GCE

Mathematics

Advanced Subsidiary GCE **4722**

Core Mathematics 2

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

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1 (i)	f(2) = 8 + 4a – 2a –14 2a – 6 = 0 a = 3	**M1***		Attempt f(2) or equiv, including inspection / long division / coefficient matching
		M1d*		Equate attempt at f(2), or attempt at remainder, to 0 and attempt to solve
		A1	**3**	Obtain a = 3
(ii)	f(-1) = -1 + 3 + 3 – 14 = -9	**M1**		Attempt f(-1) or equiv, including inspection / long division / coefficient matching
		A1 ft	**2**	Obtain -9 (or 2a – 15, following their a)
			$\boxed{5}$	
2 (i)	area $\approx \frac{1}{2}\times 3\times\left(\sqrt[3]{8}+2\left(\sqrt[3]{11}+\sqrt[3]{14}\right)+\sqrt[3]{17}\right)$	**B1**		State or imply at least 3 of the 4 correct y-coords , and no others
		M1		Use correct trapezium rule, any h, to find area between x = 1 and x = 10
	$\approx$ 20.8			
		M1		Correct h (soi) for their y-values – must be at equal intervals
		A1	**4**	Obtain 20.8 (allow 20.7)
(ii)	use more strips / narrower strips	**B1**	**1**	Any mention of increasing n or decreasing h
			$\boxed{5}$	
3 (i)	$(1 + ½x)^{10} = 1 + 5x + 11.25x^2 + 15x^3$	**B1**		Obtain 1 + 5x
		M1		Attempt at least the third (or fourth) term of the binomial expansion, including coeffs
		A1		Obtain $11.25x^2$
		A1		Obtain $15x^3$
			4	
(ii)	coeff of x^3 = (3 x 15) + (4 x 11.25) + (2 x 5) = 100	**M1**		Attempt at least one relevant term, with or without powers of x
		A1 ft		Obtain correct (unsimplified) terms (not necessarily summed) – either coefficients or still with powers of x involved
		A1	**3**	Obtain 100
			$\boxed{7}$	

4 (i)	$u_1 = 6$, $u_2 = 11$, $u_3 = 16$	**B1**	1	State 6, 11, 16
(ii)	$S_{40} = {}^{40}/_2\ (2 \times 6 + 39 \times 5)$ $= 4140$	**M1**		Show intention to sum the first 40 terms of a sequence
		M1		Attempt sum of their AP from (i), with $n = 40$, a = their u_1 and d = their $u_2 - u_1$
		A1	3	Obtain 4140
(iii)	$w_3 = 56$ $5p + 1 = 56$ or $6 + (p - 1) \times 5 = 56$ $p = 11$	**B1**		State or imply $w_3 = 56$
		M1		Attempt to solve $u_p = k$
		A1	3	Obtain $p = 11$
			$\boxed{7}$	
5 (i)	$\dfrac{\sin\theta}{8} = \dfrac{\sin 65}{11}$	**M1**		Attempt use of correct sine rule
	$\theta = 41.2^\circ$	**A1**	2	Obtain 41.2°, or better
(ii) a	$180 - (2 \times 65) = 50^\circ$ or $65 \times {}^{\pi}/_{180} = 1.134$ $50 \times {}^{\pi}/_{180} = 0.873$ **A.G.** $\pi - (2 \times 1.134) = 0.873$	**M1**		Use conversion factor of ${}^{\pi}/_{180}$
		A1	2	Show 0.873 radians convincingly (**AG)**
(ii) b	area sector = ½ x 8^2 x 0.873 = 27.9 area triangle = ½ x 8^2 x sin 0.873 = 24.5 area segment = 27.9 – 24.5 = 3.41	**M1**		Attempt area of sector, using (½) $r^2\theta$
		M1		Attempt area of triangle using (½) $r^2\sin\theta$
		M1		Subtract area of triangle from area of sector
		A1	4	Obtain 3.41or 3.42
			$\boxed{8}$	

6 a	$\int_3^5 (x^2+4x)\,dx = \left[\frac{1}{3}x^3+2x^2\right]_3^5$	**M1**		Attempt integration
	$= (^{125}/_3 + 50) - (9 + 18)$	**A1**		Obtain $^1/_3x^3 + 2x^2$
	$= 64\,^2/_3$	**M1**		Use limits $x = 3, 5$ – correct order & subtraction
		A1	4	Obtain $64\,^2/_3$ or any exact equiv
b	$\int (2-6\sqrt{y})\,dy = 2y - 4y^{\frac{3}{2}} + c$	**B1**		State $2y$
		M1		Obtain $ky^{\frac{3}{2}}$
		A1	3	Obtain $-4y^{\frac{3}{2}}$ (condone absence of $+c$)
c	$\int_1^\infty 8x^{-3}\,dx = \left[\frac{-4}{x^2}\right]_1^\infty$	**B1**		State or imply $\frac{1}{x^3} = x^{-3}$
	$= (0) - (-4)$	**M1**		Attempt integration of kx^n
	$= 4$	**A1**		Obtain correct $-4x^{-2}$ $(+c)$
		A1 ft	4	Obtain 4 (or $-k$ following their kx^{-2})
			$\boxed{11}$	
7 (i)	$\frac{\sin^2 x - \cos^2 x}{1-\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x}$	**M1**		Use either $\sin^2 x + \cos^2 x = 1$, or $\tan x = {}^{\sin x}/_{\cos x}$
	$= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$			
	$= \tan^2 x - 1$	**A1**	2	Use other identity to obtain given answer convincingly.
(ii)	$\tan^2 x - 1 = 5 - \tan x$	**B1**		State correct equation
	$\tan^2 x + \tan x - 6 = 0$ $(\tan x - 2)(\tan x + 3) = 0$	**M1**		Attempt to solve three term quadratic in $\tan x$
	$\tan x = 2$, $\tan x = -3$ $x = 63.4^\circ, 243^\circ$ $\quad x = 108^\circ, 288^\circ$	**A1**		Obtain 2 and -3 as roots of their quadratic
		M1		Attempt to solve $\tan x = k$ (at least one root)
		A1ft		Obtain at least 2 correct roots
		A1	6	Obtain all 4 correct roots
			$\boxed{8}$	

8 a	$\log 5^{3w-1} = \log 4^{250}$	**M1***		Introduce logarithms throughout
	$(3w-1)\log 5 = 250 \log 4$	**M1***		Use $\log a^b = b \log a$ at least once
	$3w - 1 = \frac{250\log 4}{\log 5}$	**A1**		Obtain $(3w-1)\log 5 = 250 \log 4$ or equiv
	$w = 72.1$	**M1d***		Attempt solution of linear equation
		A1	**5**	Obtain 72.1, or better
b	$\log_x \frac{5y+1}{3} = 4$	**M1**		Use $\log a - \log b = \log {}^a/_b$ or equiv
	$\frac{5y+1}{3} = x^4$	**M1**		Use $f(y) = x^4$ as inverse of $\log_x f(y) = 4$
	$5y + 1 = 3x^4$	**M1**		Attempt to make y the subject of $f(y) = x^4$
	$y = \frac{3x^4 - 1}{5}$	**A1**	**4**	Obtain $y = \frac{3x^4 - 1}{5}$, or equiv
			$\boxed{9}$	
9 (i)	$ar = a + d, \quad ar^3 = a + 2d$ $2ar - ar^3 = a$ $ar^3 - 2ar + a = 0$ $r^3 - 2r + 1 = 0$ **A.G.**	**M1**		Attempt to link terms of AP and GP, implicitly or explicitly.
		M1		Attempt to eliminate d, implicitly or explicitly, to show given equation.
		A1	**3**	Show $r^3 - 2r + 1 = 0$ convincingly
(ii)	$f(r) = (r-1)(r^2 + r - 1)$	**B1**		Identify $(r-1)$ as factor or $r = 1$ as root
		M1*		Attempt to find quadratic factor
	$r = \frac{-1 \pm \sqrt{5}}{2}$	**A1**		Obtain $r^2 + r - 1$
	Hence $r = \frac{-1 + \sqrt{5}}{2}$	**M1d***		Attempt to solve quadratic
		A1	**5**	Obtain $r = \frac{-1 + \sqrt{5}}{2}$ only
(iii)	$\frac{a}{1-r} = 3 + \sqrt{5}$	**M1**		Equate S_∞ to $3 + \sqrt{5}$
	$a = (\frac{3}{2} - \frac{\sqrt{5}}{2})(3 + \sqrt{5})$	**A1**		Obtain $\frac{a}{1 - \left(\frac{-1+\sqrt{5}}{2}\right)} = 3 + \sqrt{5}$
	$a = {}^9/_2 - {}^5/_2$ $a = 2$	**M1**		Attempt to find a
		A1	**4**	Obtain $a = 2$
			$\boxed{12}$	

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