

Friday 27 January 2012 – Morning

A2 GCE MATHEMATICS

4727 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

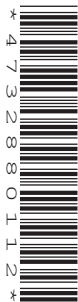
OCR supplied materials:

- Printed Answer Book 4727
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{2x^2 + y^2}{xy}. \quad (\text{A})$$

- (i) Use the substitution $y = ux$, where u is a function of x , to obtain the differential equation

$$x \frac{du}{dx} = \frac{2}{u}. \quad [3]$$

- (ii) Hence find the general solution of the differential equation (A), giving your answer in the form $y^2 = f(x)$. [4]

- 2 (i) Show that $(z^n - e^{i\theta})(z^n - e^{-i\theta}) \equiv z^{2n} - (2 \cos \theta) z^n + 1$. [1]

- (ii) Express $z^4 - z^2 + 1$ as the product of four factors of the form $(z - e^{i\alpha})$, where $0 \leq \alpha < 2\pi$. [6]

- 3 A multiplicative group contains the distinct elements e, x and y , where e is the identity.

- (i) Prove that $x^{-1}y^{-1} = (yx)^{-1}$. [2]

- (ii) Given that $x^n y^n = (xy)^n$ for some integer $n \geq 2$, prove that $x^{n-1} y^{n-1} = (yx)^{n-1}$. [3]

- (iii) If $x^{n-1} y^{n-1} = (yx)^{n-1}$, does it follow that $x^n y^n = (xy)^n$? Give a reason for your answer. [2]

- 4 The line l has equations $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z+1}{2}$ and the point A is $(7, 3, 7)$. M is the point where the perpendicular from A meets l .

- (i) Find, in either order, the coordinates of M and the perpendicular distance from A to l . [7]

- (ii) Find the coordinates of the point B on AM such that $\vec{AB} = 3\vec{BM}$. [3]

- 5 The variables x and y satisfy the differential equation

$$2 \frac{d^2y}{dx^2} + 3 \frac{dy}{dx} - 2y = 5e^{-2x}.$$

- (i) Find the complementary function of the differential equation. [2]

- (ii) Given that there is a particular integral of the form $y = pxe^{-2x}$, find the constant p . [4]

- (iii) Find the solution of the equation for which $y = 0$ and $\frac{dy}{dx} = 4$ when $x = 0$. [5]

- 6 The plane Π has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 6 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ -5 \end{pmatrix}$ and the line l has equation $\mathbf{r} = \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$.
- (i) Express the equation of Π in the form $\mathbf{r} \cdot \mathbf{n} = p$. [4]
- (ii) Find the point of intersection of l and Π . [2]
- (iii) The equation of Π may be expressed in the form $\mathbf{r} = \begin{pmatrix} 1 \\ 6 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \mu \mathbf{c}$, where \mathbf{c} is perpendicular to $\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$. Find \mathbf{c} . [3]
- 7 The set M consists of the six matrices $\begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$, where $n \in \{0, 1, 2, 3, 4, 5\}$. It is given that M forms a group (M, \times) under matrix multiplication, with numerical addition and multiplication both being carried out modulo 6.
- (i) Determine whether (M, \times) is a commutative group, justifying your answer. [2]
- (ii) Write down the identity element of the group and find the inverse of $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$. [3]
- (iii) State the order of $\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ and give a reason why (M, \times) has no subgroup of order 4. [2]
- (iv) The multiplicative group G has order 6. All the elements of G , apart from the identity, have order 2 or 3. Determine whether G is isomorphic to (M, \times) , justifying your answer. [2]
- 8 (i) Use de Moivre's theorem to prove that
- $$\tan 5\theta \equiv \frac{5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}. \quad [4]$$
- (ii) Solve the equation $\tan 5\theta = 1$, for $0 \leq \theta < \pi$. [3]
- (iii) Show that the roots of the equation
- $$t^4 - 4t^3 - 14t^2 - 4t + 1 = 0$$
- may be expressed in the form $\tan \alpha$, stating the exact values of α , where $0 \leq \alpha < \pi$. [5]

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE



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