OCR
RECOGNISING ACHIEVEMENT

Wednesday 23 January 2013 – Morning

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hours 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 For each of the following curves, find the gradient at the point with x-coordinate 2.

(i) $y = \dfrac{3x}{2x+1}$ **[3]**

(ii) $y = \sqrt{4x^2 + 9}$ **[3]**

2 The acute angle A is such that $\tan A = 2$.

(i) Find the exact value of $\operatorname{cosec} A$. **[2]**

(ii) The angle B is such that $\tan(A + B) = 3$. Using an appropriate identity, find the exact value of $\tan B$. **[3]**

3 **(a)** Given that $|t| = 3$, find the possible values of $|2t - 1|$. **[3]**

(b) Solve the inequality $|x - \sqrt{2}| > |x + 3\sqrt{2}|$. **[4]**

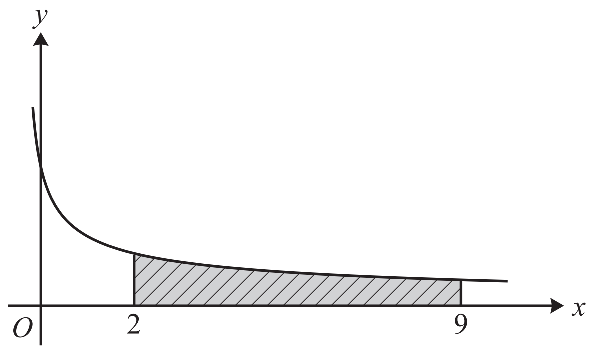
4 The mass, m grams, of a substance is increasing exponentially so that the mass at time t hours is given by

$$m = 250e^{0.021t}.$$

(i) Find the time taken for the mass to increase to twice its initial value, and deduce the time taken for the mass to increase to 8 times its initial value. **[3]**

(ii) Find the rate at which the mass is increasing at the instant when the mass is 400 grams. **[3]**

5



The diagram shows the curve $y = \dfrac{6}{\sqrt{3x+1}}$. The shaded region is bounded by the curve and the lines $x = 2$, $x = 9$ and $y = 0$.

(i) Show that the area of the shaded region is $4\sqrt{7}$ square units. **[4]**

(ii) The shaded region is rotated completely about the x-axis. Show that the volume of the solid produced can be written in the form $k\ln 2$, where the exact value of the constant k is to be determined. **[5]**

6 **(i)** By sketching the curves $y = \ln x$ and $y = 8 - 2x^2$ on a single diagram, show that the equation

$$\ln x = 8 - 2x^2$$

has exactly one real root. **[3]**

(ii) Explain how your diagram shows that the root is between 1 and 2. **[1]**

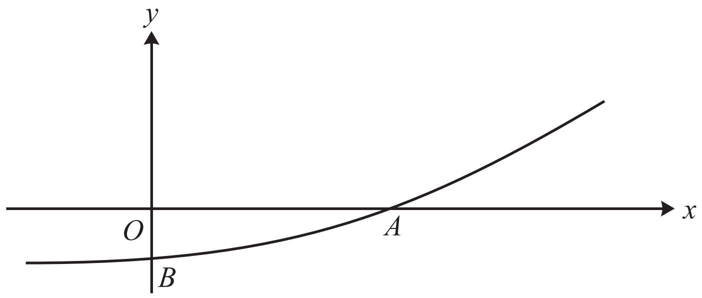
(iii) Use the iterative formula

$$x_{n+1} = \sqrt{4 - \tfrac{1}{2}\ln x_n},$$

with a suitable starting value, to find the root. Show all your working and give the root correct to 3 decimal places. **[4]**

(iv) The curves $y = \ln x$ and $y = 8 - 2x^2$ are each translated by 2 units in the positive x-direction and then stretched by scale factor 4 in the y-direction. Find the coordinates of the point where the new curves intersect, giving each coordinate correct to 2 decimal places. **[3]**

7



The diagram shows the curve with equation

$$x = (y + 4)\ln(2y + 3).$$

The curve crosses the x-axis at A and the y-axis at B.

(i) Find an expression for $\frac{dx}{dy}$ in terms of y. **[3]**

(ii) Find the gradient of the curve at each of the points A and B, giving each answer correct to 2 decimal places. **[5]**

8 The functions f and g are defined for all real values of x by

$$f(x) = x^2 + 4ax + a^2 \quad \text{and} \quad g(x) = 4x - 2a,$$

where a is a positive constant.

(i) Find the range of f in terms of a. **[4]**

(ii) Given that $fg(3) = 69$, find the value of a and hence find the value of x such that $g^{-1}(x) = x$. **[6]**

9 **(i)** Prove that

$$\cos^2(\theta + 45°) - \tfrac{1}{2}(\cos 2\theta - \sin 2\theta) \equiv \sin^2\theta.$$ **[4]**

(ii) Hence solve the equation

$$6\cos^2(\tfrac{1}{2}\theta + 45°) - 3(\cos\theta - \sin\theta) = 2$$

for $-90° < \theta < 90°$. **[3]**

(iii) It is given that there are two values of θ, where $-90° < \theta < 90°$, satisfying the equation

$$6\cos^2(\tfrac{1}{3}\theta + 45°) - 3(\cos\tfrac{2}{3}\theta - \sin\tfrac{2}{3}\theta) = k,$$

where k is a constant. Find the set of possible values of k. **[3]**