

Mathematics

Advanced GCE

Unit 4727: Further Pure Mathematics 3

Mark Scheme for June 2013

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Question		Answer	Marks	Guidance
1	(i)	vectors in plane: two of $\begin{pmatrix} -4 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$ $\mathbf{r} = \begin{pmatrix} 1 \\ 6 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$	M1 A1 [2]	Differences between two pairs Aef of parametric equation Must have “ $\mathbf{r} = \dots$ ”
1	(ii)	$\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} \times \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ -12 \end{pmatrix}$ $\left(\mathbf{r} - \begin{pmatrix} 1 \\ 6 \\ 2 \end{pmatrix} \right) \cdot \begin{pmatrix} 5 \\ 8 \\ -12 \end{pmatrix} = 0$ $5x + 8y - 12z = 29$ <p>Alternate method</p>	M1 A1 M1 A1 [4] M1 A1 M1A1 M1 A1 M1 A1	Calculate vector product or multiple Or Cartesian form = d with attempt to compute d Aef of cartesian equation, isw. EITHER x, y, z in parametric form both parameters in terms of e.g. x, y substitute into parametric form of z OR x, y, z in parametric form 2 equations in x, y, z and one parameter eliminate parameter

Question		Answer	Marks	Guidance																										
2	(i)	<table style="border-collapse: collapse; margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;"></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">7</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">7</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">5</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">3</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">1</td> </tr> </table> <p>From table clearly closed</p> <p>1 is identity</p> <p>$3^{-1} \equiv 3, 5^{-1} \equiv 5, 7^{-1} \equiv 7 \pmod{8}$</p>		1	3	5	7	1	1	3	5	7	3	3	1	7	5	5	5	7	1	3	7	7	5	3	1	<p>B2</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>[5]</p>	<p>-1 each error</p> <p>Superfluous fact/s gets -1</p>	<p>Must be clear they are referring to tabulated results</p> <p>Or "1 appears in every row"</p>
	1	3	5	7																										
1	1	3	5	7																										
3	3	1	7	5																										
5	5	7	1	3																										
7	7	5	3	1																										
2	(ii)	1 has order 1 and 3, 5, 7 all have order 2	<p>B1</p> <p>[1]</p>																											
2	(iii)	{1, 3}, {1, 5}, {1, 7} (and {1})	<p>B1</p> <p>[1]</p>	<p>All correct, no extras</p> <p>Allow {1} included or omitted</p>																										
2	(iv)	<p>in H $3^2 \equiv 9 \pmod{10}$ so 3 not order 2</p> <p>no element of order > 2 in G so not isomorphic</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Shows and states that 3 or that 7 is not order 2 (or is order 4)</p> <p>Completely correct reasoning</p> <p>Or, if zero, then SC1 for merely stating comparable orders and then saying that "orders don't correspond, so not isomorphic"</p> <p>Or</p> <p>table for H with saying "not all elements self inverse, so not isomorphic"</p>																										

Question	Answer	Marks	Guidance	
3	$u = y^3 \Rightarrow \frac{du}{dx} = 3y^2 \frac{dy}{dx}$ <p>in DE gives $x \frac{du}{dx} + 2u = \frac{\cos x}{x}$</p> $\frac{du}{dx} + \frac{2}{x}u = \frac{\cos x}{x^2}$ $I = \exp\left(\int \frac{2}{x} dx\right) = e^{2 \ln x}$ $= x^2$ $x^2 \frac{du}{dx} + 2xu = \cos x$ $\frac{d}{dx}(x^2 u) = \cos x$ $x^2 u = \sin x \quad (+A)$ $u = \frac{\sin x + A}{x^2}$ $y = \left(\frac{\sin x + A}{x^2}\right)^{\frac{1}{3}}$	<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[8]</p>	<p></p> <p>Divide</p> <p>Correctly integrates</p> <p></p> <p>Integrate</p> <p>Or gives GS in implicit form</p>	<p>Or $\frac{dy}{dx} = \frac{1}{3}u^{-\frac{2}{3}} \frac{du}{dx}$</p> <p>Both sides</p> <p>Must have form $\frac{du}{dx} + f(x)u = g(x)$</p> <p>Can be implied by subsequent work</p> <p>Must include constant at this stage</p>

Question		Answer	Marks	Guidance
4	(i)	Sketch $OA = 3 = 3, OB = \left 3e^{\frac{1}{3}\pi i} \right = 3$ and $\angle BOA = \frac{1}{3}\pi$ hence $\triangle OAB$ equilateral	B1 M1 A1 [3]	Must have axes, A shown 3 across and either scale (or co-ordinates) with B in rough position, or angle and distance on argand diagram. No inconsistencies Can be seen on diagram Alt. Attempts AB or second angle
4	(ii)	$3e^{-\frac{1}{3}\pi i}$	M1A1 [2]	Or $3e^{\frac{5}{3}\pi i}$. Isw M1 for evidence they are considering BA, or for $\frac{3}{2} - \frac{3}{2}\sqrt{3}i$ For full marks can use CiS form, or clear polar co-ordinates, in radians. Not C-iS
4	(iii)	$\left(3 - 3e^{\frac{1}{3}\pi i} \right)^5 = 3^5 e^{-\frac{5}{3}\pi i}$ $= 243 \left(\cos \frac{5}{3}\pi - i \sin \frac{5}{3}\pi \right)$ $= \frac{243}{2} + \frac{243}{2}\sqrt{3}i$	M1 A1ft B1 [3]	For mod^5 and $\text{arg} \times 5$ aef “Hence” so must use ‘their $3e^{-\frac{1}{3}\pi i}$, Condone use of “121.5”.

Question	Answer	Marks	Guidance
5	AE: $\lambda^2 + 2\lambda + 5 = 0$ $\lambda = -1 \pm 2i$ CF: $e^{-x}(A \cos 2x + B \sin 2x)$ PI: $y = ae^{-x}$ $ae^{-x} - 2ae^{-x} + 5ae^{-x} = e^{-x}$ $4a = 1$ $a = \frac{1}{4}$ GS: $y = e^{-x}\left(\frac{1}{4} + A \cos 2x + B \sin 2x\right)$ $\frac{dy}{dx} = -e^{-x}\left(\frac{1}{4} + A \cos 2x + B \sin 2x\right)$ $+ e^{-x}(-2A \sin 2x + 2B \cos 2x)$ $x = 0, \frac{dy}{dx} = 0 \Rightarrow -\left(\frac{1}{4} + A\right) + 2B = 0$ $x = 0, y = 0 \Rightarrow \frac{1}{4} + A = 0$ $A = -\frac{1}{4}, B = 0$ $y = \frac{1}{4}e^{-x}(1 - \cos 2x)$	M1 A1 A1ft B1 M1 A1 A1ft M1* *M1 A1ft A1 [11]	Differentiate & substitute Differentiate their GS of form $y = e^{-x}(P + A \cos nx + B \sin nx)$ where P is constant or linear term, n not 0 or 1 Use conditions From their GS
6	(i) $x = 2t + 1, y = 5t - 1, z = t + 2$ $(2t + 1) + 2(5t - 1) - 2(t + 2) = 5$ $\Rightarrow 10t = 10 \Rightarrow t = 1$ Intersect at (3, 4, 3)	B1 M1 A1 [3]	Parameterise Substitute into plane Solve cao or B1 for y and z correctly in terms of x e.g. $2y = 5x - 7, 2z = x + 3$ Then M1 for full simultaneous equations method. Accept vector form

Question	Answer	Marks	Guidance
6 (ii)	$\cos\left(\frac{1}{2}\pi - \theta\right) = \frac{\begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}}{\begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}} = \frac{10}{3\sqrt{30}}$ $\theta = 0.654$	M1A1 A1 [3]	Attempt to find angle or its complement or 37.5°
6 (iii)	<p>If P is point of intersection and Q is required point,</p> $\overline{PQ} = \lambda \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \text{ so } \sin \theta = \frac{2}{PQ} = \frac{2}{ \lambda \sqrt{30}}$ $\frac{10}{3\sqrt{30}} = \frac{2}{ \lambda \sqrt{30}}$ $\lambda = \pm \frac{3}{5}$ <p>points have position vectors $\begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} \pm \frac{3}{5} \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$</p> <p>points at (1.8, 1, 2.4) and (4.2, 7, 3.6)</p> <p>Alternative:</p> $\text{Distance} = \frac{ 2t + 1 + 2(5t - 1) - 2(t + 2) - 5 }{\sqrt{1^2 + 2^2 + 2^2}} = 2$ $\Rightarrow t = 0.4 \text{ or } 1.6$ <p>(1.8, 1, 2.4) and (4.2, 7, 3.6)</p>	M1* M1 A1 *M1 A1 M1* A1 *M1 A1 A1 [5]	or $\overline{PQ} \cdot \hat{\mathbf{n}} = \pm 2$ where $\mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ Use \overline{PQ} with right angled triangle or consider component of \overline{PQ} in direction of normal vector. Valid method to set up equation in λ alone. (May work from general point on original equation) Dep on 1 st M1 cao Zero if formula used without substitution in of parametric form. Solve At least one value found

Question		Answer	Marks	Guidance	
7	(i)	$(ab)^6 = abab\dots ab = a^6b^6$ as commutative $= (a^2)^3 (b^3)^2 = e^3e^2 = e$ So ab has order 1, 2, 3, or 6 $(b \neq a \Rightarrow ab \neq a^2 \Rightarrow ab \neq e$ so ab not order 1) $(ab)^2 = a^2b^2 = eb^2 = b^2$ and b not order 2, so ab not order 2 $(ab)^3 = a^3b^3 = aa^2e = aee = a \neq e$, so ab not order 3 (So must be order 6)	M1 A1 M1 A1 [4]	Must give reason Using orders of a and b Consider other cases AG Complete argument	Some demonstration that they understand commutativity Condone absence of this line Insufficient to merely have simplified all $(ab)^n$
7	(ii)	ac has order 18 18 is LCM of 2 and 9, (so we can use a similar argument to part (i)) So as G has an element of order 18 it must be cyclic.	B1 M1 A1 [3]	or explicit consideration of other cases AG Complete argument	Or abc or generator
8	(i)	$\cos 5\theta + i \sin 5\theta = (\cos \theta + i \sin \theta)^5$ $= c^5 + 5ic^4s - 10c^3s^2 - 10ic^2s^3 + 5cs^4 + is^5$ $\cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$ $= c^5 - 10c^3(1-c^2) + 5c(1-c^2)^2$ $= c^5 - 10c^3 + 10c^5 + 5c - 10c^3 + 5c^5$ $\cos 5\theta = 16c^5 - 20c^3 + 5c$	B1 M1 M1 M1 A1 [5]	Or $\cos 5\theta = \operatorname{re}\{(\cos \theta + i \sin \theta)^5\}$ Take real parts AG	No more than 1 error, can be unsimplified

Question		Answer	Marks	Guidance
8	(ii)	Multiplying by x gives $16x^5 - 20x^3 + 5x = 0$ letting $x = \cos \alpha$ gives $\cos 5\alpha = 0$ hence $5\alpha = \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi, \frac{7}{2}\pi, \frac{9}{2}\pi$ $\alpha = \frac{1}{10}\pi, \frac{3}{10}\pi, \frac{5}{10}\pi, \frac{7}{10}\pi, \frac{9}{10}\pi$ $\cos \frac{5}{10}\pi = 0$ which is not a root so roots $x = \cos \frac{1}{10}\pi, \cos \frac{3}{10}\pi, \cos \frac{7}{10}\pi, \cos \frac{9}{10}\pi$	M1 A1 A1 A1 [4]	Hence, so no marks for using quadratic at this stage.
8	(iii)	$16x^4 - 20x^2 + 5 = 0 \Leftrightarrow x^2 = \frac{20 \pm \sqrt{80}}{32}$ \cos decreases between 0 and π so $\cos \frac{1}{10}\pi$ is greatest root so $\cos \frac{1}{10}\pi = \sqrt{\frac{20 + \sqrt{80}}{32}} = \sqrt{\frac{5 + \sqrt{5}}{8}}$	B1 M1 A1 [3]	Can be gained if seen in (ii) Dep on full marks in (ii)

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