 4 and 7 proving to be more testing. Candidates generally answered the questions sequentially and there was no evidence of candidates being short of time.
As has been mentioned in previous reports, when answers are given in the question, candidates must show sufficient working to justify their answer. Failure to do this was very common in questions 7, 8 and 9.

Comments on Individual Questions

1) Most candidates multiplied by the correct conjugate and obtained the correct answer. The most common errors occurred in the denominator, which candidates found to be 25 –1 or 5 ± 1.

2) (i) This was answered correctly by most candidates, the most common error being omission of the determinant.

(ii) This was answered correctly by most candidates, but a significant minority thought that 2**A** meant $\mathbf{A}^2$.

3) The standard results were generally well known. Too many candidates tried to expand to obtain a quartic, before trying to factorise, rather than using the factor $n(n + 1)$ as their first step in factorisation.

4) Many candidates did not know that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$, and did not appreciate that matrix multiplication is not commutative. Many confused 1 with **I** and a significant number did not clearly state that their final answer was the zero matrix, rather than 0.

5) Most candidates used the determinant of the coefficients, rather than trying to solve the equations algebraically. There were a few arithmetic errors in finding the determinant, but in general the method was clearly demonstrated. Some candidates found the complete inverse matrix, often correctly, which was not actually needed.

6) (i) Most candidates answered this part correctly.

(ii) This reflection was generally recognised and usually described correctly, the incorrect ‘mirror line’ being the most common error.

(iii) The most common error was to multiply the matrices in the incorrect order.

(iv) Most candidates made a reasonable attempt to describe the matrix found in part (iii).

7) (i) Most candidates could write down a correct expression for $u_n + u_{n+1}$, but then found the factorisation difficult, despite the answer being given.

(ii) Most could establish the truth of the result for $n = 1$, but a large number made no progress in establishing that the expression in (i) being divisible by 7 and u_n being divisible by 7 implies that u_{n+1} is divisible by 7, many attempting to prove by induction that u_{n+1} was $13^{n+1} + 6^n$.

8) (i) Most candidates established this result correctly.

(ii) Most candidates used the correct value for the sum and product of the roots, with sign errors being the usual problem.

(iii) This part proved to be quite testing. Most candidates could find the value of the sum of the new roots, but algebraic errors in expanding the product of the new roots were legion. A significant proportion of candidates gave a quadratic **expression** as their final answer rather than a quadratic **equation**.

9) (i) Most candidates established the given result correctly, but a significant minority failed to show sufficient working to justify the given answer.

(ii) Most candidates realised that the process started at $r = 2$, rather than 1, while some candidates started at $r = 1$, and then removed the first term of their sum to obtain a correct answer. Those who tried this approach, but failed to remove the first term did not seem to realise that the answer to part (iii) indicated that something had gone wrong.

(iii) Most candidates knew how to find the sum to infinity.

10) (i) Most candidates showed the correct algebraic processes for finding the square roots, but many failed to include both values, i.e. $\pm$, for x and y.

(ii) Many candidates solved to find the correct values for z^2, but then thought these were the values of z. A large number of candidates were not able to see the connection of the conjugate root for z^2 i.e. $2 - i\sqrt{5}$ with the values found in part (i), and so only found 2 roots for the quartic, instead of 4.

(iii) Most candidates showed their roots correctly.

(iv) Only a small number of candidates knew that the locus was the perpendicular bisector of the line joining O to α , most sketching a circle or pair of circles.

4726 Further Pure Mathematics 2

General Comments

In general, the candidates answered the questions in the order set and were able to gain marks in every question, so that no question proved to be too difficult. A majority of candidates picked up a number of marks in question 2, which turned out to be a number-crunching question for most candidates, and in question 6, a standard hyperbolic question. Other questions produced more variable answers, often as a result of indifferent algebraic manipulation or a poor choice of method. These then led, in some cases, to a lack of time, so that answers to question 9 were sometimes rushed.
There were fewer very poor scripts than usual, but it was noticeable that most candidates were not sufficiently precise and careful enough to gain the higher marks. There appeared to be a lack of preparation in depth, resulting in marks being thrown away. Candidates could complete the “set” methods but were often less confident in the follow-up part of the question. Nevertheless, there were some outstanding scripts often showing a thoughtful approach and some flair in the answers.

Comments on Individual Questions

1) (i) This part provided a sound start for candidates, with the vast majority gaining both marks. Most candidates used their knowledge or the Formulae Booklet to write down the answer at once. As in part (ii), a significant minority opted to derive the Maclaurin series from scratch, but they were usually successful.

(ii) Most candidates used part (i) to produce $\ln(2+4x^2)$. Unfortunately, they then stopped, even though this was not a Maclaurin series. The most successful candidates went on to use $(\ln2+\ln(1+2x^2))$ and then the standard expansion of $\ln(1+x)$. There were also good attempts (albeit longer in time) at differentiating twice $\ln(e^{2x}+e^{-2x})$ or $\ln(2+4x^2)$ or even $\ln(2\cosh2x)$ and then finding f(0), f′(0) and f″(0) for the standard Maclaurin expansion. Careful differentiation often produced full marks.

2) (i) There were very few incorrect answers seen.

(ii) Apart from some sign errors, most candidates could evaluate the ratios of the errors. Hence many candidates gained five marks up to the last part of the question. However, only a small number of candidates knew the relationship between the ratios and $f'(\alpha)$. Candidates who spent some time finding $f'(\alpha)$ by differentiation and then noting the connection could still gain the final mark as long as their ratios and their $f'(\alpha)$ were reasonably close.

3) (i) Most candidates gained two marks, but only a handful went on to explain why the positive root of $(1-\sin^2y)$ was to be taken. Whilst there are more pointers if the derivative of $\cos^{-1}x$ is asked for, candidates should know the bookwork well enough to answer questions fully.

(ii) The basic rules of differentiation were not applied well, with $d/dx(\sin^{-1}2x)=1/(\sqrt{(1-4x^2)})$ and $d/dx(\tfrac{1}{2}\pi)=\tfrac{1}{2}\pi$ often seen. Candidates not confident about differentiating implicitly sensibly rewrote the equation as $y = \sin(\tfrac{1}{2}\pi-\sin^{-1}2x)$ or $\cos(\sin^{-1}2x)$ or even $\sqrt{(1-4x^2)}$ in exceptional cases. Such candidates had more success, although the $2x$ continued to present problems.

4) (i) Most candidates used the correct substitution and gained both marks. Although both marks were awarded, candidates should carefully show the introduction of $d\theta$ into their answers. It was omitted on many occasions.

(ii) Again this part was generally well answered, with some candidates losing marks by careless errors in signs when expressing $\cosh^2\theta$ in terms of $\cosh 2\theta$, despite the answer being given. However, as the answer was given, it needed to be derived fully and precisely. Statements such as "$\frac{1}{2}\sinh\theta\cosh\theta + \frac{1}{2}\theta = \frac{1}{2}x\sqrt{(x^2-1)} + \frac{1}{2}\cosh^{-1}x$" lost the final mark unless it was clearly shown how one side led to the other. Candidates using the exponential form for $\cosh x$ often gained some marks, but they found it difficult to derive the required form of answer.

5) (i) Many candidates considered the Newton-Raphson method as a formula rather than as a process involving tangents to a curve. This observation has been made in previous reports. Moreover, many candidates failed to answer the question in terms of which root (if any) the process involved for the various given k, so that, for example, when $k<0$ the answer "convergent" was often given. Part (a) was often more successfully done. A common incorrect answer to part (b) was simply "β". Only a minority of candidates were able to argue a case based upon various values of k, such as those close to 1 or 2, or close to the turning points.

(ii) This part was better answered by many candidates. Marks were lost generally by not finding the y-values of the turning points (either not correctly or not at all) and by not making clear that the curve crossed the x-axis at right-angles. Most candidates produced the correct crossing points of the x-axis and symmetry in the x-axis, albeit sketchily at times. The shape of the curve for $x>\beta$ was often not precise, but this was not penalised in this case.

6) (i) This part was generally well answered.

(ii) Candidates resorting to the exponential definitions of cosh and sinh gained no marks. It has been highlighted before that candidates should expect to use earlier results in later parts of a question. In this case, if they did not use part (i), they arrived at a quartic in e^x. The majority of candidates were able to produce and solve a quadratic in $\sinh x$, usually accurately. Candidates who then used the Formulae Booklet could write down the equivalent logarithmic forms and quickly gain five marks. Other candidates resorted at this stage to the exponential definition and solved two quadratics in e^x. This wasted time but often gained full marks, although some candidates lost the final mark by not considering the problems associated with the $\pm$ in the quadratic answers.

7) (i) This part was badly answered or often not attempted. Many candidates used incorrect assumptions such as $OQ=OR$ and $OP=OS$, or $\alpha=\frac{1}{4}\pi$, whilst others missed the fact that $0\leq\theta<2\pi$. However, "negative" angles, for example for OS, were allowed as long as they were "negative". Even candidates using $(\alpha+\frac{1}{2}n\pi)$ could not simplify their answers, for example by using the addition formulae.

(ii) This part was generally well answered. Minor arithmetic errors were seen, but most candidates attempted to deal with $\int\cos^2\theta \, d\theta$ in a reasonable way. It was surprising how often ½ was missing in the formula for the required area.

8) (i) Candidates often failed to "explain why" and answers such as "LHS = rectangles, RHS = curve and LHS < RHS" were seen. Candidates should expect, for five marks, to explain clearly and fully how each side is derived and to what each side refers. Statements such as "rectangles = ½ + ⅓ + ¼ + …" are merely imprecise copies of what is given, with no reference to areas or the limits of x. Basically, two marks were awarded for an explanation of the LHS, two marks for full working with clear limits to show the RHS as the area under the curve, with a final mark for explaining the inequality. There were some excellent answers, but these were in the minority.

(ii) This part was answered better, although there was a lack of clarity as to the final rectangle. A diagram with both limits clearly seen was often the best way to gain both marks.

(iii) Again there was sometimes a lack of precision, but most candidates gained at least one mark.

(iv) The majority of candidates ignored the results in part (iii) and wrote that the series was convergent as $1/r \to 0$ as $r \to \infty$.

9) (i) "Explain" proved difficult for a number of candidates. It was expected that a statement for the condition for a curve to have asymptotes parallel to the y-axis would be given, together with a reason as to why this condition was not met in this case. Answers such as "because $x^2 + a^2 > 0$" gained no marks. A significant number of candidates believed it related to the relative orders of the numerator and denominator.

(ii) The majority of candidates produced a quadratic in x, though often with arithmetic errors, and then attempted an inequality involving $b^2 - 4ac$. It was often apparent that candidates were unsure what their inequality related to, but, as long as they made a reasonable attempt to solve their inequality, marks were awarded. Again, basic errors caused complications for many, so that full marks were relatively rare. Candidates using differentiation were less successful, often because of inaccuracies applying the quotient rule.

(iii) There were some excellent answers involving splitting the integral into two parts and then recognising and integrating them at once. Candidates who attempted other methods had varying degrees of success. Integration by parts or attempting to write down an answer at once proved unsuccessful, but some candidates gained marks by attempting to substitute $x = a\tan\theta$ in the original integral.

4727 Further Pure Mathematics 3

General Comments

As usual in the January session, this paper attracted only a small entry. Many of those who did enter were well prepared and their work was of a high or very high standard. But a small minority were unable, probably through lack of experience, to make satisfactory attempts at several questions. All the questions were accessible for those who had covered the work fully, and there did not appear to be any problems with the time allocated for the paper.

Comments on Individual Questions

1) This question was designed to test candidates' knowledge of the structure of finite groups up to order 7.

   (i) This tested the cyclic property, subgroups and order of the elements. Although mistakes were made, there was generally a satisfactory level of knowledge of the properties and how they applied to groups of orders 3, 4 and 6 in particular.

   (ii) This part tested the fact that there are two distinct, non-isomorphic, groups of order 4 and 6. Some candidates found it difficult to understand that this was what was being asked, and it was quite common for no attempt to be made. The answer of 5 and 7 was seen several times, perhaps because there is only one group of each of these orders, but in that case order 3 might have been expected as well.

2) (i) The methods for converting between the cartesian and polar or exponential forms of complex numbers were well known. Most answers did the division first in cartesian form, then converted the answer into the required exponential form. It was less common to see the alternative method of converting both numerator and denominator into polar form first, and those who used this method were more likely to make numerical errors.

   (ii) Although some were unable to start, most realised that multiples of 3 for the value of n would help, following their answer to part (i). This earned a method mark, with the accuracy mark being given for $n = 6$.

3) This was a very standard problem, the two parts being naturally linked. Most candidates answered well, although there were more arithmetical errors than usual. Amongst the less well prepared candidates there were some attempts to use the vector product instead of the scalar product, and some were unsure about how to calculate a scalar or a vector product.

4) This was a straightforward second order differential equation, and nearly all candidates showed familiarity with the method of finding the complementary function and a particular integral. The most common errors were, firstly, to simplify the roots of the auxiliary equation incorrectly: $\frac{-4 \pm \sqrt{-4}}{2}$ often became $-2 \pm 2\mathrm{i}$. Secondly, the C.F. was frequently left in complex form as $A\mathrm{e}^{(-2+\mathrm{i})x} + B\mathrm{e}^{(-2-\mathrm{i})x}$ instead of being changed into trigonometrical form: such answers lost a mark at the beginning, although they were not penalised at the end when the C.F. and P.I. were added together. The appropriate form of the P.I. was well known and the correct values of the constants were usually found.

5) Both parts of this first order differential equation question were straightforward for those well practised in the appropriate techniques.

(i) It was an easy piece of work to differentiate the given substitution and to replace the variable y by u, and most did it well. Some less experienced candidates were unsure about how to differentiate the substitution with respect to x.

(ii) The resulting equation in u and x was easy to solve by separating the variables; then the arbitrary constant had to be included, u made the subject and back-substitution carried out. The whole process was done quite well, although the inclusion of the arbitrary constant as a multiplicative constant inside the logarithm was not seen as often as expected, and the algebra of the final rearrangement was sometimes incorrect. Several candidates rearranged the given equation so that they could use the integrating factor method: this does work, but such answers usually missed integrating 0 on the RHS to a constant. It is worth remarking that the original equation (A) can be solved directly by the integrating factor method, but any who tried this (none did, in fact) would have found the integration demanding.

6) (i) The first part of this vector question was usually done accurately. Most tackled it by finding the vector product of two vectors in the plane. In general this is the most reliable and quickest method, but in this case the numbers were such that writing down the 2-parameter form, going into cartesians and then eliminating the parameters was perhaps even quicker, provided the final stage of changing into vector form was done. Using the coordinates of three of the points also led rapidly to the cartesian equation and hence to the vector equation.

(ii) Most candidates knew about finding the angle between the two normals, by using the scalar product method. Such errors as there were usually came at the end, by giving instead the complement of the angle required.

(iii) It had been expected that this part might be found difficult, but in fact it was answered well, and mistakes were usually arithmetical. The parametric method shown in the mark scheme was almost always used. It would also be possible to use the cartesian equations of the plane and the line instead, leading to the same algebra.

7) Candidates' answers to the more demanding questions on infinite groups are not usually done as well as those on finite groups, and this question was no exception. Nevertheless, most candidates showed that they knew the basic properties of groups and attempted to use them in proofs and properties.

(i) (a) All answers indicated that the four essential properties of groups were known, but there was less certainty about justifying them in this case. For closure it was only necessary to note that the result of $x * y$ was a real number, and most answers gained the mark. The identity element was usually stated correctly as a, either by obtaining it from the definition or by guesswork. But it was disturbing to find scripts in which associativity was muddled up with commutativity, some answers proving the latter property here and calling it "associativity". Those who knew what associativity meant usually expanded the alternative sets of brackets correctly to obtain the identical results. The inverse of the element x was not obvious, and had to be obtained from the definition. Some answers claimed, correctly, that inverses existed because of the relationship $x + x^{-1} = 2a$.

(i) (b) Those who had not muddled commutativity with associativity answered this part correctly.

(i) (c) This was more demanding and it was frequently omitted. But those who realised that $x * x = e = a$ was necessary were able to show that x had to be the identity and so obtained a contradiction. In a few cases "order 2" was thought to imply that $x * x * x = e$.

(ii) Potential lack of closure was often justified correctly, by giving values to x and y which were in the range $0 < x + y \leq 5$. But the potential lack of inverse elements was seldom seen: perhaps, given more time to think about it, candidates might have realised that the identity was 5, as in part (i), and associativity unaffected, so the lack of an inverse was the one to investigate: any $x \geq 10$ has no inverse in this case.

8) (i) The identity for $\sin^6 \theta$ may not have been familiar, but the method was well known: most answers progressed fairly confidently through the procedure of using a binomial expansion and collecting terms and using multiple angles, to obtain the required result. There was, however, plenty of scope for errors, especially in signs, and not all answers scored all the marks that the writers might have expected. It was pleasing to find the correct expression for $\sin\theta$ in terms of exponentials stated at the outset in most answers, but not all realised that the – sign came from i^6, and a certain amount of working backwards was detected. There was also some crafty adjustment of a factor of 2 in the final stage of obtaining the $\cos n\theta$ terms.

(ii) The mark allocation for this part was generous, in that 2 marks were awarded for substituting $\left(\frac{1}{2}\pi - \theta\right)$ correctly throughout the identity. But the simplification of this to a similar identity proved to be beyond the ability of most. It should be well known at this level that the expression $\cos(n\pi - m\theta)$ simplifies easily to $\pm\cos m\theta$, depending on n, but it was not.

(iii) In view of the difficulties encountered in part (ii), much credit was given here for using previous answers correctly but, again, sign errors were frequent. Only the very best candidates made no errors at all and obtained the correct final answer.

Chief Examiner Report - Mechanics

Most candidates were well prepared for the examinations they sat, and it was unfortunate that some achieved less than they might, through a needless loss of marks. A quite common fault is to have a calculator working in the wrong angular mode. More common is a failure to give the answer requested (or to give it to the wrong degree of accuracy). Misreading of the questions seemed less common this session.

When candidates lose marks through such errors they inevitably gain a lower score than their knowledge and understanding warrants.

4728 Mechanics 1

General Comments

The quality of scripts seen at this session was high, candidates displaying a good knowledge of the syllabus, and competence in using their understanding. The only widespread weaknesses were in calculating normal components of contact force – questions 4(ii) and 6(ii) - and in dealing with possible constants of integration in question 5. It seemed to many examiners that there were more instances of this variable acceleration question being tackled with constant acceleration formulae than has been the case in the recent past. If this was the case, it may have been a result of the simplicity of the formula given for the acceleration.

Comments on Individual Questions

1) (i) Nearly all candidates obtained full marks.

(ii) Very few instances of the inclusion of g in the momentum terms were seen; nor were there many occasions when the particles "passed through" each other.

2) (i) Though most candidates were able to find the driving force, fewer were able to write down the value of the tension. Many candidates found part (i) harder than part (ii).

(ii) Completely correct solutions were seen. However, many candidates found it difficult to identify the forces to be included in their equations for Newton's Second Law. The equation most successfully used was for the car/trailer combination, giving the driving force. The least successful was finding the tension from an equation for the car, generally because both the tension and the resistance of the trailer were included.

3) (i) A lenient view was taken where candidates gave negative values for the magnitudes of the components of the 5 N force.

(ii) Very few candidates attempted to use sine and cosine rule, the majority following part (i) by using the result to find first the perpendicular components of the resultant force. Some candidates lost marks by giving answers which were slightly inaccurate and others through finding the angle with the y-axis.

4) (i) Most candidates answered the question correctly, but some lost a mark by giving the answer as 5.8 ms^{-2}.

(ii) The frequency of error in (ii) was higher than in (i). The commonest mistakes were in finding the normal component of reaction. The vertical component of the 20 N force was either subtracted from the weight of the block, or else was ignored. In a few cases, the weight of the block was itself ignored.

5) (i) A significant minority of candidates approached the entire question as a constant acceleration problem. However, the most common error was to ignore the initial velocity of 13 ms^{-1}.

(ii) Though correct values for the distance were often obtained, candidates who had ignored the initial velocity in (i) – or who had incorporated it in their answer on an *ad hoc* basis – were in the majority. Candidates were expected to show explicitly that no additional distance was involved, by giving evidence of considering the value of a "+c" term.

(iii) The sketch graph most often rejected was fig.3, candidates explaining how it illustrated a deceleration. Far less often was it understood that the initially horizontal graph in fig. 1 showed a velocity at t=0 of zero, and consequently the candidates were almost as likely to regard fig. 1 correct as fig. 2. Though there were no scales given on the graph axes, attempts at quantitative calculations were often seen.

6) (i) Both parts of this question were usually answered correctly, though the many candidates who thought 0.71 ms^{-1} was correct to 3 significant figures lost a mark. It was sensible of candidates to answer the two questions independently, so that an error in one part would not contaminate their answer to the other.

(ii) Correct answers were often seen. Being given an angle with the vertical made the question more difficult for many candidates, who made trigonometric errors in finding the components of weight parallel and perpendicular to the plane. In contrast, it was pleasing to see nearly all candidates realising that the parcel would travel 5 m before reaching the trolley.

7) (i) Nearly all candidates calculated correctly the speeds of the particles after two seconds. The mistakes in finding the speed of the combined particle arose either from using the initial velocities of P and Q in the momentum conservation equation, or from having the two particles moving in the same direction before their collision.

(ii) Nearly all diagrams showed for Q a graph which reached the t-axis. The common fault was for this line to start with a positive v intercept, as though the diagram were for time-speed graphs.

Very often both the Q graph in a) and the distance calculation in b) showed a lack of understanding that particle Q, once brought to rest by friction, could not start to move again.

4729 Mechanics 2

General Comments

The majority of candidates were well prepared for this examination. Candidates who did not score highly often lost marks numerically or algebraically in solving equations. The general principles of mechanics were well understood although question 3 often caused difficulties. There was an improvement in the use of diagrams and there was no evidence of the inappropriate use of radians.

Comments on Individual Questions

1) This question was generally well answered. The majority of candidates found θ in one step from first principles. A small number of candidates answered the question by quoting the formula for maximum height and others answered the question in two stages using time. Occasionally candidates used cos rather than sin or failed to take the square root.

2) Many candidates complicated finding the distance to the centre of mass from A by using medians. Some were successful using this method but most weren't. Candidates who realised that the distance was two thirds of twelve reached the answer quickly. A small number of candidates failed to take moments and attempted to find the tension by resolving vertically.

3) (i) The two parts of this question were independent, although many candidates did not appear to think so. Many started the question by finding the position of the centre of mass of the semicircular section. Many candidates over complicated the problem by taking moments about an inappropriate point. Again, good candidates achieved correct answers concisely and quoted their directions of the forces on the door clearly.

(ii) Most candidates realised the need to select a centre of mass formula from the tables. However, many selected the formula for a semicircular arc rather than for a lamina. It was also common to use degrees in the formula rather than radians. As in previous examinations with non uniform laminas, some candidates treated the problem as if the combined shape had uniform density and that it was necessary to calculate areas. Another common error was to give the distance from AE to the centre of mass of the semicircular section as 117 rather than of 217.

4) (i) There were many perfect solutions to this problem. However, some candidates confused the two situations and did not distinguish between $P/10$ and $P/20$. There was some evidence of confusion between driving force and power.

(ii) This part of the question was well answered although a significant number of candidates used P rather than $1.5P$.

5) (i) There was some evidence of fudging to achieve the given answer, but this part of the question was well answered.

(ii) Generally well answered although there were errors in the use of $mr\omega^2$ and in solving the simultaneous equations. Most candidates did not round to the requested 1 decimal place but they weren't penalised for this.

6) (i) Candidates had obviously been very well trained in deriving the equation of motion. There were just a few minus sign errors and algebraic fiddles.

(ii) Most candidates were sensible and took the hint to substitute the given values in the derived equation. However, the success rate at solving the equation for h was less good.

(iii) The majority of candidates knew what to do and at least gained some follow through marks.

(iv) This part was also well done. Very few followed the energy route.

7) (i) The vast majority of candidates immediately calculated P's first speed. Without this, the question could not progress. The fact that the objects were moving in a circle confused some and there was the occasional inappropriate use of angular speed. In using the formula for the coefficient of restitution, it was important to realise that Q was moving faster than P after the first impact. There were many perfect solutions.

(ii) Some realised that the total momentum was still 0.8 and saved some time in calculation. This time it was important to realise that P was travelling faster than Q after the collision.

(iii) The distinct topic threw many candidates. Irrelevant momentum equations were frequently quoted.

4730 Mechanics 3

General Comments

A wide range of performance of candidates is reflected by the fact that, for each question, every possible total mark was scored by some candidates. Questions 2 and 5 were the best attempted questions, with more than half of the candidates scoring full marks in each case. Questions 4 and 6 proved to be the most difficult questions, with a very significant minority of candidates failing to score more than half of the available marks in each case.

There were a number of places where candidates lost marks that were not particularly topic related. These included failing to answer the specific question asked in question 1(ii), finding the values of X and Y in question 2(iii) but ignoring the request to find the magnitude, finding the angle between OP and the horizontal in degrees, in question 4(iii), but ignoring the request to find the value of θ, and omitting the weight or the weight component in using Newton's second law in questions 4(iii), 6(i) and 7(ii)(c).

Comments on Individual Questions

1) (i) This part of the question was very well attempted, most candidates using the cosine rule in the relevant impulse-momentum triangle.

(ii) Most candidates found the angle opposite to the side of magnitude 0.5×2.5 in the impulse-momentum triangle, but a significant minority failed to proceed to the specific request for the angle between the impulse and the original direction of motion.

2) This question was very well attempted, losses of marks usually arising from a failure to finish the question, after finding the values of X and Y. Some candidates did not exploit the candidate friendly way in which the relevant distances are given, preferring instead to use distances in a form such as $\sqrt{52}\sin 33.7^{\circ}$.

3) (i)(ii) These parts of the question were well attempted.

(iii) Some candidates applied Newton's second law upwards, obtaining $a = 80gx/12$, but without using $a = -\ddot{x}$ or other device to confirm that the sign of a is positive upwards whereas x is positive downwards. This is of course essential to the confirmation that the motion is simple harmonic. Some candidates omitted the weight, despite the prompt implied by the correct execution of part (i).

(iv) There was quite a lot of confusion in dealing with this part, mainly relating to the values of A and x needed in applying $v^2 = n^2(A^2 - x^2)$.

4) (i)(ii) Part (i) was reasonably well attempted, as was the first part of (ii). However there was much muddled working in the attempts to find the transverse acceleration. Most such attempts involved differentiation of v^2 or v with respect to θ.

(iii) Nearly half of the candidates were able to find the given expression for T in terms of θ correctly, but those who couldn't usually made no mark worthy progress with this part. Very few candidates found the required answer of $\theta = 3.8$.

5) This question was very well attempted.

6) (i) Almost all of the candidates recognised the need to use Newton's second law, and the need to undertake some integration at some stage. However many mistakes were made en route to the given answer. These include:

- failure to recognise the need to find the initial speed in the medium
- omission of the weight in applying Newton's second law
- using $v\,\mathrm{d}v/\mathrm{d}x$ instead of $\mathrm{d}v/\mathrm{d}t$ and making no useful progress thereafter
- poor execution of separating the variables
- omission of the constant of integration or finding its value by using $v(0) = 0$
- poor execution of the inverse logarithmic process.

(ii) Many candidates scored all 4 marks in this part, including a significant number who made very poor attempts, or no attempt, in part (i). Common errors included obtaining $7e^{-0.2t}$ instead of $175e^{-0.2t}$ in the expression for x, and omitting the constant of integration. A significant minority used kinematic formula that relate only to motion with constant acceleration.

7) (i) This part was very well attempted.

(ii) Part (a) was also very well attempted, but in part (b) many candidates used $x = 0$ in the expression for v^2. There were very many failed attempts in part (c), but these did not reveal any commonly held misconceptions.

Chief Examiner's Report - Statistics

Similar comments can be made on the present set of examinations as in the past. Many candidates were able to produce work of high quality, although most are much stronger on numerical calculations than on demonstrating understanding through verbal responses.
Some Centres have acted on the notice given in previous Reports concerning statements of hypotheses and that over-assertive conclusions to hypothesis tests (for instance, "the time taken has changed") would be penalised. (Preferable is "there is insufficient evidence that the time taken has changed".)

In all statistics units, the incorrect use of formulae given in MF1 continues to be an issue. With the increase in statistical functions available on many calculators, it needs to be emphasised that answers obtained by a calculator with no justifying working risk scoring no marks.

There seems to be a continuing decline in standards of answers to routine questions on hypothesis tests. Only a very few candidates seem to be comfortable with logical issues such as the difference between " is it necessary to use the Central Limit Theorem?" and "is it possible to use the Central Limit Theorem". In any case the theorem itself seems very poorly understood.

4732 Probability & Statistics 1

General Comments

The paper was accessible to almost all candidates. Very few candidates scored below 20 and many very good scripts were seen, including a few with full marks. Many candidates showed a good understanding of most of the mathematics in this paper. The greatest difficulty was found in sorting out the various possibilities in questions 6 and 8. Responses to question 8 parts (ii) and (iii) suggested that conditional probability was not well understood. There were several questions that required an interpretation to be given in words, and these were generally answered fairly well. The most common inadequacy in these answers was in question 7(i) where many candidates quoted general conditions for the binomial distribution rather than the particular assumptions in the given context. There were some questions in which a partially correct method led to an incorrect answer but some marks could be gained if the working was seen. However, in many cases such an answer was seen with no working so no marks could be awarded. Candidates need to be reminded of the need to show working.

This year a significant number of candidates ignored the instruction on page 1 and rounded answers to fewer than three significant figures, thereby losing marks. Also, in some cases, marks were lost through premature rounding of intermediate answers.

There were no questions that made a significant call upon candidates' knowledge of Pure Mathematics.

Hardly any candidates appeared to run out of time.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae

The formula booklet, MF1, was useful in questions 2(i) and 4(i) and also 3(iii) and 7(iii) (for the binomial formulae) and 3(iii) and 7(iii) (for binomial tables). However, as usual, a few candidates appeared to be unaware of the existence of MF1. Other candidates tried to use the given formulae, but clearly did not understand how to do so properly. A few candidates found Σxp correctly in question 1(ii) but then divided by 5. Others attempted to use $\Sigma(x - \bar{x})^2 p$ for Var(X); these generally made arithmetical errors. In question 2(i)(a) a few candidates thought that, eg, $S_{xy} = \Sigma xy$. In the same question some candidates used the less convenient version, $b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{(\Sigma x-\bar{x})^2}$ from MF1. Most of those who used this formula either got lost in the arithmetic or misinterpreted the formula as $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{(\Sigma x-\bar{x})^2}$. Some candidates' use of the binomial tables showed that they understood the entries to be individual, rather than cumulative, probabilities.

It is worth noting yet again, that candidates would benefit from direct teaching on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1 (except in the case of b, the regression coefficient). They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

Comments on Individual Questions

1) (i) Some candidates included (1,0) but not (0,1). Others counted (1,1) twice. A few candidates attempted to use the binomial formula np and managed to arrive at the given answer by evaluating $2 \times 0.1 \times (1 - 0.1)$.

(ii) A few candidates divided Σxp by 5. In finding Var(X) some subtracted $\bar{x}$ without squaring it. Candidates who attempted to use $\Sigma(x - \bar{x})^2 p$ were far less likely to succeed than those who used $\Sigma x^2 p - \{E(X)\}^2$. A few candidates used Σxp^2.

2) (i)(a) A few candidates omitted this part, although they were able to find the equation of the regression line in (ii). Many gave correct working and arithmetic but failed to show the value of b to more than 3 significant figures before rounding to the given answer. A few candidates quoted the correct formula, without showing any figures substituted into it and then just wrote "= 1.13 to 3 sfs". These scored no marks. Some candidates were misled by MF1 into ignoring the help given in the question, and used the formula $b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ rather than the simpler $\frac{\Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$. Most of these candidates either got lost in the arithmetic or misinterpreted the formula as $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{(\Sigma x-\bar{x})^2}$. Some candidates found r.

(i)(b) Most candidates answered this part correctly, although a few saw no connection with part (i)(a) and started again.

(ii)(ab) These parts were well answered.

(iii) Some candidates referred only to the high value of r and concluded that both estimates were reliable. Others asserted that the second was unreliable, but gave no reason. Many candidates appeared not to have met a question of this sort before. They seemed to be unaware of the relevant issues, and just used native wit. This produced answers like "They seem to be in line with the data." Candidates should note that the reliability of an estimate depends on two factors: the value of r and whether it involves interpolating or extrapolating. Even the small sample size is not relevant in this case where the value of r is so high.

3) (i)(a) This part was well answered. A few candidates created a binomial distribution with a bogus value of n.

(i)(b) Common errors were $1 - (^7/_8)^3$, $1 - (^7/_8)^2$ and $(^7/_8)^2$. Candidates who used the long method $(1-(^1/_8+^7/_8 \times ^1/_8+(^7/_8)^2 \times ^1/_8)$ often omitted a term or included an extra term. Some even included $^1/_8(^7/_8)^{-1}$, which suggests rote use of a formula. It is worth noting that questions involving a geometric distribution are generally better answered by using common sense rather than by quoting the formula.

(ii) This was well answered by almost all candidates although a few tried to use np.

(iii) The change to a binomial distribution was noted only by some candidates. Some others continued to use some sort of geometric formula. A very common error was to start "from scratch" and try to find P(2 out of 15) by common sense. Many of these candidates obtained the correct powers of $^1/_8$ and $^7/_8$, but omitted the binomial coefficients.

4) (i) A few candidates calculated ranks incorrectly or calculated them in opposite directions. A more serious error was finding differences of the original data rather than of ranks and consequently obtaining a value of about -95 for r_s.

(ii) There was considerable confusion between sets of ranks that have little relationship and sets that are nearly opposite. Some candidates opted for tutors 2 and 3 because their value of r_s is closest to zero. Some chose (correctly) 1 & 3 but gave as their reason that -0.9 was the furthest from zero. Others wrote that r_s = -0.9 showed that the strongest disagreement, but did not explain that this was because the value of r_s is the largest negative value of the three or that -0.9 is close to -1.

5) (i) Text books vary as to the method for finding the median and quartiles of a discrete data set. With 23 pieces of data, this question was designed so that any of these methods would yield the same answer. The straightforward method requires no fractions at all, and with little or no effort, gives the 6th, 12th and 18th items (ie 59, 68 and 75). Candidates, however, managed to create all sorts of difficulties for themselves. Many used $^{23}/_2$ instead of $^{(23+1)}/_2$ for the median and similarly for the quartiles, and then hunted for the 5.75th item etc, often by interpolation. Centres are advised to use the method given in the OCR endorsed text book.

(ii) Some common answers which did not gain the mark were these: “The IQR uses, or shows, the actual data”, “The IQR shows the real range whereas the SD shows the spread about the mean”, “The IQR shows the position of the middle 50%” and “The IQR is easier to calculate.” The answer “The IQR is not affected by anomalies” was not accepted since the word “anomaly” does not necessarily imply “outlier”.

(iii) This “wordy” question was well answered on the whole. Some incorrect answers were: “S & L does not show the spread, or skew, as well as B &W”, “B & W is easier to compare with other data”, “S & L does not show the mean whereas B & W does” and “S & L shows the results more clearly”. An inadequate answer was “S & L does not show key values as B & W does”.

(iv) Some candidates reverted to the original table for one or both of the mean and standard deviation. These lost at least one mark. Some candidates gave 18.1 + 5 = 23.1 for the mean or 9.7 + 50 = 59.7 for the standard deviation.

6) (i)(a) This was well answered.

(i)(b) Many candidates found $4! \times 5!$ or $4! \times {}^5P_2$ or similar expressions. Some candidates derived this from considering AGAGAGAGA, which is incorrect. Others correctly found $^{4!\times4!}/_{8!}$ but failed to multiply by 2. Some found the correct answer but subtracted it from 1. As usual, some candidates found the number of arrangements but did not proceed to find the probability.

(ii)(a) Some candidates found $4! \times 4! \times 2$.

In part (ii) most candidates used arrangements although the direct probability methods are, arguably, simpler.

(ii)(b) This was well answered.

(ii)(c) Instead of using more than 4 spaces, many candidates used at least 4 or exactly 4 or exactly 5 or exactly 6 spaces. Thus although many candidates correctly found 3! × 3!, they then either failed to multiply by any factor or multiplied by 2 or 4 or 6 instead of by 3. The expression $3!\times2!$ was frequently seen. Some candidates used a complement method, which is longer than necessary. Few candidates used the straightforward probability method ($3\times(\frac{1}{4})^2$) which is probably easier than using arrangements. Many candidates showed incorrect working but without any diagram or explanation. It was therefore difficult to award them any marks.

7) (i) A few candidates used the standard notation for the binomial distribution incorrectly, e.g $X(12, 0.1)$. Some candidates did not understand the word "parameters". In stating assumptions, the most common error was to ignore the context. The second most common error was to give conditions which are inherent in the context, rather than giving assumptions. Examples of inherent conditions are (in context): "Plates can only be seconds or good" or (without context): "There must be fixed number of trials". Some candidates gave assumptions, eg "The probability is constant", but without reference to context. Another error was to give assumptions referring to the batches, rather than the plates, for example: "The probability that a batch contains a faulty plate is constant for all batches."

(ii)(a) Some candidates just gave the value for $X = 3$ from the binomial table for $n = 12$ (i.e. $P(X \le 3)$). Many gave the correct expression as a formula but evaluated it incorrectly.

(ii)(b) Some candidates found 0.6590 – 0.2824 or 1 – 0.6590. Both of these errors arise from misunderstandings of the binomial table.

(iii) Many candidates used B(4, 0.1) either using the formula or the tables. Others saw that p needed to be derived, but instead of recognising that this had already been done in part (ii), they found, for example, $P(X = 1) = 0.6590 – 0.2824$ from the tables.

8) (i) Many candidates answered this part correctly. Common errors were to omit the case where the first throw gives a 4 or to count the (2, 2) route twice.

(ii) The most common answer was $^1/_6$, sometimes without working. This arises from misinterpreting the required conditional probability as an AND probability.

(iii) The same misunderstanding in this part led to the most frequent answer of $^1/_{12}$, either from $(^1/_6)^2 \times 3$ or from $^1/_4 - {}^1/_6$. With working this answer could score a mark, but without working it scored 0.

In both parts (ii) and (iii) some candidates attempted to use the formula for conditional probability, $P(A|B) = \frac{P(A\cap B)}{P(B)}$. This sometimes yielded the correct answers, but often did not because candidates thought that the AND probability in the numerator had to be evaluated by a multiplication, i.e. (P(throw twice) × P(4)) rather than by considering what it actually meant, i.e. P((3,1) or (1,3) or (2,2)). Centres should note that this formula is not in the specification for S1. The understanding of conditional probability required in this question is limited to what "Given that" means. Both parts (ii) and (iii) can be answered by listing all the possibilities defined by the "Given that" and choosing those that are required.

4733 Probability & Statistics 2

General comments

Headlines:

- Conclusions to hypothesis tests need some indication of uncertainty. Thus *not* "the average time taken for the journey is 13.1 minutes" but something like "There is insufficient evidence that the average time taken for the journey is greater than 13.1 minutes".
- The Central Limit Theorem is very poorly understood.
- The concept of probability density functions is poorly understood.
- Questions about why one distribution can be *approximated* by another should usually be answered in terms of parameter values (e.g., "n large, p small"). Otherwise, questions about the validity of a distribution in the context of a real-life scenario should be answered by considering aspects of that scenario, and not by parameter values.

In general the calculations on this paper were found to be straightforward and many scripts were very good at this aspect of the specification. Good candidates found it easy to score about 58 marks out of 72. Questions that require understanding or interpretation were, however, less well answered. A large number of candidates appear to have difficulty understanding the questions; if a statistics examination is to have any relationship to the practical use of the subject, candidates will have to understand situations explained in English, and indeed this is true of the work of professional statisticians.

It is pleasing that almost all candidates now state hypotheses without specifically being told to do so, and conclusions to tests are generally interpreted in context, but over-assertive conclusions must be avoided, as mentioned above.

Comments on Individual Questions

1) Those who used Po(4) almost always got the right answer. The justification for this approximation is either "n large, p small" or, as given in the specification, "$n > 50$, $np < 5$". If numerical inequalities are used, they must be the ones quoted in the specification, and not different inequalities as quoted in some textbooks.

2) Almost everyone correctly used $\Phi^{-1}(0.9772) = 2$. A common mistake was $\sqrt{n} = 16 \Rightarrow n = 4$.

3) (i)(ii) These were generally very well answered. A few used $P(R > 3) = 1 - P(R \le 2)$ in (ii).

   (iii) By contrast, the majority of candidates wrongly thought that this was an application of the Central Limit Theorem and drew a normal curve. A frequency histogram should have its heights roughly proportional to the Poisson probabilities, and $P(R = 0)$ and $P(R = 1)$ were found in part (i) of the question.

4) (i) The hypothesis test for a binomial probability was perhaps better answered than in recent years, despite the need to calculate probabilities using the formula. This suggests that candidates are confused, rather than helped, by tables of cumulative probabilities. However, there were still many who converted to a normal approximation (which is not valid), or who calculated $P(R = 2)$ or $P(R < 2)$ instead of the necessary $P(R \le 2)$.

(ii) This question referred to the link between the properties of random sampling and the conditions for a binomial distribution. So answers needed to discuss the *selection* of the sample, not whether adults were equally likely to watch the programme. Among weaker candidates there is plainly confusion between the "equal probability" condition for a binomial distribution and the "constant rate" condition, which applies only to a Poisson distribution.

5) (i) Generally well done, although the number of candidates who ignore the restriction "$-2 \le x \le 2$" and continue their graphs beyond 2 or –2 remains disappointingly high.

(ii) An almost identical question has been asked twice in recent papers but answers continued to provide evidence of widespread misunderstanding. Answers such as "The probability of S is constant" were very common. Those who wrote this seem to have a vague idea that S is an event and x is a parameter that determines how likely S is to "occur". Many added that this probability was ¼. It perhaps needs to be spelt out that x denotes the possible values that the random variable can take, and that the PDF gives information about the probability that the random variable takes these values.

(iii) Often very well answered; but those who calculated μ by integration often made sign mistakes, particularly when they tried to do it as part of a complicated formula such as $\int x^2 f(x)dx - [\int x f(x)dx]^2$. Use of this formula is not recommended for weaker candidates.

6) (i) Good candidates found this question a rich source of marks. The difficult concepts of Type I and Type II errors are clearly well understood. However, some do not know what the term "critical region" means. Having found the critical values to be 49.02 and 50.98, many gave "$49.02 \le W \le 50.98$" as their answer.

This is a question in which the use of more than 3 significant figures is mandatory. Those who rounded their answers to 49.0 and 51.0 lost marks.

(ii) Many got this right with ease. However, it was perplexing that some who had got both 49.02 and 50.98 in part (i) used only one of these in part (ii). Weaker candidates attempted to find $P(> 50.2 \mid \mu = 50.0)$, or vice versa, which scored no marks.

(iii) The easiest way to answer this question is to note that a bigger sample gives a better test, and if the probability of a Type I error remains the same, the test can only become better by the probability of a Type II error becoming smaller.

7) (i) Many did this well. However, there is still a worryingly large number of candidates who confuse the roles of the sample mean (here 13.7) and the hypothesised population mean (here 13.1) in a hypothesis test. This wrecks the whole logical basis of the test and is heavily penalised, even though the calculations are almost identical.

As usual, common errors included omitting the $n/(n-1)$ factor for the unbiased variance estimate, omitting the $\sqrt{64}$ factor in the standardisation, and comparing wrong tails, or comparing z with a probability.

(ii) To judge by answers to this question, very few candidates seem to know what "necessary" means. The correct answer is "it is necessary to use the CLT because the distribution of T is not stated in the question", but many said "you need to use the CLT because n is large". That is the reason why the CLT *can* be used, not the reason why it *needs* to be used. Another common answer revealing misunderstanding of the CLT was "It is not necessary to use the CLT because you can assume that the distribution is normal". Those who gave this answer did not seem to realise that this assumption *is* the CLT.

8) (i) The criteria for a normal approximation are either "n large, p close to ½" or, as given in the specification, "$np > 5$, $nq > 5$". Those who attempted to use $npq > 5$ often did not know what to do. In any case, if these numerical criteria are used, the values of np and npq have to be stated. Most who used the normal approximation could get the right answer, with a good proportion of correct continuity corrections.

(ii) This particular question has not been asked before on S2 examinations. There are two possible approaches: to use N(14.7, 4.41/36), or to multiply everything by 36 and use the fact that the *total* number has the distribution B(756, 0.7). The latter is easier to understand and to handle, particularly in view of the continuity correction, which by the first method is 1/72 (and needs to be included, though its omission lost only 1 mark). More got this question completely right by the second method than by the first. However, the majority were groping in the dark. The most common answer was to use N(14.7, 4.41/36) but with a continuity correction of 0.5; candidates who used this had often not appreciated the fact that the distribution of the sample mean is not binomial, and their justification for the normal approximation (basically the same as in part (i)) was inadequate. By the first method, the Central Limit Theorem, or the statement that if K has a normal distribution then so has $\overline{K}$, were needed. By the second method, all that was needed was the familiar "$np > 5$, $nq > 5$" applied to $n = 756$.

Comparison of the two methods should illustrate the way to find the continuity correction by the first method. This is a recommended teaching technique.

4734 Probability & Statistics 3

General Comments

There was a similar size of entry to that of January 2008, but the performance on the paper was not as good.

There were three questions involving hypothesis tests for which the responses were varied. Statements of hypotheses in questions 5 and 7 were often given in terms of sample statistics and the definition of the parameters used rarely included the words population mean. The conclusion of a test should be preceded by a specific comparison of the test statistic with a critical value (or equivalent using a critical region) or credit will be lost.

The presentation of candidates' work was mostly satisfactory and easy to read. Only in a few cases were figures overwritten rather than replaced.

Comments on Individual Questions

1) This was generally well answered. Several, however, stated that the distribution of T was normal and some forgot to give the variance.

2) Most candidates were happier with part (ii) than part (i), where several integrated F to find f. It was hoped that $\mathrm{F}(q_3) = 0.75$ would be used, but this was rarely seen.

3) (i) Many candidates were familiar with the procedure for finding a confidence interval, but there was often difficulty in handling the percentages. Many candidates gave confidence limits rather than an interval. The Course Book uses a closed interval $[a, b]$, others use an open interval (a, b). Either of these is acceptable.

(ii) In this part errors were made with the interval width and use of an acceptable variance estimate. The sample size in Part (i) was large enough to use $0.28\times0.72/n$.
A reason for the approximate nature of the answer is that the variance is an estimate, but others were acceptable.

4) Parts (i) and (iii) were well done, but in Part (ii) a majority of candidates calculated $\mathrm{E}(X)$ in order to find the expected profit. This was not acceptable.

5) This was the most searching question, and was least well answered. Candidates did not start well. About 2/3 gave their hypotheses in terms of the sample means and very few could define their parameters adequately. There were some good attempts at parts (ii) and (iii) but many had forgotten how to find P(Type II error). Only the best were confident enough to comment on their answer.

6) This was not an easy question but it had been structured so as to give some clues. Most were, at least, able to calculate $\mathrm{E}(F)$ but many had difficulty with $\mathrm{Var}(F)$. The fact that F had an approximate normal distribution depended on B and G having approximate normal distributions. In justifying this, candidates were required to demonstrate tha,t in both distributions, $np > 5$ and $nq > 5$. This was rarely seen.
Most candidates realised that a normal calculation was required to find the required number of calculators. Many forgot a continuity correction and, more seriously, used 10.25/55 as the variance.

7) Candidates usually recognised this as a paired-sample test but could not quote the required necessary condition, namely that the population of differences should be normal. Only a very few tried a two-sample test and this scored very little.

(i) This was straightforward and there were many good solutions.

(ii) This was testing but many realised that increasing each of the T_2 marks by k increased the mean difference by k.

8) This was generally well answered in all parts. Most could show convincingly that $p = 0.2$ and parts (ii) and (iii) yielded good scores. A majority was aware that the final two cells needed to be combined in order not to give an inflated value of the test statistic. However, many did not calculate the required value of v correctly.

4736 Decision Mathematics 1

General Comments

There was no evidence to suggest that candidates did not have enough time to complete the paper.

Fewer candidates produced scruffy work, but there were still some instances that were almost illegible. Some candidates struggled with very basic mathematics, such as plotting straight line graphs, and several dropped marks through not answering everything that had been asked for, but those who had learnt the terminology involved in Decision Mathematics and understood when and how to apply the standard algorithms generally performed well.

Comments on Individual Questions

1) (i) Most candidates were confident in tracing through the algorithm, although some candidates spread their working out to cover a page or more. Candidates should be encouraged to present their results in a clear and concise form, such as a table showing the values of the variables at the end of each pass.

(ii) Many correct answers were seen. A few candidates misread the flow chart and went back to the start of the algorithm, they then had problems when A became 0.

(iii) Few candidates appreciated that the algorithm needed a stopping condition to prevent it from continuing forever. Most candidates effectively answered that the counter was there to count the passes.

2) (i) Most candidates drew an appropriate graph. A few could not count the vertices or arcs correctly. Some candidates appeared to think that when an arc starts and ends at the same vertex (a ‘loop’) it only counts as one arc ending at that vertex, when in fact it has two arc endings there.

(ii) The majority of candidates knew that their graph was semi-Eulerian but generally gave incomplete explanations of how they knew this. The simplest answer was to say that it had exactly two odd nodes.

(iii) There were many incomplete or confused answers to this part, often candidates just referred to the specific case drawn rather than a general case. Several candidates tried to explain why the graph must have a cycle, but usually their explanations were incomplete or relied on the graph being simply connected. The most convincing explanations came from the candidates who used the vertex orders to deduce that the graph must have 6 arcs and then stated that a tree with five vertices only has 4 arcs.

3) (i) As in previous sessions, several candidates did not show their working for Kruskal’s algorithm on the list of arcs although they got the correct tree and its weight. In a few cases candidates had clearly used just used Prim’s algorithm on the diagram.

(ii) Most candidates were able to find both the weight of the minimum spanning tree on the reduced network and to add the two least weights from E to find a lower bound. Some missed the arc EF and added in ED instead. A few candidates only answered one of the two requests in this part, even though the insert provided space for both values.

(iii) Several candidates were able to apply the nearest neighbour method correctly as far as vertex E but then could not give an adequate reason for its failure. Some candidates continued beyond E to repeat vertices or try to pick up vertex C.

(iv) Most candidates were able to apply the nearest neighbour algorithm but then omitted to complete the cycle by returning to the start, or sometimes returned by a longer, indirect, route. Some candidates found the weight of their route from B to A and then doubled it to get an upper bound. Whilst it is true that this strategy gives a value that must equal or exceed the weight of the optimum travelling salesperson route, it does not generally give a useful upper bound.

(v) Dijkstra's algorithm is now generally well understood and many candidates scored full marks on this part. Some candidates lost marks for writing down all the temporary labels at vertices instead of just updating when the calculated value is an improvement on the current value.

(vi) Apart from arithmetic errors, most candidates knew how to carry out the route inspection algorithm. Some just wrote down the weights of the six paths joining odd vertices instead of forming three pairs and giving their totals.

4) (i) Usually answered correctly, although some candidates said that nine passes would be required instead of eight.

(ii) This was nearly always correct.

(iii) Most candidates were able to write out the list after the second pass, a few miscounted the comparisons though.

(iv) Many candidates could correctly apply the shuttle sort algorithm, recording the results at the end of each pass. Some candidates repeated the second pass and others omitted the third pass, in which no swaps were made. A number of candidates gave a spurious ninth pass in which nothing happened.

Some candidates tried to write out every swap, in these cases it was unusual to find the results at the end of each pass clearly identified.

Counting the comparisons and swaps caused far more problems for candidates. Those who used tally marks or just gave totals instead of recording the number of comparisons and the number of swaps in each pass were penalised. In some instances candidates claimed that the number of swaps in a pass exceeded the number of comparisons.

(v) Most candidates identified that shuttle sort was more efficient than bubble sort, but often, even when they had counted the comparisons and swaps correctly, they forget to include those from the first and second passes.

5) (i) Quite a few candidates appreciated that once the first batch had been prepared subsequent batches could be prepared while the previous batch was baking. Usually these candidates were also able to show that four batches needed at least 52 minutes, and hence four batches could be made but five could not.

Some candidates said that $60 \div 12 = 5$, but she would need 'turnaround' time between getting one batch out of the oven and putting the next one in. Others claimed that $60 \div 12 = 4$. A few candidates claimed that only three batches could be made, having not appreciated that Katie did not need to stand and watch the cookies while they baked.

(ii) Many candidates recognised that the given constraint came from $8x+12y+10z \le 48$, but several though that the 48 was the total baking time for the four batches identified in part (i), rather than the maximum available preparation time, bearing in mind that the last batch needs 12 minutes to bake.

(iii) Several candidates realised that the variables needed to be integer-valued.

(iv) Most candidates gave a correct objective function, usually $P = 5x+4y+3z$. Some candidates tried adding the constraints together or gave an inequality instead of an objective function.

Some candidates realised that this objective could only be realised if all the cookies that had been made were sold. Others incorrectly suggested that the demand for the three types needed to be equal or that customers needed to buy complete batches.

(v) Most candidates were able to set up the initial Simplex tableau and perform an iteration of the Simplex algorithm. Some candidates chose an incorrect pivot and then achieved negative values in the column for RHS, and others lost at least one basis column (columns with all 0's apart from a single 1).

Many candidates correctly read off the values of x, y, z and P from their tableau but they did not always interpret the values in context.

(vi) The graph work was often poorly done. Few candidates were able to correctly draw the three lines $x+y = 4$, $4x+6y = 24$ and $y = 2x$, and fewer still could identify the feasible region. Several candidates lost marks for failing to label and scale their axes, some did not use rulers to draw the lines and some did not use graph paper.

Having drawn their graphs, very few candidates calculated the vertices of the feasible region, as instructed in the question. Most candidates tried to calculate the profit at certain points, although not always feasible points and not always integer-valued points. The question had told candidates what to do but many chose to answer some other problem of their own instead.

4737 Decision Mathematics 2

General Comments

Most candidates achieved good marks on this paper. The candidates were, in general, well prepared and were able to show what they knew.

Comments on Individual Questions

1) (i) Many candidates scored full marks on this question. A few made arithmetic errors but the majority of the wrong answers came from candidates who were either not able to transfer the values from stage 1 into the correct rows in stage 2 or who had found a maximum path.

(ii) Most candidates were able to trace back through the table to find the appropriate route. Some candidates wasted time drawing the network.

2) (i) The majority of the candidates were able to write down the precedences for all the activities except *H*. Often *G* was omitted as a preceding activity for *H*.

(ii) Most candidates achieved a reasonable attempt at a forward pass, in the backward pass several candidates seemed to ignore the dummy activities.

Most candidates were able to list the critical activities correctly.

(iii) Although some candidates seemed to misunderstand what was being asked here, the majority got at least some of the numbers of workers correct and several got them all correct.

(iv) Several candidates found the minimum delay and quite a few found the maximum delay too. Some candidates seemed to just be guessing or had lost the story by now.

3) (i) Several candidates made slips in calculating the capacity of the given cut. Sometimes an arc was omitted but more often candidates had either mistaken the direction on the arc *EB* or had incorrectly dealt with the arcs flowing from sink to source across the cut.

(ii) Most candidates could say why arc *SB* had to be at its lower capacity, and several were able to say why arc *CE* had to be at its upper capacity. Several of the explanations were confused and often candidates just wrote down everything they could think of about the named vertices. Fewer candidates successfully explained why arc *HT* had to be at its lower capacity, often it was assumed that 3 litres per second flowed through arc *CH* with no explanation about why the flow could not be either more or less than this.

(iii) Some candidates tried to show excess capacities and potential backflows instead of the flow that had been asked for. Others showed the flow and then replaced it with the augmented flow after the flow augmenting route had been applied, although this had not been asked for in the question. Several candidates thought that the flow augmenting route should use arc *SB*, even when their flow meant that this was not possible.

The cut was described in various ways, but several candidates still confuse the flows with the arc capacities when calculating a cut.

(iv) Several candidates gave an explanation that was at least partially correct. Many identified that vertex *B* needed at least 5 litres per second flowing from it but could receive at most 3 litres per second.

4) (i) Nearly all the candidates were able to draw the bipartite graph correctly; those who did not had usually omitted the arc *DP*. Some candidates drew a second bipartite graph to show the matching; others superimposed it on the first graph. A few candidates seemed to think that they had shown the matching but whatever they had done was not visible to the examiners. The errors that occurred were usually because *D* was initially matched with *S* instead of *W*.

(ii) Several candidates wrote down the shortest alternating path *E-P-A-R-B-S* and hence the corresponding complete matching. Several more wrote down an alternating path, but not the shortest such path. Most candidates wrote their alternating path out as a string, a few candidates gave a list of which arcs had 'gone in' without really saying about the arcs that had been removed. Some candidates just wrote the numbers by the vertices on the graph or showed their alternating path on their graph, this was not regarded as an acceptable answer.

(iii) Some candidates omitted to add a dummy column, a few added a dummy column but did not make the entries in the dummy column large enough and a minority of candidates decided that they had a maximisation problem and subtracted all the entries in the table from 60.

Most candidates made a reasonable attempt at reducing the rows and columns, a few only reduced rows and some did not reduce at all but went straight into augmenting.

Quite a few candidates made errors in the augmenting, particularly when augmenting by 2. Some appeared to think that the entries that were crossed out twice should be increased by 1 irrespective of the value being used for the augmenting. There were several numerical errors in this part and some overly ambitious claims about the number of lines needed to cross through all the zeros.

Some candidates only carried out one augmentation, and some none at all, often these candidates then tried to form a matching using zeros and 'other small values'.

Most candidates were able to calculate the cost of whatever matching they had chosen, only a few included the costs for the dummy.

5) (i) Several candidates were able to identify that Sanjeev won 5 games and that Euan won 3 games. Some candidates gave Sanjeev's total against Euan instead of Euan's result.

(ii) Most candidates showed the row minima and column maxima, but others just gave play-safe choices with no workings seen. Some candidates marked the row maximin and column minimax but did not state that Sanjeev and Fiona, respectively, were the play-safe choices.

A few candidates omitted to show that the game was not stable, and others referred in a rather vague way to the play-safes not being equal. Ideally candidates should have either identified the row maximin value as -2 and the column minimax value as 0 and said that the game was not stable since $-2 \neq 0$, or argued it in words by considering what the other club would do if they knew that a play-safe strategy was going to be used.

(iii) This part tested the implications of the game not being stable. Several candidates were able to write down the correct choices for their play-safe choices. Some candidates just repeated the play-safe choices.

(iv) This part was testing dominance. Tom should not be chosen because the rugby club always do better by choosing Sanjeev. Some candidates thought that Tom should not be chosen because ‘two times out of three he loses’. A few candidates seemed to then forget that low entries in the table were advantageous for the cricket club but quite a few candidates were able to identify that Doug should not be chosen because he never did any better than Fiona, and indeed once Tom has been eliminated Doug’s column is dominated by Fiona’s column.

(v) Some candidates wrote down probability expressions for all three choices, and some did not write separate probability expressions but just set their expressions equal to one another as the first line of their answer. There were several arithmetic slips in solving to find p.

(vi) Several candidates did not attempt this part. Those who did rarely gave convincing explanations. Often Doug’s column was removed or Tom’s row left in, and frequently the figures seemed to have been squeezed out from the given inequalities rather than genuinely derived from the pay off matrix for the game.

Candidates needed to remove Tom’s row, as instructed in the question, then multiply through by -1, to get pay offs for the cricket club, and add 4, to remove all the negative values. The two constraints then came from the cricket club’s expected pay offs when Sanjeev is chosen and when Ursula is chosen by the rugby club.

(vii) Some candidates made numerical mistakes, but several were able to put the values through the constraints, remembering that there will be no slack on the final constraint in the optimum case, to get a maximum value for m of 5 and a corresponding maximum value of M of 1.

One or two candidates worked all the way through the Simplex algorithm, which was totally correct but very time consuming for a 2 mark answer.

Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)
January 2009 Examination Series

Unit Threshold Marks

7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	57	50	43	37	31	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	59	51	44	37	30	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	55	48	41	34	28	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	62	54	46	38	31	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	49	44	39	34	30	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	54	47	40	33	27	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	62	54	46	38	30	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	61	51	41	31	21	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	57	48	40	32	24	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	58	50	43	36	29	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	58	49	41	33	25	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	50	43	37	31	25	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	58	51	45	39	33	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	60	53	46	39	33	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	24.1	50.4	72.7	85.8	95.1	100	960
3892	28.1	59.4	78.1	90.6	93.8	100	32
7890	26.8	58.1	84.4	92.2	96.6	100	205
7892	33.3	75.0	91.7	91.7	100	100	12

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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