GCE

Mathematics

Unit **4726**: Further Pure Mathematics 2

Advanced GCE

Mark Scheme for June 2014

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
Cao	Correct answer only
Oe	Or equivalent
Rot	Rounded or truncated
Soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers ~~(?)~~ are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a ~~single~~ candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular ~~M or B~~ mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B ~~or B~~ mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader~~. [This may vary depending on the specification, strand and unit you are marking.]~~ .

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h ~~For a~~ *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. ~~(Note that a miscopy of the candidate's own working is not a misread but an accuracy error.)~~ A penalty is then applied; ~~of~~ 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.~~)~~

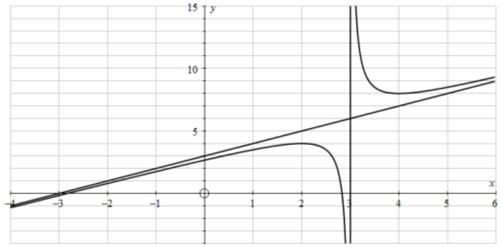
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$\int_0^2 \frac{1}{\sqrt{4+x^2}}\,\mathrm{d}x = \left[\sinh^{-1}\left(\frac{x}{2}\right)\right]_0^2$ $=\sinh^{-1}1-\sinh^{-1}0$ $=\ln\ 1+\sqrt{1+1}\ -0$ $=\ln\ 1+\sqrt{2}$ **cao isw**	**M1** **M1** **A1**	Standard form Use of log form and substitute limits dep on 1st M	
			Alternative: $\int_0^2 \frac{1}{\sqrt{4+x^2}}\,\mathrm{d}x = \left[\ln\ x+\sqrt{x^2+4}\ \right]_0^2$ $=\ln\ 2+\sqrt{8}\ -\ln 2$ $=\ln\ 1+\sqrt{2}$	**[3]** **M1** **M1** **A1**	 Standard form Substitute limits	

Question			Answer	Marks	Guidance	
2	(i)		$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$ $\Rightarrow \ln(1+x^2) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} \ldots$ Validity: $-1 \le x \le 1$ or $\lvert x \rvert \le 1$	**B1** **B1** **B1** **[3]**	2 or 3 terms correct unsimplified All terms correct	
	(ii)		$\ln(1+x^2) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} - \ldots$ Substitute $x = \frac{1}{2}$ $\Rightarrow \ln\left(\frac{5}{4}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2}\left(\frac{1}{2}\right)^4 + \frac{1}{3}\left(\frac{1}{2}\right)^6 - \frac{1}{4}\left(\frac{1}{2}\right)^8 + \ldots$ $= \frac{1}{4}\left(1 - \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{3}\left(\frac{1}{2}\right)^4 - \frac{1}{4}\left(\frac{1}{2}\right)^6 + \ldots\right)$ $\Rightarrow \left(1 - \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{3}\left(\frac{1}{2}\right)^4 - \frac{1}{4}\left(\frac{1}{2}\right)^6 + \ldots\right)$ $= 4\ln\left(\frac{5}{4}\right)$ **isw**	**M1** **A1** **[2]**	Sub $x = \frac{1}{2}$ into *their* ans to (i) Single ln expression	Alt: divide by x^2 then sub

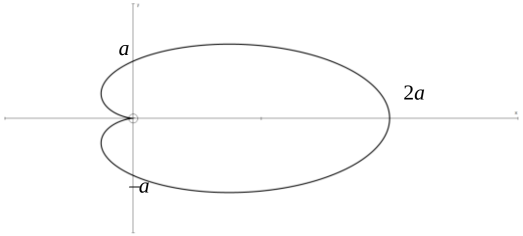
Question			Answer	Marks	Guidance	
3	(i)		Heights of rectangles = $\left(\frac{1}{2}\right)^3, \left(\frac{1}{3}\right)^3, \left(\frac{1}{4}\right)^3, \ldots, \left(\frac{1}{n}\right)^3$	M1	Heights, with at most one extra and/or one omitted	
			Width of rectangles $=1$			
			$\Rightarrow$ Sum of areas = $\left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3+\left(\frac{1}{4}\right)^3+\ldots+\left(\frac{1}{n}\right)^3$	A1	**isw**	No limits M0
			or $\sum_{r=2}^{n}\left(\frac{1}{r}\right)^3$ or $\sum_{r=1}^{n}\left(\frac{1}{r}\right)^3-1$	**[2]**		
	(ii)		Area = $\int_1^\infty \frac{1}{x^3}\,dx=\left[-\frac{1}{2x^2}\right]_1^\infty=\frac{1}{2}$	M1 A1	Integrate correct function: seen by x^2 in denominator	Or with upper limit of *n*
			Since sum of areas of rectangles approximates, but is less than, the area under the curve	M1	**www**	
			$\left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3+\left(\frac{1}{4}\right)^3+\ldots=\sum_{r=2}^{\infty}\left(\frac{1}{r^3}\right)<\frac{1}{2}$	M1	Compare *their* answer to (i) (taken to ∞) with *their* integral dep on 1st M	
			$\Rightarrow \sum_{r=1}^{\infty}\left(\frac{1}{r^3}\right)<\frac{1}{2}+1=\frac{3}{2}$	A1	Dealing with 1 Dep on previous 2 Ms	
				[5]		

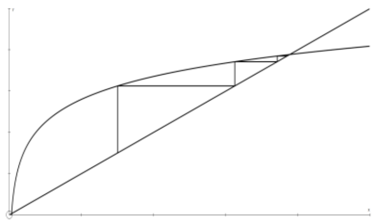
Question			Answer	Marks	Guidance	
4	(i)		For 1st curve $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$	**B1**		Alt: M1 Set up quadratic in sin or cos and solve A1 Both values correct
			For 2nd curve $\tan^{-1}\left(\sqrt{2}\times\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$	**B1**		
				[2]		
	(ii)		For 1st curve $y=\cos^{-1}x$, $\frac{dy}{dx}=\frac{-1}{\sqrt{1-x^2}}$	**B1**	**soi**	
			For 2nd curve $y=\tan^{-1}x$, $\frac{dy}{dx}=\frac{\sqrt{2}}{1+2x^2}$	**B1**	**soi**	
			For 1st curve, when $x=\frac{1}{\sqrt{2}}$, $\frac{dy}{dx}=\frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}}=\frac{-1}{\frac{1}{\sqrt{2}}}=-\sqrt{2}$	**M1**	Substituting value into *their* derivatives and using $m_1 \times m_2$ = ~(1) (i.e. evidence of finding the product of gradients)	
			For 2nd curve, when $x=\frac{1}{\sqrt{2}}$, $\frac{dy}{dx}=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$ Since $m_1\times m_2=-1$ then Yes	**A1**	Depends on exact correct numerical values being seen	Acceptable reason: One the negative reciprocal of the other. Condone: One the negative inverse of the other
				[4]		

Question			Answer	Marks	Guidance	
5	(i)		$y=\frac{x^2-8}{x-3}$ Vertical asymptote $x=3$	B1		
			$y=\frac{x^2-8}{x-3}=\frac{x^2-9+1}{x-3}=\frac{x-3\quad x+3\ +1}{x-3}$ $=x+3+\frac{1}{x-3}$	M1	Seen by an answer of $x+a+\left(\frac{b}{x-3}\right)$	Allow if fraction missing
			$\Rightarrow$ Oblique asymptote: $y=x+3$	A1 **[3]**	Condone incorrect *b*	
	(ii)		$xy-3y=x^2-8\Rightarrow x^2-xy+\ 3y-8\ =0$	M1	Attempt to get quad	Alternative: $\frac{\mathrm{d}y}{\mathrm{d}x}=1-\frac{1}{x-3\ ^2}$ $=0$ when $x-3\ ^2=1\Rightarrow x=2,4$ $x=2\Rightarrow y=4$ $x=4\Rightarrow y=8$ $\Rightarrow$ No values in range $4<y<8$
			Discriminant is $y^2-4\ 3y-8$	M1	Finding discriminant	
			$\Rightarrow y^2-12y+32<0\Rightarrow(y-8)(y-4)<0$	M1	Dealing with inequality to find result	
			$\Rightarrow 4<y<8$	A1 **[4]**		
	(iii)			B1	Asymptotes	x = 3 is identified and the other line has +ve gradient.
				B1 **[2]**	Correct shape	Must include a vertical and oblique (with +ve gradient) asymptotes and curve must approach them.

Question			Answer	Marks	Guidance	
6	(i)		$x=\cosh y=\dfrac{e^y+e^{-y}}{2}\Rightarrow e^y+e^{-y}=2x$	M1	Finding 3 term quadratic in e^y	
			$\Rightarrow e^{2y}-2xe^y+1=0$			
			$\Rightarrow e^y=\dfrac{2x\pm\sqrt{4x^2-4}}{2}=x\pm\sqrt{x^2-1}$	A1	Correct solution	Condone ignoring -ve sign at this point.
			$\Rightarrow y=\ln\left(x\pm\sqrt{x^2-1}\right)$			Condone interchange of *x* and *y* but final ans must be correct
			Reject $-$ sign as principal value taken	B1	Including reason **oe**	
			$\Rightarrow y=\ln\left(x+\sqrt{x^2-1}\right)$	A1		
				[4]		
	(ii)		$y=\ln\left(x+\sqrt{x^2-1}\right)\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{x+\sqrt{x^2-1}}\times\left(1+\dfrac{2x}{2\sqrt{x^2-1}}\right)$	M1	Alt: $x=\cosh y\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{\sinh y}$	
			$=\dfrac{1}{x+\sqrt{x^2-1}}\times\dfrac{x+\sqrt{x^2-1}}{\sqrt{x^2-1}}=\dfrac{1}{\sqrt{x^2-1}}$	A1	$=\dfrac{1}{\sqrt{x^2-1}}$	
				[2]		
	(iii)		$x=\cosh^{-1}3$	M1	Use of $\cosh^{-1}$	
			$=\ln\left(3+\sqrt{8}\right)$	A1		
			$=-\ln\left(3+\sqrt{8}\right)$ **oe**	A1	**ft**, -ve the first answer	
				[3]		

Question			Answer	Marks	Guidance	
7	(i)		$I_n = \int_0^{\pi/2} \sin^n x \, dx = I_n = \int_0^{\pi/2} \sin^{n-1} x \sin x \, dx$ $\Rightarrow I_n = \left[\sin^{n-1} x \times -\cos x\right]_0^{\pi/2} + \int_0^{\pi/2} \cos x (n-1) \sin^{n-2} x \cos x \, dx$ $= 0 + (n-1) \int_0^{\pi/2} \sin^{n-2} x \; 1 - \sin^2 x \; dx = (n-1) \; I_{n-2} - I_n$ $\Rightarrow nI_n = I_{n-2} \Rightarrow I_n = \frac{n-1}{n} I_{n-2}$	M1 M1 A1 [3]	Correct start for reduction Deal with $\cos^2$ dep on 1st M www	
	(ii)		$I_n = \frac{n-1}{n} I_{n-2} \Rightarrow I_{2n+1} = \frac{2n+1 \; -1}{2n+1} I_{2n-1} = \frac{2n}{2n+1} I_{2n-1}$ and $\frac{2n}{2n+1} < 1$ Alternative $I_n = \frac{n-1}{n} I_{n-2} . \frac{n-1}{n} < 1 \Rightarrow I_n < I_{n-2}$ for all $n \Rightarrow I_{2n+1} < I_{2n-1}$	M1 A1 [2]	Allow using n instead of $2n+1$	Alt: M1 $y = \sin^n x < y = \sin^{n-2} x$ in range A1 means that the area underneath is less and therefore... This can be argued one step at a time instead of 2
	(iii)		$I_{11} = \frac{256}{693}, \quad I_{10} = \frac{63}{512}.\pi, \quad I_9 = \frac{128}{315}$ $\Rightarrow \frac{256}{693} < \frac{63}{512}.\pi < \frac{128}{315}$ $\Rightarrow \frac{131072}{43659} < \pi < \frac{65536}{19845}$ $\Rightarrow 3.0022 < \pi < 3.3024$	B1 B1 M1 M1 A1 A1 [6]	For I_1 soi For I_0 soi Applying reduction formula for at least one of I_9 and I_{11} Applying reduction formula for I_{10} Lhs fraction or decimal equivalent correct to 4dp Likewise Rhs	Allow for pa. Both correct but both only to 3sf give A1 only

Question			Answer	Marks	Guidance	
8	(i)		$a(1+\cos\theta)=0$ when $\cos\theta=-1$	M1	soi	
			$\Rightarrow \theta=\pi$	A1	Only this answer: A0 if anything else	
				[2]		
	(ii)			B1	Correct shape, correct orientation, roughly symmetric	
				B1	All 3 intersections on axes indicated, cusp at pole dep on 1st B.	
				[2]		
	(iii)		$r=a(1+\cos\theta)$	M1	Use of formula with limits	
			$A=\frac{1}{2}\int_0^{2\pi} r^2\,\mathrm{d}\theta=\frac{1}{2}\int_0^{2\pi} a^2(1+\cos\theta)^2\,\mathrm{d}\theta$			
			$=\frac{a^2}{2}\int_0^{2\pi}(1+2\cos\theta+\cos^2\theta)\,\mathrm{d}\theta$	A1	Condone omission of a^2	
			$=\frac{a^2}{2}\int_0^{2\pi}(1+2\cos\theta+\frac{1}{2}(\cos 2\theta+1))\,\mathrm{d}\theta$	M1	Dealing with $\cos^2$	
				A1	Condone omission of a^2	
			$=\frac{a^2}{2}\left[\theta+2\sin\theta+\frac{1}{2}\left(\frac{1}{2}\sin 2\theta+\theta\right)\right]_0^{2\pi}=\frac{a^2}{2}\left(2\pi+0+\frac{1}{2}(0+2\pi)\right)$	M1	Substitute limits dep on 2nd M	
			$=\frac{a^2}{2}(3\pi)=\frac{3\pi a^2}{2}$	A1		
				[6]		

Question			Answer	Marks	Guidance	
9	(i)		3.8 3.868001 3.868001 3.882190 3.882190 3.885120 3.885120 3.885723 3.885723 3.885847 3.885847 3.885873 3.885873 3.885878 Root = 3.88588	**M1** **A1** **A1** **A1** **[4]**	 For x_2 For x_3	N.B. Working must be seen
	(ii)			**B1** **B1** **[2]**	Curve and line Iterations showing staircase from below. At least two seen	Concave curve initially above $y = x$ Only [3,4] required so ignore behaviour at origin
	(iii)		3.8 3.868001 0.068001 $\square_1$ 3.868001 3.88219 0.014189 $\square_2$ 3.88219 3.88512 0.002929 $\square_3$ 3.88512 3.885723 0.000603 3.885723 3.885847 0.000124 3.885847 3.885873 3.885873 3.885878 $\square_3$ = 0.00293	 **M1** **A1** **[2]**	 Working differences Anything that rounds to 0.00293	

Question			Answer	Marks	Guidance	
	(iv)		$g'(\alpha)=\dfrac{0.8}{3.88588}=0.20587$ $g'(\alpha)^{n-1}<10^{-6}$ $\Rightarrow\ n-1\ \log 0.20587<\log 10^{-6}$ $\Rightarrow n-1>\dfrac{6}{.68640}=8.74...$ $\Rightarrow n>9.74....$ i.e. least $n=10$	M1 A1 M1 A1 [4]	Attempt to find g' by differentiating g(x) correctly. Condone = Take logs If = has been used then the answer must include a justification	S.C. by successive evaluations B4 S.C. answer only seen B2 If ans wrong: M1 for g', M1 for successive multiplication by g' Or: M1 for continuation of table to find d4, d5, etc and a comparison with 10^{-6}d1

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