OCR
Oxford Cambridge and RSA

Tuesday 26 June 2018 – Morning

A2 GCE MATHEMATICS

4731/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4731/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

1 A uniform rectangular lamina, of mass m kg and sides 1.2 m and 0.6 m, is rotating about a fixed vertical axis which is perpendicular to the lamina and passes through its centre. A stationary particle of mass 2 kg becomes attached to the lamina at one of its corners, and this causes the angular speed of the lamina to change instantaneously from $4\,\text{rad s}^{-1}$ to $\frac{1}{8}m\,\text{rad s}^{-1}$.

(i) Find m. **[4]**

The lamina then slows down with constant angular deceleration. The lamina turns through 52 radians as its angular speed reduces from $\frac{1}{8}m\,\text{rad s}^{-1}$ to zero.

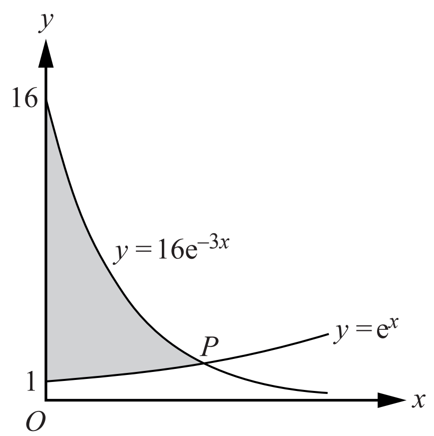
(ii) Find the time taken for the lamina to come to rest. **[2]**

2 Plane A is flying with constant speed $520\,\text{km h}^{-1}$ on a course with bearing 060°. Plane B is flying at the same altitude as A with constant speed $1010\,\text{km h}^{-1}$ on a course with bearing 310°. At 9 am the planes are 450 km apart with B on a bearing of 110° from A.

(i) Find the shortest distance between A and B in the subsequent motion. **[6]**

(ii) Find the time when A and B are at the point of closest approach. **[3]**

3



The diagram shows the curves $y = e^x$ and $y = 16e^{-3x}$, which intersect at the point P. The shaded region, bounded by the two curves and the y-axis, is occupied by a uniform lamina.

(i) Show that the x-coordinate of P is $\ln 2$. **[2]**

(ii) Find the x-coordinate of the centre of mass of the lamina, giving your answer in the form $a + b\ln 2$ where the values of a and b are to be stated as exact fractions. **[8]**

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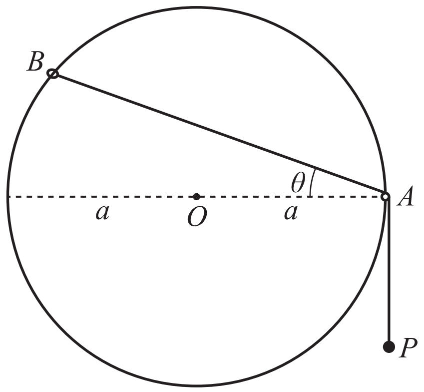
4 A uniform circular hoop has mass $4m$ and radius $2a$. The points A and B are at opposite ends of a diameter of the hoop. The hoop is free to rotate in a vertical plane about a fixed horizontal axis passing through A. A particle of mass m is attached to the hoop at B. The hoop is released from rest with AB horizontal and its rotation is opposed by a frictional couple of magnitude $kmga$, where k is a positive constant. At time t, before the hoop first comes to instantaneous rest, the angle turned through by AB is θ.

(i) Show that at time t the angular acceleration of the hoop is given by $\frac{g}{48a}(12\cos\theta - k)$. **[6]**

(ii) Given that the hoop first comes to instantaneous rest when $\theta = \frac{5}{6}\pi$, find, in terms of a and g, the angular acceleration of the hoop at the first instant when $\theta = \frac{1}{3}\pi$. **[4]**

5



A smooth circular wire, with centre O and radius a, is fixed in a vertical plane, and the point A is on the wire at the same horizontal level as O. A small ring B of mass m can move freely on the wire. A light inextensible string of length l, where $l > 2a$, has one end attached to B. The string passes over a small smooth pulley at A and carries at its other end a particle P of mass λm, where λ is a positive constant. The part AP of the string is vertical and the part AB of the string makes an angle θ radians with the horizontal, where $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$ (see diagram). You may assume that the string does not become slack.

(i) Taking A as the reference level for gravitational potential energy, show that the total potential energy V of the system is given by

$$V = 2mga\cos\theta(\lambda + \sin\theta) + k,$$

expressing k in terms of λ, g, l and m. **[4]**

It is given that $\lambda = \frac{1}{6}$.

(ii) Show that there are two possible positions of equilibrium. **[4]**

(iii) By considering the values of $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}$, determine whether these two positions are stable or unstable. **[6]**

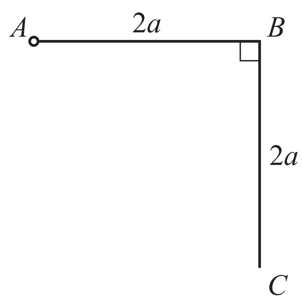
6 The region bounded by the curve $y = \frac{2a^2}{x}$ for $a \leqslant x \leqslant 2a$ (where a is a positive constant), the x-axis, and the lines $x = a$ and $x = 2a$, is rotated through 2π radians about the x-axis to form a uniform solid of revolution of mass m.

(i) Show that the moment of inertia of this solid about the x-axis is $\frac{7}{6}ma^2$. **[7]**

The solid is free to rotate about a fixed horizontal axis along the line $y = ka$.

(ii) Given that the solid makes small oscillations of approximate period $\frac{\pi}{3}\sqrt{\frac{83a}{g}}$ about this axis, find the possible values of k. **[4]**

7



A compound pendulum consists of two uniform rods AB and BC, each of length $2a$ and mass m. The rods are rigidly joined together so that AB is perpendicular to BC. The pendulum is freely hinged to a fixed point at A. The pendulum can rotate in a vertical plane about a smooth fixed horizontal axis through A (see diagram).

(i) Show that the moment of inertia of the pendulum about the axis of rotation is $\frac{20}{3}ma^2$. **[3]**

The pendulum is released from rest in the position with B vertically above C.

(ii) Find the vertical component of the force exerted by the axis on the pendulum as the pendulum passes through its equilibrium position. Give your answer as an exact multiple of mg. **[9]**

END OF QUESTION PAPER

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