

**ADVANCED SUBSIDIARY GCE**  
**MATHEMATICS**  
Further Pure Mathematics 1

**4725**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

None

**Friday 5 June 2009**  
**Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Evaluate  $\sum_{r=101}^{250} r^3$ . [3]

2 The matrices  $\mathbf{A}$  and  $\mathbf{B}$  are given by  $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$  and  $\mathbf{I}$  is the  $2 \times 2$  identity matrix. Find the values of the constants  $a$  and  $b$  for which  $a\mathbf{A} + b\mathbf{B} = \mathbf{I}$ . [4]

3 The complex numbers  $z$  and  $w$  are given by  $z = 5 - 2i$  and  $w = 3 + 7i$ . Giving your answers in the form  $x + iy$  and showing clearly how you obtain them, find

(i)  $4z - 3w$ , [2]

(ii)  $z^*w$ . [2]

4 The roots of the quadratic equation  $x^2 + x - 8 = 0$  are  $p$  and  $q$ . Find the value of  $p + q + \frac{1}{p} + \frac{1}{q}$ . [4]

5 The cubic equation  $x^3 + 5x^2 + 7 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

(i) Use the substitution  $x = \sqrt{u}$  to find a cubic equation in  $u$  with integer coefficients. [3]

(ii) Hence find the value of  $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$ . [2]

6 The complex number  $3 - 3i$  is denoted by  $a$ .

(i) Find  $|a|$  and  $\arg a$ . [2]

(ii) Sketch on a single Argand diagram the loci given by

(a)  $|z - a| = 3\sqrt{2}$ , [3]

(b)  $\arg(z - a) = \frac{1}{4}\pi$ . [3]

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z - a| \geq 3\sqrt{2} \quad \text{and} \quad 0 \leq \arg(z - a) \leq \frac{1}{4}\pi. \quad [3]$$

7 (i) Use the method of differences to show that

$$\sum_{r=1}^n \{(r+1)^4 - r^4\} = (n+1)^4 - 1. \quad [2]$$

(ii) Show that  $(r+1)^4 - r^4 \equiv 4r^3 + 6r^2 + 4r + 1$ . [2]

(iii) Hence show that

$$4 \sum_{r=1}^n r^3 = n^2(n+1)^2. \quad [6]$$

8 The matrix  $\mathbf{C}$  is given by  $\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ .

(i) Draw a diagram showing the image of the unit square under the transformation represented by  $\mathbf{C}$ . [3]

The transformation represented by  $\mathbf{C}$  is equivalent to a transformation  $S$  followed by another transformation  $T$ .

(ii) Given that  $S$  is a shear with the  $y$ -axis invariant in which the image of the point  $(1, 1)$  is  $(1, 2)$ , write down the matrix that represents  $S$ . [2]

(iii) Find the matrix that represents transformation  $T$  and describe fully the transformation  $T$ . [6]

9 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 2 \end{pmatrix}$ .

(i) Find, in terms of  $a$ , the determinant of  $\mathbf{A}$ . [3]

(ii) Hence find the values of  $a$  for which  $\mathbf{A}$  is singular. [3]

(iii) State, giving a brief reason in each case, whether the simultaneous equations

$$ax + y + z = 2a,$$

$$x + ay + z = -1,$$

$$x + y + 2z = -1,$$

have any solutions when

(a)  $a = 0$ ,

(b)  $a = 1$ .

[4]

10 The sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_1 = 3$  and  $u_{n+1} = 3u_n - 2$ .

(i) Find  $u_2$  and  $u_3$  and verify that  $\frac{1}{2}(u_4 - 1) = 27$ . [3]

(ii) Hence suggest an expression for  $u_n$ . [2]

(iii) Use induction to prove that your answer to part (ii) is correct. [5]



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