OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4737

Decision Mathematics 2

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet
  (sent with general stationery)
- Insert for Questions 4 and 6 (inserted)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Wednesday 26 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- The insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- There is an **insert** for use in Questions **4** and **6**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

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1 Four friends, Amir (A), Bex (B), Cerys (C) and Duncan (D), are visiting a bird sanctuary. They have decided that they each will sponsor a different bird. The sanctuary is looking for sponsors for a kite (K), a lark (L), a moorhen (M), a nightjar (N), and an owl (O).

Amir wants to sponsor the kite, the nightjar or the owl; Bex wants to sponsor the lark, the moorhen or the owl; Cerys wants to sponsor the kite, the lark or the owl; and Duncan wants to sponsor either the lark or the owl.

(i) Draw a bipartite graph to show which friend wants to sponsor which birds. **[1]**

Amir chooses to sponsor the kite and Bex chooses the lark. Cerys then chooses the owl and Duncan is left with no bird that he wants.

(ii) Write down the shortest possible alternating path starting from the nightjar, and hence write down one way in which all four friends could have chosen birds that they wanted to sponsor. **[2]**

(iii) List a way in which all four friends could have chosen birds they wanted to sponsor, with the owl not being chosen. **[1]**

2 Amir, Bex, Cerys and Duncan all have birthdays in January. To save money they have decided that they will each buy a present for just one of the others, so that each person buys one present and receives one present. Four slips of paper with their names on are put into a hat and each person chooses one of them. They do not tell the others whose name they have chosen and, fortunately, nobody chooses their own name.

The table shows the cost, in £, of the present that each person would buy for each of the others.

		To			
		Amir	Bex	Cerys	Duncan
From	Amir	–	15	21	19
	Bex	20	–	16	14
	Cerys	25	12	–	16
	Duncan	24	10	18	–

As it happens, the names are chosen in such a way that the total cost of the presents is minimised.

Assign the cost £25 to each of the missing entries in the table and then apply the Hungarian algorithm, reducing rows first, to find which name each person chose. **[7]**

3 The table lists the duration, immediate predecessors and number of workers required for each activity in a project.

Activity	Duration (hours)	Immediate predecessors	Number of workers
A	3	–	1
B	2	–	1
C	2	A	2
D	3	A, B	2
E	3	C	3
F	3	C, D	3
G	2	D	3
H	5	E, F	1
I	4	F, G	2

(i) Represent the project by an activity network, using activity on arc. You should make your diagram quite large so that there is room for working. **[3]**

(ii) Carry out a forward pass and a backward pass through the activity network, showing the early event times and late event times clearly at the vertices of your network.

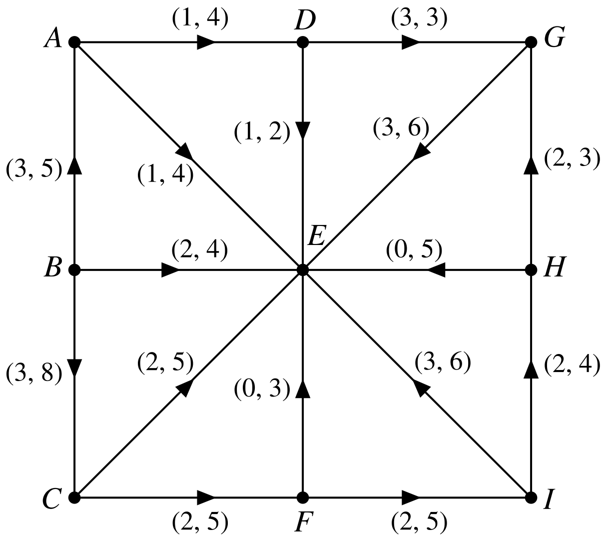
State the minimum project completion time and list the critical activities. **[5]**

(iii) Draw a resource histogram to show the number of workers required each hour when each activity begins at its earliest possible start time. **[2]**

(iv) Show how it is possible for the project to be completed in the minimum project completion time when only six workers are available. **[2]**

4 Answer parts (v) and (vi) of this question on the insert provided.

The diagram represents a system of pipes through which fluid can flow. The weights on the arcs show the lower and upper capacities of the pipes in litres per second.



(i) Which vertex is the source and which vertex is the sink? **[1]**

(ii) Cut α partitions the vertices into the sets $\{A, B, C\}$, $\{D, E, F, G, H, I\}$. Calculate the capacity of cut α. **[2]**

(iii) Explain why partitioning the vertices into sets $\{A, D, G\}$, $\{B, C, E, F, H, I\}$ does not give a cut. **[1]**

(iv) (a) How many litres per second must flow along arc DG? **[1]**

(b) Explain why the arc AD must be at its upper capacity. Hence find the flow in arc BA. **[2]**

(c) Explain why at least 7 litres per second must flow along arc BC. **[2]**

(v) Use the diagrams in the insert to show a minimum feasible flow and a maximum feasible flow. **[4]**

The upper capacity of BC is now increased from 8 to 18.

(vi) (a) Use the diagram in the insert to show a flow of 19 litres per second. **[1]**

(b) List the saturated arcs when 19 litres per second flows through the network. Hence, or otherwise, find a cut of capacity 19. **[2]**

(vii) Explain how your answers to part **(vi)** show that 19 litres per second is the maximum flow. **[2]**

5 A card game between two players consists of several rounds. In each round the players both choose a card from those in their hand; they then show these cards to each other and exchange tokens. The number of tokens that the second player gives to the first player depends on the colour of the first player's card and the design on the second player's card.

The table shows the number of tokens that the first player receives for each combination of colour and design. A negative entry means that the first player gives tokens to the second, zero means that no tokens are exchanged.

		Second player		
		Square	Triangle	Circle
First player	Red	2	−1	1
	Yellow	−2	0	−3
	Blue	−5	1	3

(i) Explain how you know that the game is zero-sum. Describe what zero-sum means for the way in which the players play the game. **[2]**

(ii) Find the play-safe choice for each player, showing your working. Explain how you know whether the game is stable or unstable. Describe what 'stable' and 'unstable' mean for the way in which the players play the game. **[6]**

The first player decides not to risk playing a blue card.

(iii) Show that in this reduced game the circle strategy dominates the square strategy, and explain what this means for the way in which the second player plays the game. **[2]**

The first player uses random numbers to choose between the other two colours, where the probability of choosing a red card is p and the probability of choosing a yellow card is $1 - p$.

(iv) Write down an expression for the expected number of tokens that the first player is given in each round for each choice of design. Calculate the optimal value of p, showing your working. **[3]**

The entries in the row for 'Blue' in the original table are now all multiplied by −1. So, for example, when the first player chooses blue and the second chooses square, instead of the first player giving the second player 5 tokens, the second player now gives the first player 5 tokens.

The first player now uses random numbers to choose between the three colours, letting x, y and z denote the probabilities of choosing red, yellow and blue respectively.

The problem of choosing between the three colours is modelled as the following LP.

Maximise $M = m - 3$,

subject to
$m \leqslant 5x + y + 8z$,
$m \leqslant 2x + 3y + 2z$,
$m \leqslant 4x$,
$x + y + z \leqslant 1$,

and $m \geqslant 0,\ x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$.

(v) Explain how the expression $5x + y + 8z$ was formed. **[2]**

The Simplex algorithm is used to solve the LP problem. The solution has $x = 0.6$, $y = 0.4$ and $z = 0$.

(vi) Calculate the value of each of the expressions $5x + y + 8z$, $2x + 3y + 2z$ and $4x$. Hence write down the optimal value of M. **[3]**

6 Answer this question on the insert provided.

Four friends have decided to sponsor four birds at a bird sanctuary. They want to construct a route through the bird sanctuary, starting and ending at the entrance/exit, that enables them to visit the four birds in the shortest possible time. The table below shows the times, in minutes, that it takes to get between the different birds and the entrance/exit. The friends will spend the same amount of time with each bird, so this does not need to be included in the calculation.

	Entrance/exit	Kite	Lark	Moorhen	Nightjar
Entrance/exit	–	10	14	12	17
Kite	10	–	3	2	6
Lark	14	3	–	2	4
Moorhen	12	2	2	–	3
Nightjar	17	6	4	3	–

Let the stages be 0, 1, 2, 3, 4, 5. Stage 0 represents arriving at the sanctuary entrance. Stage 1 represents visiting the first bird, stage 2 the second bird, and so on, with stage 5 representing leaving the sanctuary. Let the states be 0, 1, 2, 3, 4 representing the entrance/exit, kite, lark, moorhen and nightjar respectively.

(i) Calculate how many minutes it takes to travel the route

$$(0; 0) – (1; 1) – (2; 2) – (3; 3) – (4; 4) – (5; 0).$$ **[1]**

The friends then realise that if they try to find the quickest route using dynamic programming with this (stage; state) formulation, they will get the route (0; 0) – (1; 1) – (2; 2) – (3; 3) – (4; 1) – (5; 0), or this in reverse, taking 27 minutes.

(ii) Explain why the route (0; 0) – (1; 1) – (2; 2) – (3; 3) – (4; 1) – (5; 0) is not a solution to the friends' problem. **[1]**

Instead, the friends set up a dynamic programming tabulation with stages and states as described above, except that now the states also show, in brackets, any birds that have already been visited. So, for example, state 1(234) means that they are currently visiting the kite and have already visited the other three birds in some order. The partially completed dynamic programming tabulation is shown opposite.

(iii) For the last completed row, i.e. stage 2, state 1(3), action 4(13), explain where the value 18 and the value 6 in the working column come from. **[2]**

(iv) Complete the table in the insert and hence find the order in which the birds should be visited to give a quickest route and find the corresponding minimum journey time. **[9]**

Stage	State	Action	Working	Suboptimal minimum
4	1(234)	0	10	10
	2(134)	0	14	14
	3(124)	0	12	12
	4(123)	0	17	17
3	1(23)	4(123)	17 + 6 = 23	23
	1(24)	3(124)	12 + 2 = 14	14
	1(34)	2(134)	14 + 3 = 17	17
	2(13)	4(123)	17 + 4 = 21	21
	2(14)	3(124)	12 + 2 = 14	14
	2(34)	1(234)	10 + 3 = 13	13
	3(12)	4(123)	17 + 3 = 20	20
	3(14)	2(134)	14 + 2 = 16	16
	3(24)	1(234)	10 + 2 = 12	12
	4(12)	3(124)	12 + 3 = 15	15
	4(13)	2(134)	14 + 4 = 18	18
	4(23)	1(234)	10 + 6 = 16	16
2	1(2)	3(12) 4(12)	20 + 2 = 22 15 + 6 = 21	21
	1(3)	2(13) 4(13)	21 + 3 = 24 18 + 6 = 24	24
	1(4)			
	2(1)			
	2(3)			
	2(4)			
	3(1)			
	3(2)			
	3(4)			
	4(1)			
	4(2)			
	4(3)			
1	1			
	2			
	3			
	4			
0	0	1 2 3 4		

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