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Oxford Cambridge and RSA

AS/A LEVEL GCE

Examiners' report

MATHEMATICS

3890-3892, 7890-7892

4727/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4727/01 series overview

This paper is one of the optional pure papers for the Further Maths legacy examination.

Most candidates demonstrated competence in applying the standard methods necessary for this paper, although a significant minority appeared unfamiliar with the basics of group theory. Many candidates benefitted from the ability to keep track of lengthy mathematical arguments; stronger ones also showed the ability to communicate concisely. Questions that required both good communication and problem solving skills gave the very strongest candidates the opportunity to demonstrate their grasp of the subject.

Group Theory continues to cause candidates a disproportionate amount of difficulties compared to other topics. Part of this is due to the required rigour of logic and communication, but there was evidence also of an element of unfamiliarity with basic concepts among some candidates. Furthermore, candidates rarely seem to 'show familiarity with the structure of finite groups up to order 7', as the specification requires.

Key point

Candidates should develop methods to minimise trivial calculation errors and transcription errors, since these can often result in the later marks for a question becoming inaccessible. In particular, showing workings at each stage means that such errors will often only result in loss of accuracy marks.

There was some evidence that a few candidates found difficulty in completing the paper in the time available.

Question 1 (i)

1 **(i)** Find the shortest distance from the point $(3, -1, -2)$ to the plane with equation $x - 2y + 4z = 11$. **[2]**

Compared to the previous sitting of this paper, more candidates used the standard formula for perpendicular distance from a point to a plane, which is given in the List of Formulae (MF1). Those who did were mostly successful although some got the sign of '*d*' wrong. Using other (lengthier) methods such as finding the parameter for the base of the perpendicular worked well for some candidates, but others made numerical errors or did not have a complete method to hand.

Question 1 (ii)

(ii) Find a cartesian equation of the plane which passes through the point $(3, -1, -2)$ and is parallel to the plane $x - 2y + 4z = 11$. **[2]**

This question was well answered by most candidates, although a few missed out on full marks due to neglecting to give their answer in cartesian form (as requested).

Question 2 (i)

2 A multiplicative group G consists of the elements $\{1, z, z^2, z^3, z^4, z^5\}$.

(i) State the order of the element z^4. **[1]**

While determining the order of z^4 was straightforward for most candidates, there were a significant minority who appeared unaware of how to approach this and unfamiliar with the basic structure of $\mathbb{Z}_6$.

Question 2 (ii)

(ii) List all the subgroups of G. **[3]**

Most candidates were able to give at least two correct subgroups; although there still appears some confusion as to whether trivial subgroups should be included (they should, unless explicitly excluded by the demand). Common errors were to include the set $\{1,z,z^5\}$, or to give a list of all sets of the form $\{1,g\}$, $g \in G$.

Question 2 (iii)

The group H consists of the set $\{1, 2, 3, 4, 5, 6\}$ with the operation of multiplication modulo 7.

(iii) Determine whether G is isomorphic to H. **[2]**

In determining whether the groups were isomorphic, the majority of candidates determined the orders (1,6,3,2,3,6) of the elements of G and (1,3,6,3,6,2) of *H* and then asserted that the two groups are isomorphic since the orders match up. As highlighted in last year's report to centres, this argument is only true for small groups so these candidates did not gain full marks unless they went on to discuss the only two possible groups of order six. Those candidates who stated that the groups were isomorphic and supported this with a correct explicit isomorphism between *G* and *H* were given full credit. There was evidence that some candidates were unaware that an isomorphism is not simply mapping where the orders of the elements match up.

Correct isomorphic mappings	***Mappings that do not form an Isomorphism despite the orders of elements being matched up***
$1 \to 1$ $z \to 3$ $z^2 \to 2$ $z^3 \to 6$ $z^4 \to 4$ $z^5 \to 5$	$1 \to 1$ $z \to 5$ $z^2 \to 2$ $z^3 \to 6$ $z^4 \to 4$ $z^5 \to 3$
$1 \to 1$ $z \to 5$ $z^2 \to 4$ $z^3 \to 6$ $z^4 \to 2$ $z^5 \to 3$	$1 \to 1$ $z \to 3$ $z^2 \to 4$ $z^3 \to 6$ $z^4 \to 2$ $z^5 \to 5$

Besides demonstrating the cyclic nature of *H* (by identifying generator), the easiest way to gain full marks was to reference some unique characteristic of the two groups of order six. Therefore, to answer that both groups are abelian by definition and that there is only one abelian group of order 6 was a concise, sufficient answer. Those who drew up group tables rarely made any progress.

Question 3

3 It is given that the differential equation

$$2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 3y = 10e^{-x}$$

has a particular integral of the form axe^{-x}, where a is a constant. Solve the differential equation subject to the conditions $y = 0$ and $\frac{dy}{dx} = -\frac{9}{2}$ when $x = 0$. **[10]**

Most candidates had been well prepared for solving second order differential equations and their arguments were clearly presented. Most of the marks dropped were due to accuracy marks and sign errors rather than method ones. A few candidates incorrectly attempted to use the boundary conditions before finding the general solution. A small number did not use the product rule when differentiating axe^{-x}. Just about all candidates who reached a particular solution were careful to include '$y =$' in this solution, the absence of which had been noted in past reports.

Question 4 (i)

4 The operation $*$ is defined by $x * y = xy + k(x + y) + 12$, where x and y are real numbers and k is a real constant. It is given that the operation $*$ is associative.

(i) Show that there are two possible values for k, one of which is 4. **[4]**

The majority of candidates were familiar with the concept of associativity and this question produced some excellent examples of well-communicated extended mathematical argument. In particular, many candidates were able to correctly expand $x*(y*z)$ and almost all of these had a serious attempt at simplifying the ensuing equation. The very best answers justified their progression, for example from $k^2x - kx - 12x = k^2z - kz - 12z$ either by factorising to get $(x - z)(k^2 - 12k - 12) = 0$ and/or by explicitly stating that 'since z and x can take any values', so $k^2 - 12k - 12 = 0$.

A small number used particular numerical values for a, b, c that, while leading to the correct values of k, fails to establish that these work in general. Other considered less general identities, such as $a*(b*b) = (a*b)*b$ that had the same drawback. A few candidates falsely assumed that a group had already been defined; instead of using the property of associativity, they erroneously assumed the existence of an identity element leading them to make no valid progress at all.

Question 4 (ii)

(ii) In the case where $k = 4$, determine whether the set of real numbers, under the operation $*$, forms a group. **[4]**

Familiarity with the criteria for a set to be a group under an operation was evident in most scripts, with many candidates explicitly considering associativity, closure, identity and inverse. Many candidates attempted to find a unique value for *e* but some of these did not spot how to eliminate *x* from their expression for *e*. Where that was the case, some drew the (not unreasonable, but invalid) conclusion that there was no (unique) identity element, where others ignored the problem of *e* equalling a variable function.

Those who found $e = -3$ were often successful in finding x^{-1} as a function of *x*. However, not all of these noticed the significance of the fact that the denominator could potentially equal zero, so they went on to (mistakenly) claim both that inverses existed and that a group was formed.

Exemplar 1

Closure: group is closed since
$xy + 4x + 4y + 12$ is real when $\{x, y \in \mathbb{R}\}$

associativity is know

Identity $xy + 4x + 4y + 12 = x$

$xy + 4y = -(12 + 3x)$

$y(x+4) = -(12+3x)$ M1

$y = \frac{-(12+3x)}{x+4} = -3$ A1

Inverse $xy + 4x + 4y + 12 = -3$

$xy + 4y = -4x - 15$

$y(x+4) = -(4x+15)$ M1

$y = \frac{-(4x+15)}{x+4}$

there is no inverse for when $x = -4$ A1

$\therefore$ not a group.

Exemplar 1 gives an example of a good answer worthy of full marks

Question 5

5 The differential equation

$$\frac{dy}{dx} + \frac{2y}{1-x} = 4(1-x^2)\sqrt{y}$$

is to be solved for $x < 1$. Use the substitution $u = \sqrt{y}$ to find the general solution of the differential equation, expressing your answer in the form $y = f(x)$. **[8]**

Most candidates were able to form a correct differential equation in *x* and *u*, but from that point on there were a wide variety of levels of response. Most candidates appeared aware of the structure that their workings should take and attempted to find an integrating factor (I), multiplied all terms by their I and assumed that their LHS was equal to the derivative of *u* times their I. They then attempted to integrate their RHS, although if their integrating factor was incorrect the method would have broken down.

The commonest errors included:

- omitting to simplify down to the form $\frac{du}{dx} + Pu = Q$ before attempting to find an integrating factor,
- inability to correctly integrate $\frac{1}{1-x}$,
- inability to simplify $\frac{1-x^2}{1-x}$, leading to difficulties in integrating,
- transcription errors.

Omission of a constant of integration was rare and the final stage of substitution back to form an equation of the form *y = f(x)* was generally done well.

Question 6 (i)

6 **(i)** Use de Moivre's theorem to find an expression for $\cot 7\theta$ in terms of $\cot\theta$ and hence find the exact roots of the equation $u^6 - 21u^4 + 35u^2 - 7 = 0$. **[7]**

The first part of this type of question is frequently posed with the command phrase 'show that'. Since candidates here were asked simply to find an expression for $\cot 7\theta$, a less rigorous approach could be taken. The question required the use of de Moivre's theorem however, so any candidate who merely wrote down the required formula without some evidence of de Moivre's use could not be credited marks. The best answers started with a reference to the fact that $\cos 7\theta + i\sin 7\theta = (\cos\theta + i\sin\theta)^7$ whether in this form or in relation to real and imaginary parts, before applying this fact to the question in hand.

Many candidates then realised that the next step should be to set $\cot 7\theta$ equal to zero, but not all solved this correctly and some appeared to believe that if $\cot 7\theta = 0$, then $\tan 7\theta = 0$ too. Among those who did solve this correctly, some stopped with solely the angles as their proposed roots, while some gave roots in the correct form but omitted one of them and/or included $\cot\frac{7\pi}{14}$. Others completed the question successfully and gave exactly six correct roots.

Key point

A clear understanding of the command words within Further Maths examination papers can improve candidates' chances of accessing all marks and equally of not wasting time with excessive detail.

Exemplar 2

$\cot 7\theta = \frac{\cos 7\theta}{\sin 7\theta}$

$\cos 7\theta + i\sin 7\theta = (\cos\theta + i\sin\theta)^7$ ✓

~~cos7θ~~ Let $\cos\theta = c$ and $\sin\theta = s$

$\therefore \cos 7\theta + i\sin 7\theta = c^7 + {}^7C_1 c^6 si + {}^7C_2 c^5 (is)^2 + {}^7C_3 c^4 (is)^3 + {}^7C_4 c^3 (is)^4 + {}^7C_5 c^2 (is)^5 + {}^7C_6 c(is)^6 + (is)^7$

$\cos 7\theta + i\sin 7\theta = c^7 + 7ic^6 s - 21c^5 s^2 - 35ic^4 s^3 + 35c^3 s^4 + 21ic^2 s^5 - 7cs^6 - is^7$ ✓

~~cos7θ~~ equating real parts: $\cos 7\theta = c^7 - 21c^5 s^2 + 35c^3 s^4 - 7cs^6$ ✓

equating imaginary parts: $\sin 7\theta = 7c^6 s - 35c^4 s^3 + 21c^2 s^5 - s^7$ ✓

$\therefore \frac{\cos 7\theta}{\sin 7\theta} = \cot 7\theta = \frac{c^7 - 21c^5 s^2 + 35c^3 s^4 - 7cs^6}{7c^6 s - 35c^4 s^5 + 21c^2 s^5 - s^7}$

$\cot 7\theta = \frac{\cos^7\theta - 21\cos^5\theta\sin^2\theta + 35\cos^3\theta\sin^4\theta - 7\cos\theta\sin^6\theta}{7\cos^6\theta\sin\theta - 35\cos^4\theta\sin^3\theta + 21\cos^2\theta\sin^5\theta - \sin^7\theta}$

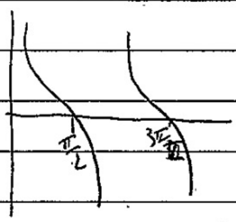
~~$\cot 7\theta = \cot^7\theta - 21\cot^2\theta + 35$~~

(continued) $\therefore \cot 7\theta = \frac{\cot^7\theta - 21\cot^5\theta + 35\cot^3\theta - 7\cot\theta}{7\cot^6\theta - 35\cot^4\theta + 21\cot^2\theta - 1}$ ✓

$\div \sin^7\theta$

Let $\cot\theta = u$ $\therefore$ $\cot 7\theta = \frac{u^7 - 21u^5 + 35u^3 - 7u}{7u^6 - 35u^4 + 21u^2 - 1}$

$\therefore \cot 7\theta = 0$



$\therefore 7\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2} \ldots$ ✓

$\theta = \frac{\pi}{14}, \frac{3\pi}{14}, \frac{5\pi}{14}, \frac{7\pi}{14}, \frac{9\pi}{14}, \frac{11\pi}{14}, \frac{13\pi}{14}$

$\cot\left(\frac{7\pi}{14}\right) = 0$ 0 is not a root

$\therefore$ exact roots of the equation

$= \cot\left(\frac{\pi}{14}\right), \cot\left(\frac{3\pi}{14}\right), \cot\left(\frac{5\pi}{14}\right), \cot\left(\frac{9\pi}{14}\right), \cot\left(\frac{11\pi}{14}\right), \cot\left(\frac{13\pi}{14}\right)$ ✓

Exemplar 2 gives an example of a good answer worthy of full marks.

Question 6 (ii)

(ii) State the exact roots of the equation $v^3 - 21v^2 + 35v - 7 = 0$, justifying your answer. Hence find the exact value of

$$\frac{\cot^2\left(\frac{1}{14}\pi\right)\cot^2\left(\frac{3}{14}\pi\right)+\cot^2\left(\frac{3}{14}\pi\right)\cot^2\left(\frac{5}{14}\pi\right)+\cot^2\left(\frac{5}{14}\pi\right)\cot^2\left(\frac{1}{14}\pi\right)}{\cot\left(\frac{1}{14}\pi\right)\cot\left(\frac{3}{14}\pi\right)\cot\left(\frac{5}{14}\pi\right)}.$$ **[4]**

The majority of candidates made little or no attempt at this part, seeming unable to make the connection with part (i). Of those who did give the correct three roots, not all adequately explained why the other three possibilities were repeats. A very small number recognised the given expression as $\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\sqrt{\alpha\beta\gamma}}$ and obtained $\frac{35}{\sqrt{7}}$. The commonest error here was to omit the square root in the denominator.

Question 7 (i)

7 The plane Π_1 passes through the points $(5, 2, -2)$, $(4, 0, -1)$ and $(2, 1, -3)$.

(i) Find a cartesian equation of the plane Π_1. **[5]**

Almost all candidates appeared familiar with one of the methods for deriving the equation of a plane through 3 (non-collinear) points. A number lost marks through simple numerical errors, however.

Using the cross product of two direction vectors to find ***n*** was the most common approach and the one least prone to difficulties. A small number expressed the plane in parametric form and then attempted to eliminate the two parameters. Even fewer substituted the points into the general form $ax + by + cz = \mathrm{d}$ and tried to solve 3 equations in 4 unknowns.

The occasional candidate confused the cartesian equation for a plane with that for a line. Good answers reduced the risk of loss of marks by showing their working for each stage (finding vectors parallel to plane, performing the cross (vector) product and substituting a point into the plane with discovered normal).

Question 7 (ii)

The line l_1 has equation $\frac{x}{2} = \frac{y-4}{-1} = \frac{z+3}{3}$.

(ii) Find the acute angle between Π_1 and l_1. **[3]**

The angle required was usually correctly found. The only notable common error was to give the angle between line and normal rather than between line and plane. A few candidates when finding angles in this paper repeatedly used the lengthier method of using the vector product rather than the scalar product and, although this is acceptable, centres may wish to stress the time-saving benefits of the latter.

Question 7 (iii)

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} p \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} q \\ -6 \\ 12 \end{pmatrix}$ and lies in Π_1.

(iii) Find the value of p and show that $q = 12$. **[3]**

The majority of candidates found *p* by substituting $(p, 2, 4)$ into the plane equation and then set the scalar product of $(q, -6, 12)$ with the normal equal to zero to find *q*. Others substituted the general point into the plane and compared coefficients of λ and the constant, while yet others used two particular points, usually with $\lambda = 0, 1$.

Question 7 (iv)

The plane Π_2 is perpendicular to Π_1 and l_2 lies in Π_2.

(iv) Find an equation of Π_2, giving your answer in the form $\mathbf{r} . \mathbf{n} = d$. **[3]**

A fair number of correct answers were seen, the usual method being to calculate the required normal as $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} = \begin{pmatrix} 13 \\ 16 \\ -5 \end{pmatrix}$. Some tried to find the normal direction by solving $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, not appreciating that the solution to this is not unique. Some candidates lost a mark by giving the cartesian equation only. Others used a value for *q* they had derived, rather than the one given in part (iii).

Question 8 (i)

8 **(i)** Show that, if $z \neq \pm 1$ and $z \neq 0$,

$$\sum_{r=1}^{n} z^{2r-1} = \frac{1 - z^{2n}}{z^{-1} - z}.$$ **[2]**

Some candidates lost marks because they gave insufficient working to demonstrate their understanding of the route to the given answer via the formula for the sum of a geometric series. Candidates who wrote out the terms in the series were more likely to correctly identify the first term and the ratio. A number mistakenly thought that the first term was 1 or z^{-1}, or that the common ratio was *z*.

A small number of candidates were credited full marks for deriving the expression from first principles using a 'method of differences' approach.

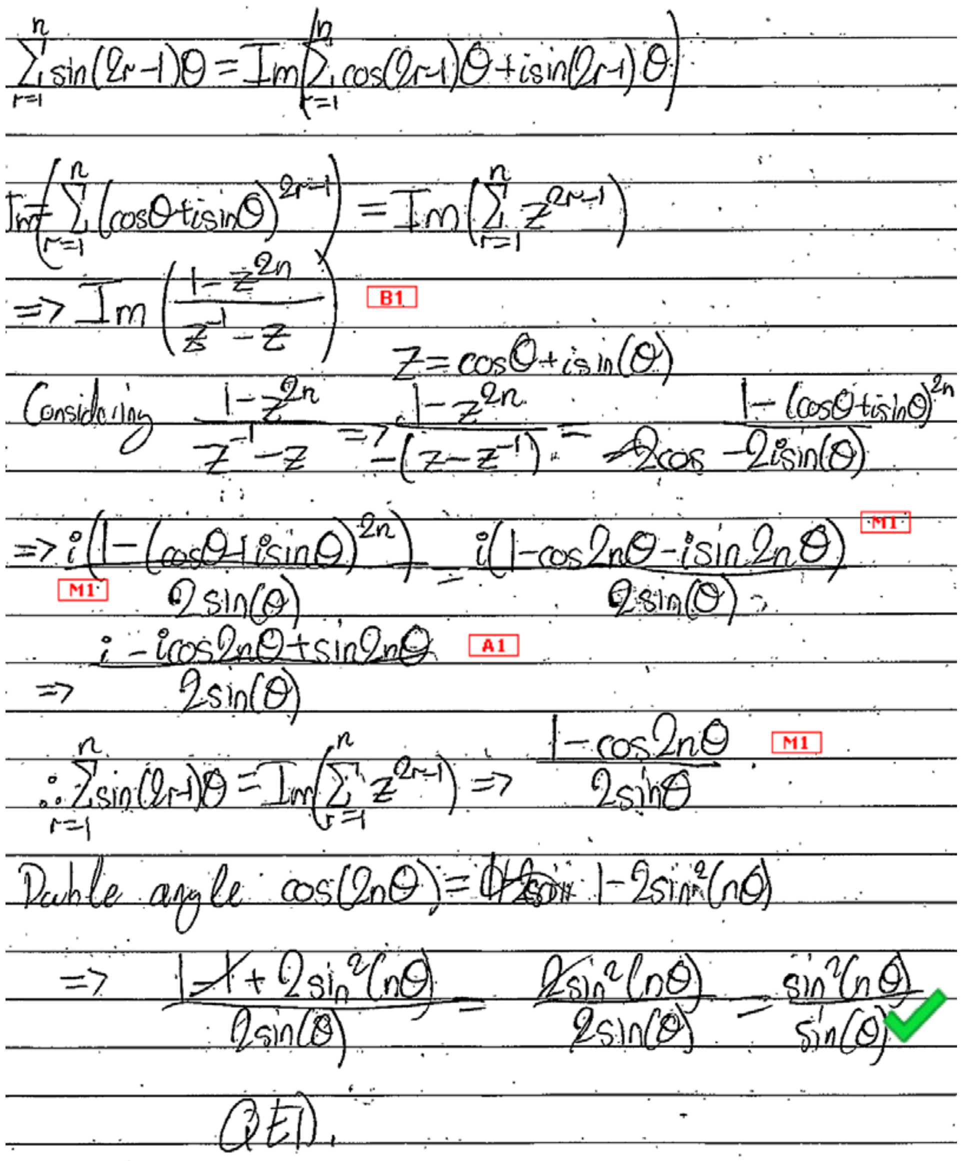
Question 8 (ii)

(ii) Hence show that, if $\sin\theta \neq 0$,

$$\sum_{r=1}^{n} \sin(2r-1)\theta = \frac{\sin^2 n\theta}{\sin\theta}. \qquad [6]$$

This question provided an opportunity for the strongest candidates to demonstrate their abilities. This they did with a variety of effective approaches (most of which can be seen in the mark scheme). The exemplar below gives one example.

Exemplar 3



$$\sum_{r=1}^{n} \sin(2r-1)\theta = \mathrm{Im}\left(\sum_{r=1}^{n} \cos(2r-1)\theta + i\sin(2r-1)\theta\right)$$

$$\mathrm{Im}\left(\sum_{r=1}^{n} (\cos\theta + i\sin\theta)^{2r-1}\right) = \mathrm{Im}\left(\sum_{r=1}^{n} z^{2r-1}\right)$$

$$\Rightarrow \mathrm{Im}\left(\frac{1-z^{2n}}{z^{-1}-z}\right) \quad \text{B1} \qquad z = \cos\theta + i\sin(\theta)$$

Considering $\dfrac{1-z^{2n}}{z^{-1}-z} \Rightarrow \dfrac{1-z^{2n}}{-(z-z^{-1})} = \dfrac{1-(\cos\theta+i\sin\theta)^{2n}}{-2i\sin(\theta)}$

$$\Rightarrow \frac{i\left(1-(\cos\theta+i\sin\theta)^{2n}\right)}{2\sin(\theta)} \ \text{M1} = \frac{i(1-\cos 2n\theta - i\sin 2n\theta)}{2\sin(\theta)} \ \text{M1}$$

$$\Rightarrow \frac{i - i\cos 2n\theta + \sin 2n\theta}{2\sin(\theta)} \quad \text{A1}$$

$$\therefore \sum_{r=1}^{n} \sin(2r-1)\theta = \mathrm{Im}\left(\sum_{r=1}^{n} z^{2r-1}\right) \Rightarrow \frac{1-\cos 2n\theta}{2\sin\theta} \quad \text{M1}$$

Double angle $\cos(2n\theta) = 1 - 2\sin^2(n\theta)$

$$\Rightarrow \frac{1 - 1 + 2\sin^2(n\theta)}{2\sin(\theta)} = \frac{2\sin^2(n\theta)}{2\sin(\theta)} = \frac{\sin^2(n\theta)}{\sin(\theta)} \ ✓$$

QED.

There were also however many who were unable to even find their way into the problem. Others could see how to use the imaginary part of (i) and how to apply de Moivre's theorem, but then struggled to convert their expression into a form in which it could easily have the imaginary part identified. The commonest error was to gloss over the problem caused by a term in *i* in the denominator.

Question 8 (iii)

(iii) Hence find the exact value of

$$\int_0^{\frac{1}{6}\pi} \frac{\sin^2 3\theta}{\sin\theta}\,\mathrm{d}\theta. \qquad \textbf{[3]}$$

Some candidates made no attempt at this question. Others did not make use of the result of (ii) and hence could not integrate. Of those who did, many scored at least the first two marks, though there were some calculation errors in the substitution.

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