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Examiners' report

MATHEMATICS

3890-3892, 7890-7892

4731/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners’ reports are produced to offer constructive feedback on candidates’ performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates’ performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4731/01 series overview

The work on this paper was generally of a very high standard. Many of the candidates were very competent and demonstrated a sound understanding of the principles of mechanics covered in this module. However, a small number of candidates struggled with the majority of the paper and were not able to apply principles appropriate to the situations. Candidates seemed to be particularly confident this series on solving problems regarding relative velocity, applying the principle of conservation of mechanical energy and using energy to investigate stability of equilibrium. The topics which provided more of a challenge this series included the work done by a frictional couple and finding the force exerted on a body by the axis of rotation. Candidates appeared to have sufficient time to complete the paper.

The standards of presentation and communication were high, though some candidates did not include necessary detail when establishing given answers.

Question 1(i)

1 A uniform rectangular lamina, of mass m kg and sides 1.2 m and 0.6 m, is rotating about a fixed vertical axis which is perpendicular to the lamina and passes through its centre. A stationary particle of mass 2 kg becomes attached to the lamina at one of its corners, and this causes the angular speed of the lamina to change instantaneously from $4\,\text{rad s}^{-1}$ to $\frac{1}{8}m\,\text{rad s}^{-1}$.

(i) Find m. **[4]**

This question was answered extremely well with the vast majority of candidates correctly applying the principle of the conservation of angular momentum to find *m*. The most common error was in the failure to correctly calculate, using the parallel axis theorem, the moment of inertia of the rectangular lamina (about the given axis of rotation) after the particle became attached. More worryingly, were the small minority of candidates who attempted to use an energy approach.

Question 1(ii)

The lamina then slows down with constant angular deceleration. The lamina turns through 52 radians as its angular speed reduces from $\frac{1}{8}m\,\text{rad s}^{-1}$ to zero.

(ii) Find the time taken for the lamina to come to rest. **[2]**

Although the most efficient method in this part was to use $\theta = \frac{1}{2}(\omega_0 + \omega_1)t$ some candidates initially calculated the angular acceleration before correctly finding the time taken for the lamina to come to rest.

Question 2(i)

2 Plane A is flying with constant speed $520\,\text{km h}^{-1}$ on a course with bearing 060°. Plane B is flying at the same altitude as A with constant speed $1010\,\text{km h}^{-1}$ on a course with bearing 310°. At 9 am the planes are 450 km apart with B on a bearing of 110° from A.

(i) Find the shortest distance between A and B in the subsequent motion. **[6]**

Relative velocity remains a difficult and challenging topic for some and a number of candidates left both parts of this question blank. However, it was pleasing to note that there were a significant number of candidates who answered both parts of this question correctly. The most succinct and efficient solutions in this part were from those candidates who applied both the cosine and sine rules. The most common error was using incorrect angle(s) and it was noticeable that a number of candidates incorrectly summed 60 and 50.

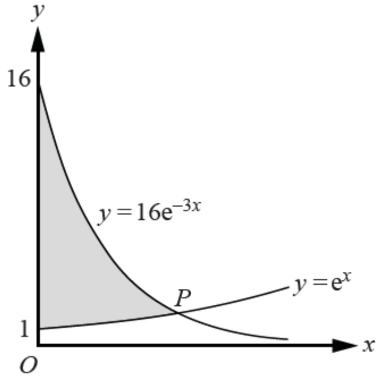
Question 2(ii)

(ii) Find the time when A and B are at the point of closest approach. **[3]**

In 2 (ii) a significant number of candidates used the sine of the correct angle rather than the cosine and a number of candidates used the shortest distance from part (i) in their attempt to work out the time to closest approach. Finally, many did not give the correct time in context (which, if correct should have been given as 9.21 am) when planes *A* and *B* were at their point of closest approach.

Question 3(i)

3



The diagram shows the curves $y = e^x$ and $y = 16e^{-3x}$, which intersect at the point P. The shaded region, bounded by the two curves and the y-axis, is occupied by a uniform lamina.

(i) Show that the x-coordinate of P is $\ln 2$. **[2]**

While the vast majority of candidates correctly showed that the *x*-coordinate of *P* was ln 2 by first stating $16e^{-3x} = e^x$ and then taking logs accurately, a number of candidates did not show insufficient working, for example, $x = \ln 16 - 3x \Rightarrow x = \ln 2$.

Question 3(ii)

(ii) Find the x-coordinate of the centre of mass of the lamina, giving your answer in the form $a + b\ln 2$ where the values of a and b are to be stated as exact fractions. **[8]**

Most candidates found the exact value of the *x*-coordinate of the centre of the mass of the lamina bounded between the two curves correctly. Two approaches were generally seen, the first was to find the centre of mass of the lamina by considering the difference between the two curves, and the second was to consider the centres of masses of the two laminae separately. The former approach was generally more successful. Most candidates were proficient in carrying out the integration by parts although sign errors were seen in the integration of the exponential terms.

Question 4(i)

4 A uniform circular hoop has mass $4m$ and radius $2a$. The points A and B are at opposite ends of a diameter of the hoop. The hoop is free to rotate in a vertical plane about a fixed horizontal axis passing through A. A particle of mass m is attached to the hoop at B. The hoop is released from rest with AB horizontal and its rotation is opposed by a frictional couple of magnitude $kmga$, where k is a positive constant. At time t, before the hoop first comes to instantaneous rest, the angle turned through by AB is θ.

(i) Show that at time t the angular acceleration of the hoop is given by $\frac{g}{48a}(12\cos\theta - k)$. **[6]**

This part was answered extremely well with the majority of candidates correctly starting the problem by first finding the moment of inertia of the combined hoop and particle about *A* as $4m(2a)^2 + 4m(2a)^2 + m(4a)^2 = 48ma^2$. Most candidates then went on to correctly apply the rotational form of Newton's second law by either considering the moment of the hoop and particle separately or combining the two and calculating the centre of mass of the combined body from *A* as $\frac{12a}{5}$.

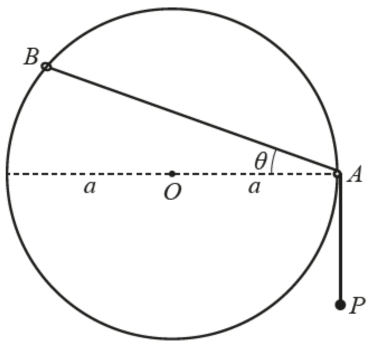
Question 4(ii)

(ii) Given that the hoop first comes to instantaneous rest when $\theta = \frac{5}{6}\pi$, find, in terms of a and g, the angular acceleration of the hoop at the first instant when $\theta = \frac{1}{3}\pi$. **[4]**

Candidates struggled with this part with many incorrectly setting the expression for the angular acceleration given in part (i) to zero. For those that realised that the most efficient method was to apply the work-energy principle by setting the loss in potential energy equal to the work done by the frictional couple (as many noted that the kinetic energy is initially zero and zero again when the hoop first comes to rest), many though could not deal accurately with the required trigonometry. A number of candidates obtained the 'correct' value for *k* and hence the angular acceleration by incorrectly stating that $\ddot{\theta} = \frac{g}{48a}(12\cos\theta - k) \Rightarrow \dot{\theta} = \frac{g}{48a}(12\sin\theta - k\theta)$. However, it was pleasing that a number of candidates did integrate the angular acceleration correctly and obtained the correct answers from using $\frac{1}{2}\dot{\theta}^2 = \frac{g}{48a}(12\sin\theta - k\theta)$.

Question 5(i)

5



A smooth circular wire, with centre O and radius a, is fixed in a vertical plane, and the point A is on the wire at the same horizontal level as O. A small ring B of mass m can move freely on the wire. A light inextensible string of length l, where $l > 2a$, has one end attached to B. The string passes over a small smooth pulley at A and carries at its other end a particle P of mass λm, where λ is a positive constant. The part AP of the string is vertical and the part AB of the string makes an angle θ radians with the horizontal, where $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$ (see diagram). You may assume that the string does not become slack.

(i) Taking A as the reference level for gravitational potential energy, show that the total potential energy V of the system is given by

$$V = 2mga\cos\theta(\lambda + \sin\theta) + k,$$

expressing k in terms of λ, g, l and m. **[4]**

The vast majority of candidates correctly derived the given result for the total potential energy of the system in this part. Nearly all candidates correctly stated the gravitational potential energy for the ring *B* as $(mg\sin\theta)(AB)$ but a minority struggled to show that $AB = 2a\cos\theta$. Nearly all candidates correctly dealt with the gravitational potential energy of particle *P* and the vast majority went on to correctly derive the given result.

Question 5(ii)

It is given that $\lambda = \frac{1}{6}$.

(ii) Show that there are two possible positions of equilibrium. **[4]**

The vast majority of candidates correctly differentiated *V* using the product rule and set their expression equal to zero. Most went on to derive a correct quadratic equation in sine. From here, some candidates simply stated that as this was a quadratic it would have two solutions or felt that it was sufficient to prove that there were two possible positions of equilibrium by only showing that the discriminant was positive. Some candidates correctly found that $\sin\theta = -\frac{3}{4}$ and $\sin\theta = \frac{2}{3}$ but again did not justify why these two values for sine implied that there would be two positions of equilibrium.

Question 5(iii)

(iii) By considering the values of $\frac{d^2V}{d\theta^2}$, determine whether these two positions are stable or unstable. **[6]**

Most candidates in this part correctly went on to find the second derivative of V and they further knew that they had to find the sign of the second derivative for their values of θ. However, many attempts were inaccurate or lacking detail (even though the question specifically asked for candidates to consider the values of the second derivative). In this type of question candidates would be wise to either state the value(s) of the second derivative either exactly or to at least 3 significant figures before comparing this to zero.

Question 6(i)

6 The region bounded by the curve $y = \frac{2a^2}{x}$ for $a \leqslant x \leqslant 2a$ (where a is a positive constant), the x-axis, and the lines $x = a$ and $x = 2a$, is rotated through 2π radians about the x-axis to form a uniform solid of revolution of mass m.

(i) Show that the moment of inertia of this solid about the x-axis is $\frac{7}{6}ma^2$. **[7]**

This part was answered extremely well with the majority of candidates scoring full marks. Most correctly derived the volume of the uniform solid of revolution as $2\pi a^3$ and then realised that $I = \frac{1}{2}\rho\pi\int y^4 dx$. The majority then went on to substitute for ρ and y and proceeded to obtain, with sufficient working, the given result.

Question 6(ii)

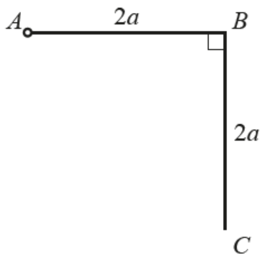
The solid is free to rotate about a fixed horizontal axis along the line $y = ka$.

(ii) Given that the solid makes small oscillations of approximate period $\frac{\pi}{3}\sqrt{\frac{83a}{g}}$ about this axis, find the possible values of k. **[4]**

The responses to this part were mixed. Some perfect solutions were seen but a number of candidates did not realise that as the axis of rotation was now about the line *y* = *ka* then the moment of inertia was no longer the given answer from part (i). Many candidates derived the period instead of simply stating that $T = 2\pi\sqrt{\frac{I}{mgh}}$ (which, according to the specification can be quoted without proof if required) and many did not replace *h* with *ka*. However, it was extremely pleasing to note that nearly all candidates who correctly stated that $\frac{\pi}{3}\sqrt{\frac{83a}{g}} = 2\pi\sqrt{\frac{\frac{7}{6}ma^2 + mk^2a^2}{mg(ka)}}$ were successful in finding the two correct values of *k*.

Question 7(i)

7



A compound pendulum consists of two uniform rods AB and BC, each of length $2a$ and mass m. The rods are rigidly joined together so that AB is perpendicular to BC. The pendulum is freely hinged to a fixed point at A. The pendulum can rotate in a vertical plane about a smooth fixed horizontal axis through A (see diagram).

(i) Show that the moment of inertia of the pendulum about the axis of rotation is $\frac{20}{3}ma^2$. **[3]**

This part was answered extremely well with nearly all candidates correctly deriving the moment of inertia of the frame about the axis of rotation through *A*.

Question 7(ii)

The pendulum is released from rest in the position with B vertically above C.

(ii) Find the vertical component of the force exerted by the axis on the pendulum as the pendulum passes through its equilibrium position. Give your answer as an exact multiple of mg. **[9]**

As expected the final part of this question was tackled with varying degrees of success as although nearly all candidates (who attempted this part) appreciated the need to apply the conservation of energy many did not calculate the change in potential energy of the frame correctly as it would seem that many candidates could not deal with the relatively straight-forward task of finding the centre of mass of the given frame. Furthermore, nearly all candidates did not seem to realise that the initial potential energy of the system relative to a zero level of potential energy through *AB* was $-mga$ and not zero. However, the calculation of the change in kinetic energy was far more successful. Even those candidates who had a correct expression for the angular velocity at the point when the pendulum past through its equilibrium position found the very last part of (ii) demanding and only a few succeeded in getting this bit correct. Only a minority of candidates, when trying to find the vertical component of the force acting on the frame at the axis of rotation, derived the correct equations of motion involving the radial component of the acceleration. Common errors included sign errors, using a mass of m rather than $2m$ and using a radius of a rather than $\frac{a\sqrt{10}}{2}$.

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